Section A
All questions should be attempted

1. P and Q are the points (2,3) and (-1,4).
What is the gradient of the line perpendicular to PQ?

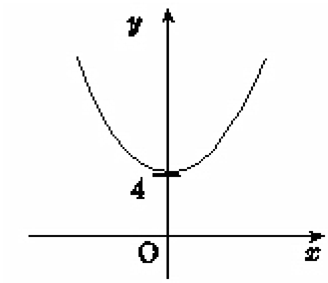
A $-\frac{8}{7}$
B 3
C 5
D 7

2. $f(x) = 2x - 1$ and $g(x) = 2x + 1$ are functions defined on the set of real numbers.
Find an expression for $f(g(x))$.

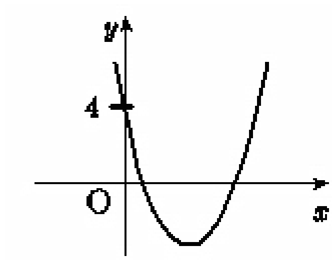
A $f(g(x)) = 4x^2 - 1$
B $f(g(x)) = 4x^2$
C $f(g(x)) = 4x$
D $f(g(x)) = 4x + 1$

3. Which of the sketches shown below is most likely to represent the graph of $y = 3x^2 - 7x + 4$?

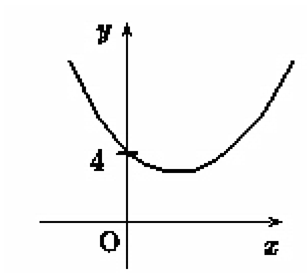
A



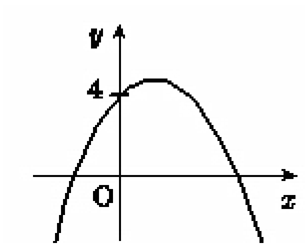
B



C



D



4. If $f(x) = 6x^3 - 2x^{-\frac{1}{2}}$ then the derivative, $f'(x)$ is

A $18x^2 + x^{-\frac{3}{2}}$

B $2x^2 + 4x^{\frac{1}{2}}$

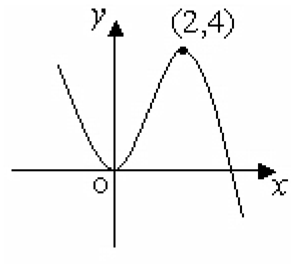
C $6x^2 - x^{-\frac{3}{2}}$

D $18x^2 + x^{\frac{1}{2}}$

5. PQ is the diameter of a circle.
P and Q have the coordinates (3,2) and (7,2) respectively.
What is the equation of the circle?

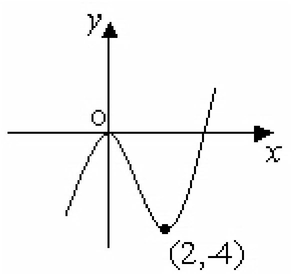
A $(x-3)^2+(y-2)^2=16$
B $(x-4)^2+y^2=2$
C $(x+5)^2+(y+2)^2=2$
D $(x-5)^2+(y-2)^2=4$

