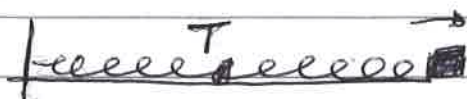


Homework 11 solutions

(1)

1a)



$$\begin{aligned}\lambda &= 4\text{N} \\ \ell &= 1\text{m} \\ m &= 0.25\text{kg}\end{aligned}$$

$$T = \frac{\lambda x}{\ell}$$

$$ma = -\frac{\lambda x}{\ell} \checkmark$$

$$\ddot{x} = -\frac{\lambda}{m\ell} x$$

$$\ddot{x} = -16x \checkmark$$

since motion is of the form $\ddot{x} = -\omega^2 x$ it is SHM

where $\underline{\omega = 4} \checkmark$

b) amplitude = $0.2\text{m} \checkmark$

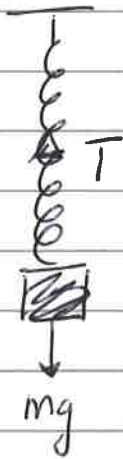
$$v_{\text{max}} = \omega a$$

$$v_{\text{max}} = 4 \times 0.2$$

$$= \underline{0.8\text{ms}^{-1}} \checkmark$$

(2)

2a)



$$m = 50 \text{ kg}$$
$$l = 0.8 \text{ m}$$
$$e = 0.14 \text{ m}$$

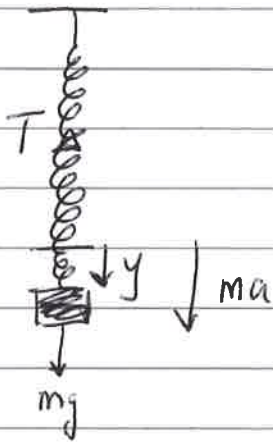
$$T = \frac{\lambda e}{l}$$

in equilibrium so $\frac{\lambda e}{l} = mg$ ✓

$$\lambda = \frac{mgl}{e}$$

$$\lambda = 2800 \text{ N} \checkmark$$

b)



$$ma = mg - T \checkmark$$

$$m\ddot{y} = mg - \frac{\lambda(e+y)}{l}$$

$$50\ddot{y} = 50g - \frac{2800(0.14+y)}{0.8}$$

$$50\ddot{y} = 490 - 3500(0.14+y)$$

$$50\ddot{y} = -3500y$$

$$\ddot{y} = -70y \checkmark$$

motion is of the form $\ddot{y} = -\omega^2 y$ where $\omega^2 = 70$

$$\omega = \sqrt{70}$$

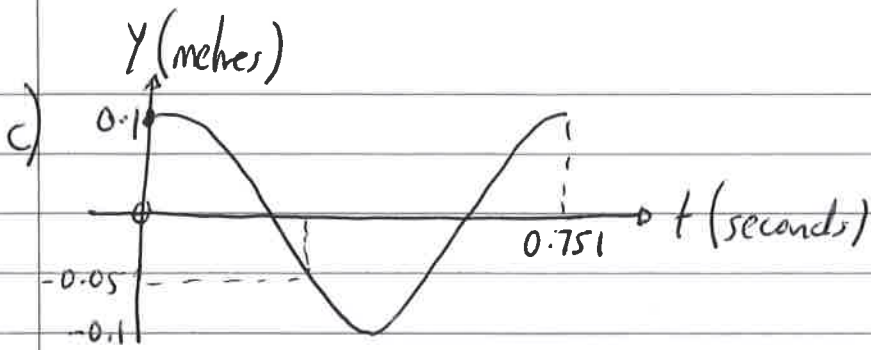
$$\text{so } \omega = \frac{2\pi}{T}$$

$$T = \frac{2\pi}{\omega}$$

$$T = \frac{2\pi}{\sqrt{70}}$$

$$T = 0.75 \text{ seconds} \checkmark$$

(3)



