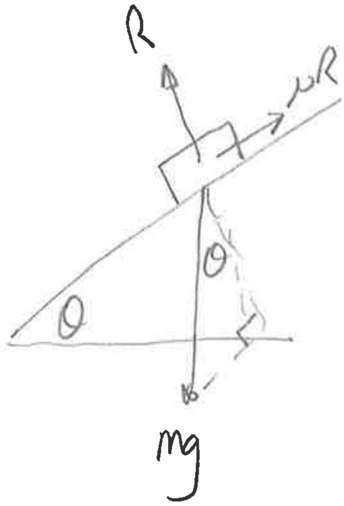
Homework 7 solutions

1)



[on the point of slipping down so friction acts upward]

$m = 4kg$

$\mu = 0.2$

system in equilibrium

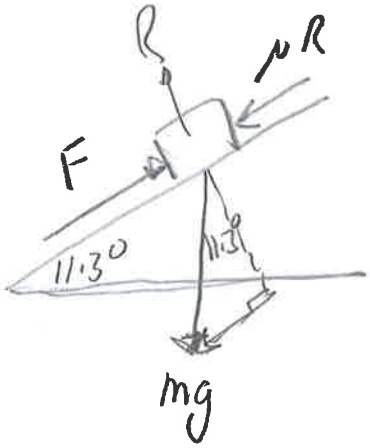
$\mu R = mg\sin\theta$ ✓ $R = mg\cos\theta$ ✓

$\mu mg\cos\theta = mg\sin\theta$ ✓

$\mu = \tan\theta$

$\tan\theta = 0.2$

$\theta = 11.3^\circ$ ✓



system in equilibrium

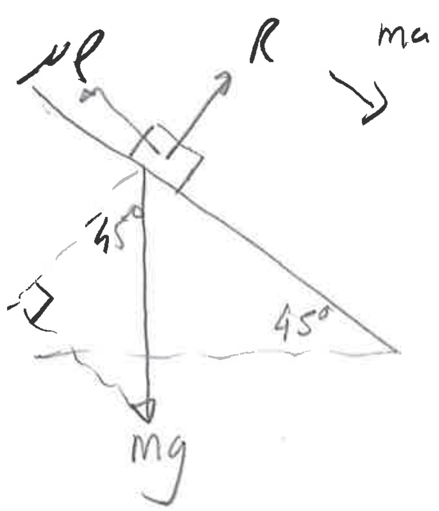
$R = mg\cos 11.3^\circ$

$F = \mu R + mg\sin 11.3^\circ$ ✓

$F = \mu mg\cos 11.3 + mg\sin 11.3$ ✓

$F = 15.4N$ ✓

2) a)



$\mu = 1/2$

$\sum F = ma$ [in the direction of motion]

$ma = mg\sin 45° - \mu R$ ✓ $R = mg\cos 45°$ ✓

$ma = mg\sin 45° - \mu mg\cos 45°$

$a = \frac{\sqrt{2}}{2}g - \frac{\sqrt{2}}{2}\mu g$ $\left[= \frac{\sqrt{2}}{4}g\right]$

$u = 0$

$s = L$

$v^2 = u^2 + 2as$

$v^2 = 2\left(\frac{\sqrt{2}}{2}g - \frac{\sqrt{2}}{2}\mu g\right)L$ ✓

$v^2 = (\sqrt{2}g - \sqrt{2}\mu g)L$

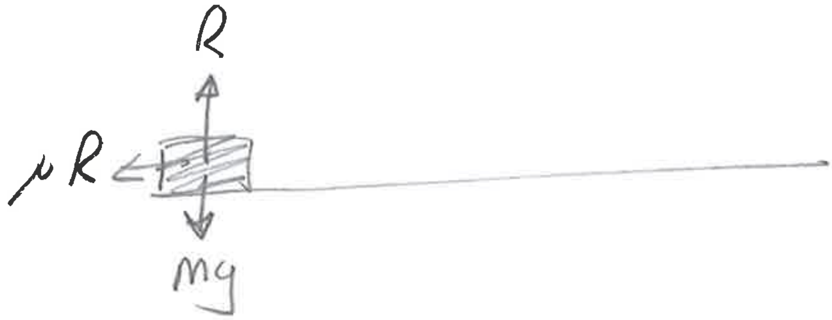
$v^2 = \sqrt{2}gL(1-\mu)$ $\mu = \frac{1}{2}$