6. Part of the graph of $y=f(x)$ is shown opposite.

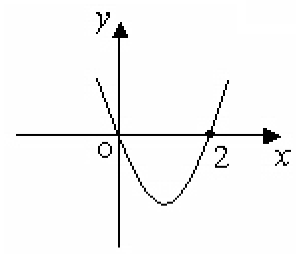


The graph of the derivative, $y=f'(x)$ could be represented by,

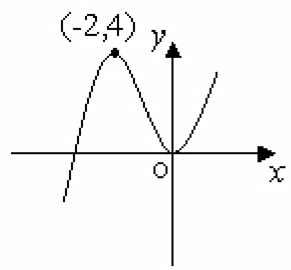
A



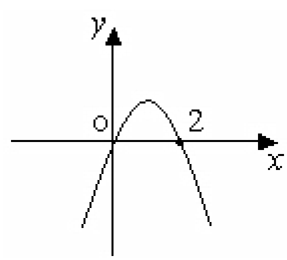
B



C



D



7. What is the remainder on dividing the polynomial $5x^3-4x+8$ by $x-2$?

A -24
B 0
C 8
D 40

8. Find $\int_{-1}^{1} x^4\,dx$

A 0

B $\frac{1}{4}$

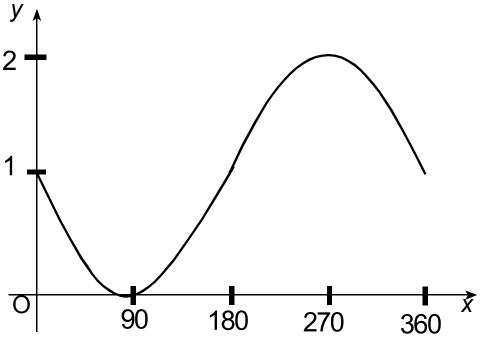
C $\frac{2}{5}$

D 8

9. The quadratic equation $4kx^2 - 8x + k = 0$ has equal roots.
The value of k, where $k > 0$ is,

A 4
B 2
C 0
D -2

10. The diagram below shows the graph of a trigonometric function.



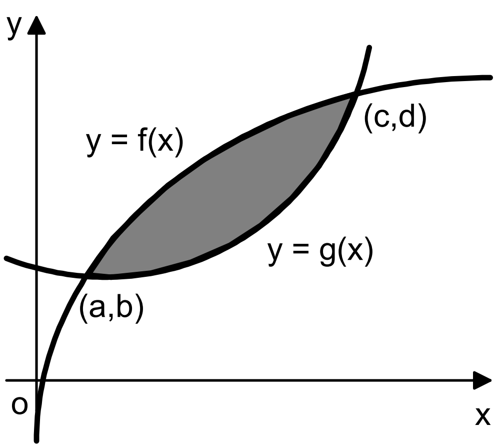
Which of the following could be the equation of the graph?

A $y = 1 + \sin x^o$
B $y = 1 - \sin x^o$
C $y = 2 - \cos x^o$
D $y = 2\cos x^o - 1$

11. A recurrence relation is defined by $U_{n-1} = 0.4U_n - 24$.
The limit of this sequence is,

A -40
B -24
C 0.03
D 50

12. The graphs $y = f(x)$ and $y = g(x)$ intersect at points (a,b and (c,d) as shown opposite.



The shaded area is given by,

A $\int_b^d (f(x) - g(x))\ dx$

B $\int_a^c (f(x) + g(x))\ dx$

C $\int_a^c (f(x) - g(x))\ dx$

D $\int_a^d (f(x) - g(x))\ dx$

13. Given that $\cos x^o = \frac{1}{\sqrt{3}}$ and $0 < x^o < 90$, then the exact value of $\cos 2x$ will be,

A $\frac{\sqrt{3}}{2}$

B $-\frac{1}{3}$

C $\frac{1}{3}$

D $\frac{1}{2\sqrt{3}}$

14. The tangent to the curve with equation $y = 2x^3 - 1$ at the point where $x = 1$ has a gradient of,

A 1
B 6
C 5
D $-\frac{1}{2}$

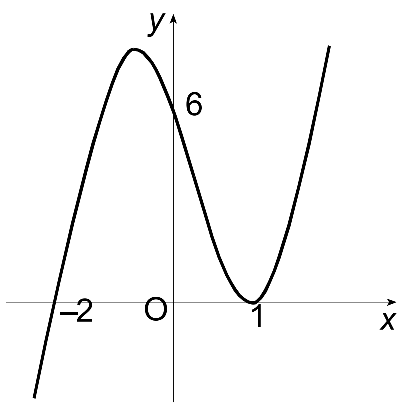
15. A circle has the equation $x^2 + y^2 + 4x - 2y - 4 = 0$. Which of the following correctly states the coordinates of its centre and the value of its radius?

