Homework 11

1) A toy car of mass 250 grams is stationary on a smooth horizontal surface. One end of a light spring is attached to the car, the other end is fixed to the surface. The natural length of the spring is 1 metre and the modulus of elasticity is 4 newtons.

The car is pulled along the surface, extending the spring by 20 centimetres, and then released.

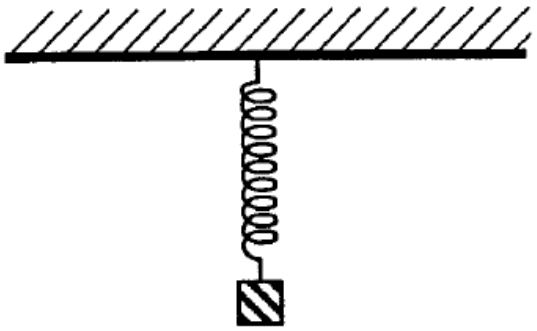
(*a*) Show that the displacement, x metres, of the car from its equilibrium position satisfies an equation of the form

$$\frac{d^2x}{dt^2} = -\omega^2 x$$

where the value of the constant ω should be stated. 3

(*b*) Calculate the maximum speed of the car. 2

2) Large springs for shock absorbers are tested at a research laboratory. A body of mass 50kg is suspended from a test spring of natural length 0·80 metres, which has the other end attached to a fixed horizontal surface.



(a) Given that, when in equilibrium, the body extends the spring by 0·14 metres, find the modulus of elasticity of the spring. 2

(b) The body is then pulled down a distance y metres vertically and released. Show that the resultant motion is simple harmonic and **find the period.** **3**

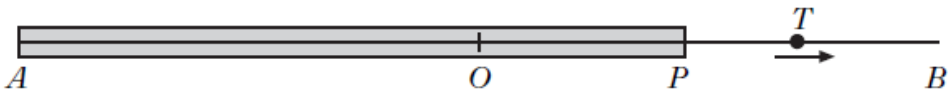
The particle has been pulled down a distance of $0{\cdot}1$ metres and released.

(c) Find an expression for y in terms of t and use this to determine the distance the particle has travelled after $0{\cdot}25$ seconds. **3**

3) In a fairground game, a small target T executes simple harmonic motion about a point O with extreme points A and B. When the target is 1 metre from O, its speed is $\frac{\pi}{\sqrt{3}}$ m s^{-1} and when it is $\sqrt{3}$ metres from O its speed is $\frac{\pi}{3}$ m s^{-1}.

(a) Show that the amplitude of the motion is 2 metres and calculate the period of the oscillation. **5**

(b) A player has to shoot at the target, but it is only visible to the player when it is to the right of the point P as shown in the diagram.



Given that the target takes 0·75 seconds to move from P to B, calculate the distance PB. **5**

4) A piston moves with simple harmonic motion. It performs 50 complete oscillations per second and has a maximum speed of 40 m s^{-1}. Calculate the amplitude of the motion. **3**

Find the speed of the piston when the displacement from the centre of oscillation is 0·08 m. **2**