$$y = 0.1 \cos \omega t$$

$$y = 0.1 \cos(\sqrt{70} t)$$

at $t = 0.25$ $y = 0.1 \cos(\sqrt{70} \times 0.25)$

$$y = -0.05 \text{ m}$$

so total distance travelled = $0.1 + 0.05 = 0.15 \text{ metres}$

3a) at $x = 1$ $v = \frac{\pi}{\sqrt{3}} \Rightarrow v^2 = \omega^2 (a^2 - x^2)$

$$\frac{\pi^2}{3} = \omega^2 (a^2 - 1)$$

at $x = \sqrt{3}$ $v = \frac{\pi}{3} \Rightarrow \frac{\pi^2}{9} = \omega^2 (a^2 - 3)$

$$\omega^2 = \frac{\pi^2}{3(a^2 - 1)} \quad \text{and} \quad \omega^2 = \frac{\pi^2}{9(a^2 - 3)}$$

$$\Rightarrow \frac{\pi^2}{3(a^2 - 1)} = \frac{\pi^2}{9(a^2 - 3)}$$

$$3(a^2 - 1) = 9(a^2 - 3)$$

$$3a^2 - 3 = 9a^2 - 27$$

$$6a^2 = 24$$

$$a^2 = 4$$

$$a = \pm 2 \quad \text{so amplitude} = 2 \text{ m}$$

(4)

$$\omega^2 = \frac{\pi^2}{3(a^2-1)}$$

$$a=2 \Rightarrow \omega^2 = \frac{\pi^2}{3(2^2-1)}$$

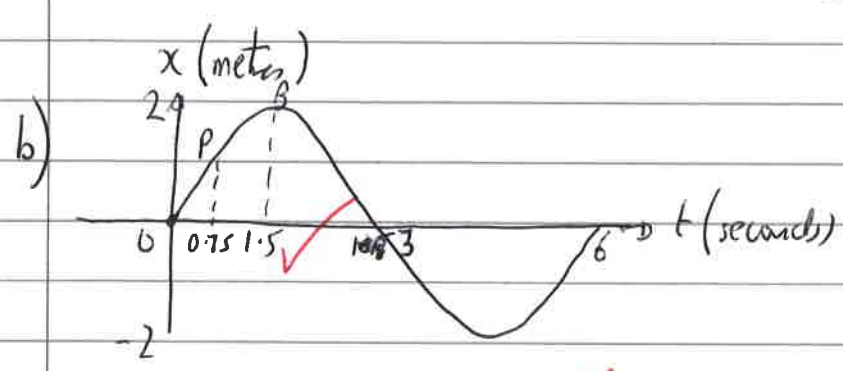
$$\omega^2 = \frac{\pi^2}{9}$$

$$\omega = \frac{\pi}{3}$$

$$T = \frac{2\pi}{\omega}$$

$$T = \frac{2\pi}{\pi/3}$$

$$T = \underline{6 \text{ seconds}} \checkmark$$



$$x = 2 \sin \frac{\pi}{3} t \checkmark \checkmark$$

$$\text{at } t = 0.75 \quad x = \sqrt{2} \text{ m } \checkmark$$

$$\text{so distance } PB = 2 - \sqrt{2} = \underline{0.586 \text{ m}} \checkmark$$

(5)

(4) $V_{\max} = 40 \text{ ms}^{-1}$

$$T = \frac{1}{50} \text{ seconds} \quad \checkmark \quad \omega = \frac{2\pi}{T}$$

$$\underline{\omega = 100\pi} \quad \checkmark$$

$$V_{\max} = \omega a$$

$$100\pi a = 40$$

$$\underline{a = 0.127 \text{ m}} \quad \checkmark$$

$$v^2 = \omega^2 (a^2 - x^2)$$

$$v^2 = (100\pi)^2 (0.127^2 - 0.08^2) \quad \checkmark$$

$$\underline{v = 31.1 \text{ ms}^{-1}} \quad \checkmark$$