A (-2,1), r = 1
B (2,-1), r = 3
C (-2,1), r = 3
D (2,-1), r = 1

16. When $x^2 + 8x + 5$ is expressed in the form $(x+a)^2 + b$, what is the value of b?

A -59
B -11
C 0
D 5

17. The diagram opposite shows part of a cubic function.
What is the equation of this graph?



A $y = 6(x+2)(x-1)^2$

B $y = 3(x-2)(x+1)^2$

C $y = 3(x+2)(x-1)^2$

D $y = 6(x-2)(x+1)^2$

18. A line **L**, is parallel to the line with equation $y=-2x+3$ and passes through the point (-3,1). What is the equation of **L**?

A $y-1=-2(x-3)$
B $y-1=4(x-3)$
C $y-1=-2(x+3)$
D $y+3=-2(x-1)$

19. Find $\int \frac{1}{5\sqrt{x}}\,dx$.

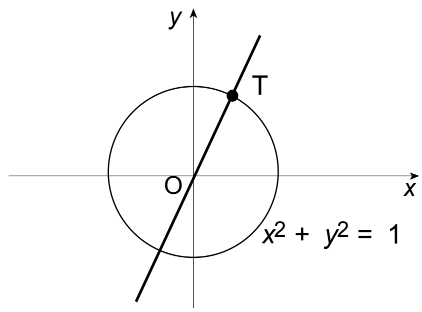
A $\frac{2}{5}x^{\frac{1}{2}}+c$

B $\frac{5}{2}x^{\frac{1}{2}}+c$

C $-\frac{1}{10}x^{-\frac{3}{2}}+c$

D $\frac{1}{10}x^{-\frac{3}{2}}+c$

20. The line with equation $y=2x$ intersects the circle with equation $x^2+y^2=1$ at the point T.



What is the x-coordinate of T?

A $\frac{1}{3}$

B $\frac{1}{\sqrt{6}}$

C $\frac{1}{\sqrt{5}}$

D $\frac{1}{2}$

Section B
All questions should be attempted

21. A function is given by $f(x) = x^3 + 3x^2 + 2x + 6$.

a) Show that (x+3) is a factor of $f(x)$.

(2)

b) Hence solve the equation $x^3 + 3x^2 + 2x + 6 = 0$
and clearly state why there is only one real root.

(3)

22. Find the equation of the **tangent** at the point (-2,-1) on the circle with equation $x^2 + y^2 - 6x - 4y - 21 = 0$.

(5)

23. a) Find the derivative of the function

$$f(x) = \tfrac{1}{3}x^3 - \tfrac{1}{2}x^2 - 12x + 11$$

(2)

b) Construct a nature table and hence determine the values of x for which the function is **decreasing**.

(3)

24. Solve algebraically the equation

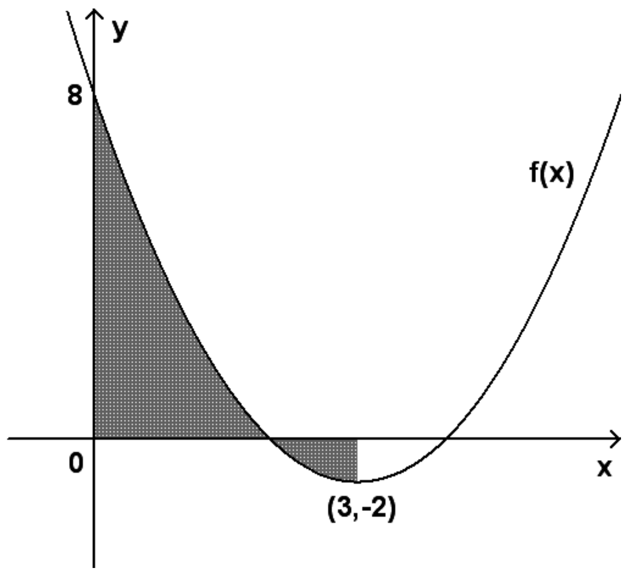
$$\cos 2x^o + 5\cos x^o - 2 = 0 \qquad \text{where } 0 \le x \le 360^o$$

(5)

25. This is the graph of $y = x^2 - 6x + 8$.

Calculate the area of the shaded region.

(5)



26. If $\sin x = \frac{5}{13}$ and $\tan y = \frac{2}{3}$

a) Determine $\cos x$, $\cos y$ and $\sin y$.

