Homework 13 solutions

1) $m = 2kg$

a)

$ma = 8t$

$a = 4t$ ✓

$v = 2t^2 + c$ $\quad c = 0$ since starts from rest $\Rightarrow 0 = 0 + c$, $c = 0$

$v = 2t^2$ ✓

b) $Wd = \Delta E$

$= \frac{1}{2}mv^2 - \frac{1}{2}mu^2$ ✓

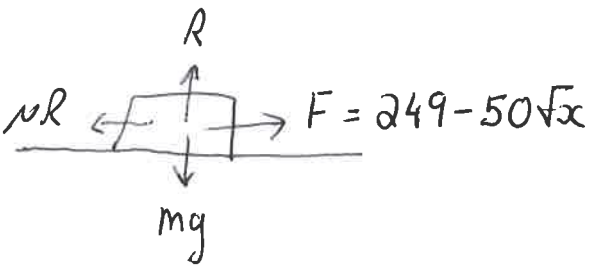
$= \frac{1}{2} \times 2 \times 2^2 - 0$

$= 4J$ ✓

at $t = 1$ $v = 2ms^{-1}$ ✓

$t = 0$ $u = 0ms^{-1}$

2a) $m = 20kg$



resultant force $= 249 - 50\sqrt{x} - 20g \times 0.25$

$F_R = 200 - 50\sqrt{x}$ ✓

$W = \int_0^{10} F_R \, dx$

$W = \int_0^{10} (200 - 50x^{1/2}) \, dx$ ✓

$= \left[200x - \frac{50x^{3/2}}{3/2}\right]_0^{10}$ ✓

$= 946J$ ✓

b) $W_D = \Delta E$

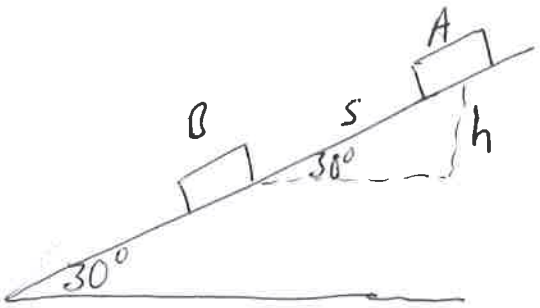
$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = 946$ ✓

$\frac{1}{2} \times 20v^2 - \frac{1}{2} \times 20 \times 12^2 = 946$

$10v^2 = 2386$

$v = 15.4 ms^{-1}$ ✓

30)



Energy at A

$E_K = 0$

$E_P = mgh$

Energy at B (immediately before collision)

$E_K = \frac{1}{2}mv^2$

$E_P = 0$

conservation of energy

$\frac{1}{2}mv^2 = mgh$ ✓

$v^2 = 2gh$

$v = \sqrt{2gh}$ ✓

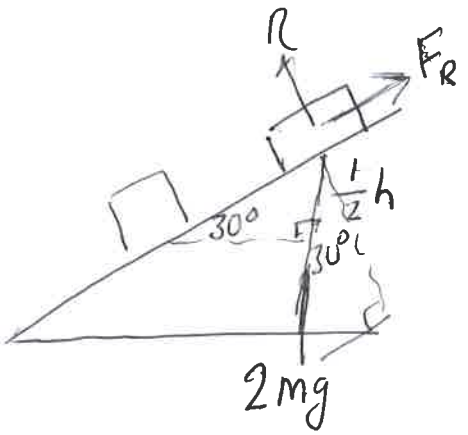
now block P hits and couples with block Q

conservation of momentum

$m \times \sqrt{2gh} + m \times 0 = 2mV$

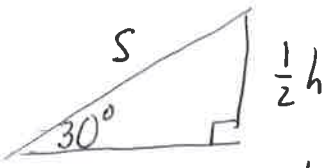
$V = \frac{1}{2}\sqrt{2gh}$ ✓

ii)



$u = \frac{1}{2}\sqrt{2gh}$

$v = 0$



$\sin 30° = \frac{\frac{1}{2}h}{s}$

$s = \frac{\frac{1}{2}h}{\sin 30°}$

$\underline{s = h}$ ✓

use work/energy principle

$(2mg\sin 30° - F_R) \times h = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$ ✓

$(2mg\sin 30° - F_R) \times h = 0 - \frac{1}{2} \times 2m \times \left(\frac{1}{2}\sqrt{2gh}\right)^2$

$(2mg\sin 30° - F_R) \times h = -\frac{1}{2} \times 2m \times \frac{1}{4} \times 2gh$ ✓

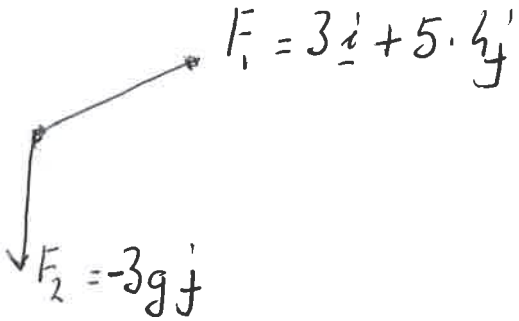
$(mg - F_R) \times \cancel{h} = -\frac{1}{2}mg\cancel{h}$

$mg - F_R = -\frac{1}{2}mg$

$F_R = \frac{3}{2}mg$ ✓

$\underline{F_R = \frac{3}{2}W}$

4)



resultant force $F_R = 3\underline{i} + 5.4\underline{j} - 3g\underline{j}$

$F_R = 3\underline{i} - 24\underline{j}$ ✓

$m = 3kg \Rightarrow \quad \underline{a} = F/m$

$\underline{a} = \underline{i} - 8\underline{j}$ ✓

$\underline{v} = t\underline{i} - 8t\underline{j} + C$

initial velocity $v = 2\underline{i} - \underline{j}$

$\Rightarrow \quad \underline{v} = (t+2)\underline{i} - (8t+1)\underline{j}$ ✓

$$W = \int F \cdot v \, dt$$

$$= \int_0^4 [3(t+2) + 24(8t+1)] \, dt$$ ✓

$$= \int_0^4 (195t + 30) \, dt$$

$$= \left[\frac{195t^2}{2} + 30t\right]_0^4$$

$$= 1680 \, J$$ ✓

or $W_D = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$

at $t = 4$ $\quad \underline{v} = 6\underline{i} - 3\underline{j}$

$v^2 = 1125$

at $t = 0$ $\quad \underline{u} = 2\underline{i} - \underline{j}$

$u^2 = 5$

so $\quad W_D = \frac{1}{2} \times 3 \times 1125 - \frac{1}{2} \times 3 \times 5$

$= 1680\text{J}$

5) $\quad a = 4 - t^{1/2}$

$v = 4t - \frac{t^{3/2}}{3/2} + C$

$v = 4t - \frac{2t^{3/2}}{3}$ ✓ $\quad C = 0$ since $v = 0$ at $t = 0$

max speed when $a = 0$ $\quad 4 - t^{1/2} = 0$

$t^{1/2} = 4$

$t = 16$ seconds ✓

increase in KE $= \frac{1}{2}mv^2 - \frac{1}{2}mu^2$ $\quad$ at $t = 16$ $v = 21.3\text{ms}^{-1}$ ✓

$= \frac{1}{2} \times 9 \times 21.3^2 - 0$

$= 2048\text{J}$ ✓