$v^2 = \frac{\sqrt{2}}{2}gL$

$v = \sqrt{\frac{\sqrt{2}}{2}gL}$ ms^{-1} ✓

$\left[v = \sqrt{\frac{gL}{\sqrt{2}}}\right]$

b) $u = \sqrt{\frac{\sqrt{2}}{2} gL}$



$\sum F = ma$

$ma = -\mu R$ $\quad R = mg$

$ma = -\mu mg$

$a = -\mu g$

$u = \sqrt{\frac{\sqrt{2}}{2} gL}$

$a = -\mu g$

$s = \frac{1}{2} L$

$v^2 = u^2 + 2as$

$v^2 = \frac{\sqrt{2}}{2} gL - 2\mu g \times \frac{1}{2} L$

$v^2 = \frac{\sqrt{2}}{2} gL - \mu g L$ $\quad \mu = \frac{1}{2}$

$v^2 = gL\left(\frac{\sqrt{2}}{2} - \frac{1}{2}\right)$

$v^2 = \frac{gL(\sqrt{2}-1)}{2}$

$v = \sqrt{\frac{gL(\sqrt{2}-1)}{2}}$

3a)

$v_c = U\underline{j}$

$V_f = -2\sqrt{2}\,U\cos 45\,\underline{i} + 2\sqrt{2}\,U\sin 45\,\underline{j}$

$V_f = -2U\underline{i} + 2U\underline{j}$

$S_c = Ut\underline{j} + c$

at $t=0$ $S_c = 0 \Rightarrow c = 0$

$S_c = Ut\underline{j}$ ✓

$S_f = -2Ut\underline{i} + 2Ut\underline{j} + c$ ✓

at $t=0$ $S_f = 10\underline{i} \Rightarrow c = 10\underline{i}$

$S_f = (10-2Ut)\underline{i} + 2Ut\underline{j}$ ✓

$_fS_c = S_f - S_c$

$= (10-2Ut)\underline{i} + Ut\underline{j}$ ✓

b) $|_fS_c|^2 = (10-2Ut)^2 + (Ut)^2$ ✓

$= 100 - 40Ut + 4U^2t^2 + U^2t^2$

$= 5U^2t^2 - 40Ut + 100$

$\dfrac{d|_fS_c|^2}{dt} = 10U^2t - 40U$ ✓

at min distance $10U^2t - 40U = 0$

$t = \dfrac{40U}{10U^2}$

$t = \dfrac{4}{U}$ ✓

$$|fSc|^2 = 5U^2t^2 - 40Ut + 100$$

$$t = \frac{4}{u} \Rightarrow |fSc|^2 = 5U^2\left(\frac{4}{U}\right)^2 - 40U\left(\frac{4}{U}\right) + 100$$

$$= 80 - 160 + 100$$

$$|fSc|^2 = 20$$

$$|fSc| = \sqrt{20}$$

$$= 2\sqrt{5}$$

4)

$$\frac{4x-3}{x(x^2+3)} = \frac{A}{x} + \frac{Bx+C}{x^2+3}$$

$$\frac{4x-3}{x(x^2+3)} = \frac{A(x^2+3) + x(Bx+C)}{x(x^2+3)}$$

$$4x - 3 = A(x^2+3) + x(Bx+C)$$

$x = 0$ $\quad -3 = 3A$

$A = -1$

$x = 1$ $\quad 1 = 4A + B + C$

$1 = -4 + B + C$

$B + C = 5$

$x = 2$ $\quad 5 = 7A + 4B + 2C$

$5 = -7 + 4B + 2C$

$4B + 2C = 12$

$2B + C = 6$

$\Rightarrow B = 1$ and $C = 4$

so $$y = \frac{-1}{x} + \frac{x+4}{x^2+3}$$