(2)

b) Hence show that $\cos(x + y) = \dfrac{2}{\sqrt{13}}$

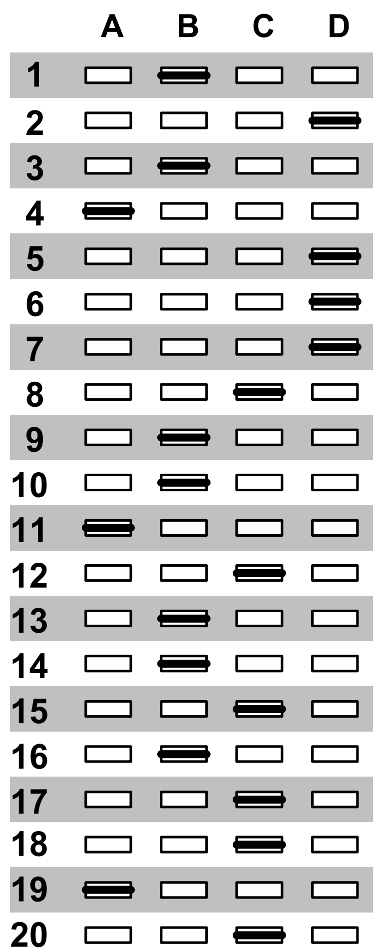
(3)

End of question paper

Marking Scheme for HIGHER MATHS PRELIM 2008 PAPER 1

Section A

1	B
2	D
3	B
4	A
5	D
6	D
7	D
8	C
9	B
10	B
11	A
12	C
13	B
14	B
15	C
16	B
17	C
18	C
19	A
20	C



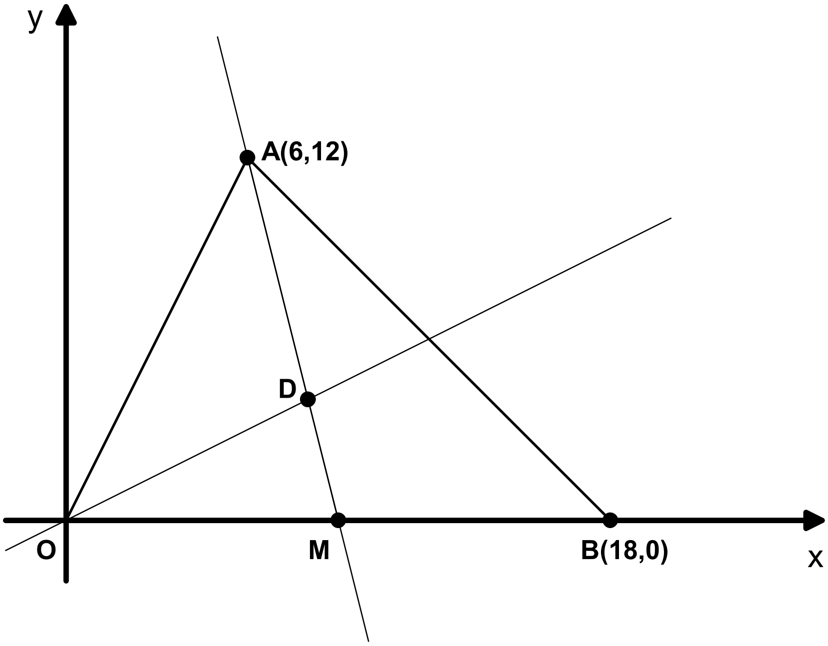
Correct answers - 2 marks each

Section A = 40 marks
Section B = 30 marks

Total for paper 1 = 70 marks

Please record pupil’s scores on the objective question answer sheet.

1. The diagram below shows a triangle OAB. A and B are the points (6,12) and (18,0) respectively.
The median from A meets OB at M as shown.



a) Show that the median, AM, has the equation $y = -4x + 36$

[3]

b) Find the equation of the median from the origin to AB.

[3]

c) Show that the two medians meet at the point D(8,4).

[2]

d) Show that the distance of AD is twice the distance of DM.

[3]

2. a) The terms of a sequence satisfy $U_{n+1} = kU_n + 5$. Find the value of k which produces a sequence with a limit of 4.

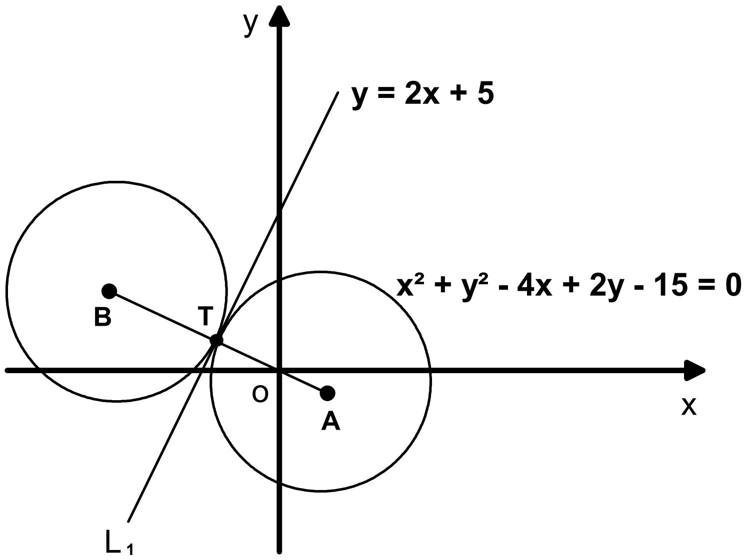
[2]

b) A sequence satisfies the recurrence relation $U_{n+1} = mU_n + 5$, $U_0 = 3$.

(i) Express U_1 and U_2 in terms of m.
(ii) Given that $U_2 = 7$, find the value of m which produces a sequence with no limit.

[5]

3. Two circles, both with the same radius, touch externally at T as shown below.



The circle with A as its centre has equation $x^2 + y^2 - 4x + 2y - 15 = 0$.
Line L_1 is the common tangent to both circles through T and has as its equation $y = 2x + 5$.

a) Find the coordinates of T, the point of tangency.

[3]

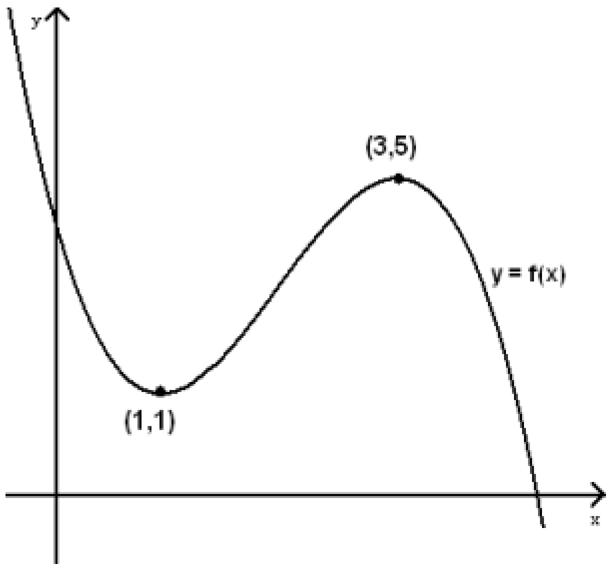
b) Find the coordinates of B and hence write down the equation of the other circle in the diagram.

[3]

4. The graph of the cubic function $y = f(x)$ is shown in the diagram.

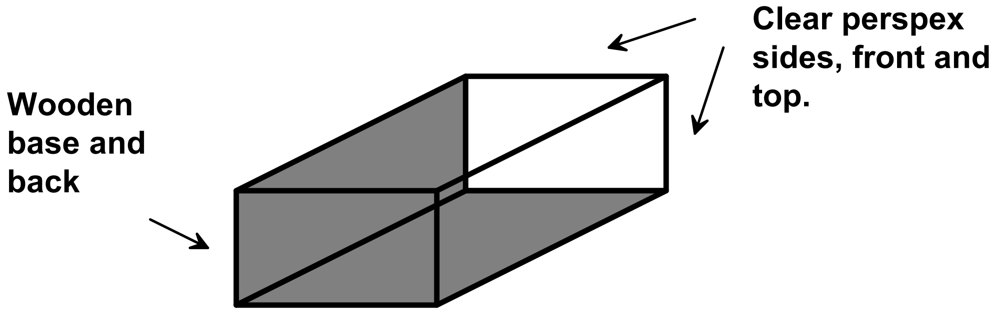
There are turning points at (1,1) and (3,5).

Sketch the graph of the derivative, $y = f'(x)$.

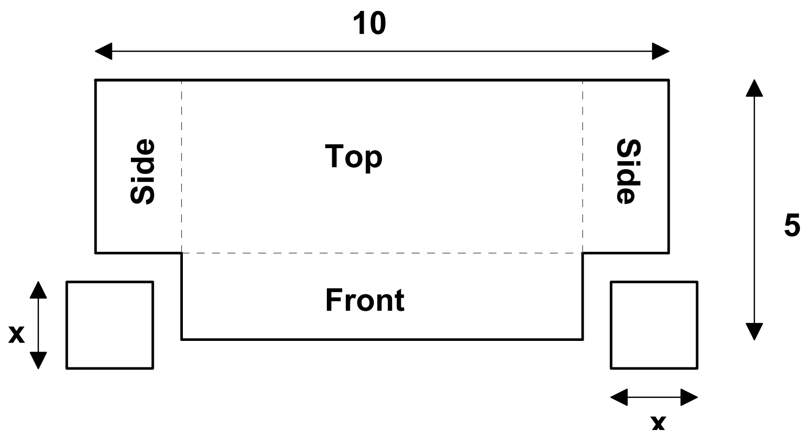


[3]

5. A museum is constructing a cabinet to hold some new exhibits. Two sides of the cabinet are made from wood (which is inexpensive and easy to obtain). The other four sides are made from a very expensive clear perspex. An illustration of the cabinet is shown below.



The four sides of perspex are to be cut from a single sheet measuring 10 feet by 5 feet. This is done by removing two squares of length **x** feet from the bottom corners of the sheet of perspex (see below).



a) Show that the volume, V(x), of the cabinet is given by

$$V(x) = 2x^3 - 20x^2 + 50x$$

[3]

b) The museum wishes to **maximise** the use of the perspex sheet. What value of **x** makes the cabinet of largest volume?

[6]

c) Calculate the largest volume and give your answer in cubic feet.

[2]

6. The point P(x,y) lies on the curve with equation $y = 6x^2 - x^3$.

a) Find the value of x for which the gradient of the tangent at P is 12.

[5]

b) Hence find the equation of the tangent at P.

[2]

7. Two functions are defined on a suitable domain as $f(x) = x^2 + a$ and $g(x) = x + 1$, where a is a constant.

a) Find the value of a given that $f(g(-2)) = -1$

[2]

b) Hence solve the equation $f(f(x)) = 2$

[5]

8. a) By expressing sin(3x) as sin(2x+x) and expanding the brackets, show that

$$\sin(3x) = 3\sin x - 4\sin^3 x$$

[5]

b) Use the above relationship to solve the equation,

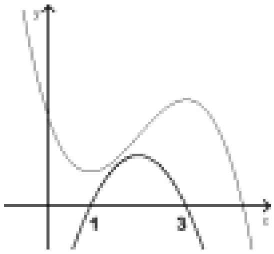
$$3\sin\theta = 4\sin^3\theta \qquad \text{where } 0 \le \theta \le \pi$$

[3]

End of question paper

Marking Scheme for HIGHER MATHS PRELIM 2008 PAPER 2

Question	1 mark for each •	Illustration of evidence for awarding a mark at each •
1a	Ans: proof •1 Finds the mid-point of OB •2 Finds the gradient of median AM •3 Substitutes into equation for straight line correctly 3 marks	• M(9,0) • $^{m}AM = \frac{12-0}{6-9} = -4$ • $y-12=-4(x-6)$ or $y-0=-4(x-9)$
1b	Ans: $y=\frac{1}{2}x$ •1 Finds mid-point of AB •2 Calculates gradient of 2nd median •3 Gives correct equation 3 marks	• midpoint of AB (12,6) • $m=\frac{6-0}{12-0}=\frac{1}{2}$ • $y=\frac{1}{2}x$
1c	Ans: Shows that D is (8,4) •1 Equates or uses simultaneous equations •2 Solves by algebra 2 marks	• $\frac{1}{2}x=-4x+36$ • $9x=72 \Rightarrow x=8$ $y=\frac{1}{2}\times 8=4$
1d	Ans: shows that $\lvert AD\rvert = 2\lvert DM\rvert$ •1 Calculates length AD •2 Calculates length DM •3 Demonstrates $\lvert AD\rvert = 2\lvert DM\rvert$ 3 marks	• $\lvert AD\rvert = \sqrt{(6-8)^2+(12-4)^2} = \sqrt{68}$ • $\lvert DM\rvert = \sqrt{(8-9)^2+(4-0)^2} = \sqrt{17}$ • $\sqrt{68} = \sqrt{4\times 17} = 2\sqrt{17}$ hence $\lvert AD\rvert = 2\sqrt{17} = 2\lvert DM\rvert$
2a	Ans: $k=-\frac{1}{4}$ •1 Know how to find limit •2 Process 2 marks	• $4=k\times 4+5$ • $k=-\frac{1}{4}$
2b	Ans: $m=-2$ •1 Interpret rec relation •2 Interpret rec relation •3 Arrange in standard quadratic form •4 Factorises quadratic •5 Uses limit condition 5 marks	• $U_1=3m+5$ • $U_2=m(3m+5)+5$ • $3m^2+5m-2=0$ • $(3m-1)(m+2)=0$ • $m=-2$
3a	Ans: (-2,1) •1 Substitutes for y in circle equation •2 Simplifies and solves for x •3 Substitutes to find y 3 marks	• $x^2+(2x+5)^2-4x+2(2x+5)-15=0$ • $5(x+2)^2=0;\ x=-2$ • $y=2(-2)+5;\ y=1$
3b	Ans: $(x+6)^2+(y-3)^2=20$ •1 Establishes coordinates of B •2 Finds r^2 •3 Substitutes into general circle equation 3 marks	• B(-6,3) • $r^2=20$ • $(x+6)^2+(y-3)^2=20$

Question	1 mark for each •	Illustration of evidence for awarding a mark at each •
4	Ans:  •1 Interprets stationary points •2 Interpret between roots •3 Know that f'(cubic) = parabola 3 marks	• Only two intercepts on the x-axis at 1 and 3 • Function is +ve between the roots and -ve outwith • A parabola (symmetrical about midpoint of x-intercepts), stated or implied by the accuracy of the diagram.
5a	Ans: Proof •1 For correct length •2 For correct breadth •3 For correct volume 3 marks	• 10 - 2x • 5 - x • $2x^3 - 20x^2 + 50x$
5b	Ans: $x = \frac{5}{3}$ •1 For knowing to differentiate •2 For correct derivative •3 For V'(x) = 0 •4 For correct solutions •5 For nature table •6 For selecting correct value for max. 6 marks	• V'(x) • $6x^2 - 40x + 50$ • $6x^2 - 40x + 50 = 0$ • $x = \frac{5}{3}$, x=5 • Justify max at $x = \frac{5}{3}$ • $x = \frac{5}{3}$
5c	Ans: $V = 37\frac{1}{27}$ *or* $\frac{1000}{27}$ *or* 37.04 cu feet •1 For correct substitution into V(x) •2 For correct volume 2 marks	• $2(\frac{5}{3})^3 - 20(\frac{5}{3})^2 + 50(\frac{5}{3})$ • $V = 37\frac{1}{27}$ *or* $\frac{1000}{27}$ *or* 37.04 cu feet
6a	Ans: $x = 2$ •1 Know to differentiate •2 Differentiate •3 Set derivative equal to gradient •4 Start to solve •5 Process 5 marks	• $\frac{dy}{dx} = ..$ • $12x - 3x^2$ • $12x - 3x^2 = 12$ • $3(x-2)^2 = 0$ • $x = 2$
6b	Ans: $y = 12x - 8$ •1 Substitutes and finds y coordinate •2 State equation of tangent 2 marks	• $y = 16$ • $y - 16 = 12(x - 2)$ no need to simplify