Homework 1 Solutions

1) $u = 0$
$v = 20$
$s = 300$
$a = ?$

stage 1 - accelerating

$v^2 = u^2 + 2as$
$20^2 = 0 + 2 \times a \times 300$
$a = \frac{2}{3}\ ms^{-2}$

$v = u + at$
$20 = a + \frac{2}{3}t$
$t = 30 sec$

stage 3 - decelerating.

$u = 20$
$v = 0$
$t = 15 sec$
$a = ?$

$v = u + at$
$0 = 20 + 15a$
$a = -\frac{4}{3}\ ms^{-2}$

$s = ut + \frac{1}{2}at^2$
$s = 20 \times 15 + \frac{1}{2} \times -\frac{4}{3} \times 15^2$
$s = 150m$

stage 2 - constant speed.

distance $= 5000 - 300 - 150 = 4550m$

$t = s/v$

$t = \frac{4550}{20}$

$t = 227.5 secs$

Total time $= 30 + 15 + 227.5$
$= 272.5 secs$

2) a) $a = 2 - \sqrt{t}$

$a = 2 - t^{1/2}$

$a = \frac{dv}{dt} = 2 - t^{1/2}$

$a = 0$ when $2 - t^{1/2} = 0$

$t^{1/2} = 2$

$t = 4$ secs

t	$\rightarrow 4 \rightarrow$
$\frac{dv}{dt}$	$+ \; 0 \; -$
	$/ \; - \; \backslash$

so increases to a max when $t = 4$ secs, and then decreases.

b) at rest when $v = 0$ $\quad a = 2 - t^{1/2}$

$v = 2t - \frac{2}{3}t^{3/2} + C$

at $t = 0$ $v = 0$ $\Rightarrow$ $\quad v = 2t - \frac{2}{3}t^{3/2}$

$2t - \frac{2}{3}t^{3/2} = 0$

$t\left(2 - \frac{2}{3}t^{1/2}\right) = 0$

$t = 0 \quad \frac{2}{3}t^{1/2} = 2$

$t^{1/2} = 3$

$t = 0, \; t = 9$ secs

5) $V_1(t) = -\frac{1}{2}t + 45$

$V_2(t) = \frac{1}{8}t + \frac{15}{2}$

to find T_B solve these as simultaneous equations

$$-\frac{1}{2}t + 45 = \frac{1}{8}t + \frac{15}{2}$$

$$\frac{5}{8}t = \frac{75}{2}$$

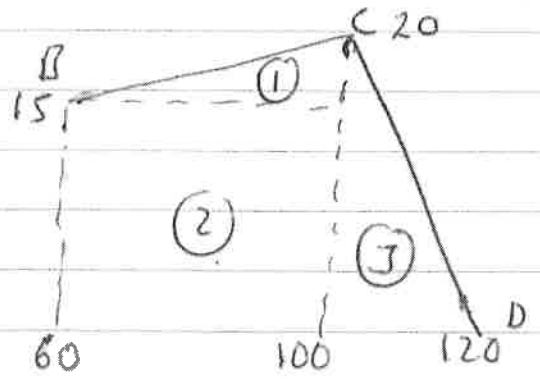
$$t = 60 \text{ seconds}$$

so at B $V = -\frac{1}{2}t + 45$

$V = -\frac{1}{2} \times 60 + 45$

$V = 15 ms^{-1}$

b)



at C

$V = \frac{1}{8}t + \frac{15}{2}$

$V = \frac{1}{8} \times 100 + \frac{15}{2}$

$V = 20 ms^{-1}$

Total distance = (1) + (2) + (3)

$= \frac{1}{2} \times 40 \times 5 + 40 \times 15 + \frac{1}{2} \times 20 \times 20$

$= 900m$

ii) passes through starting point when displacement = 0
ie when $s = 0$

$$v = 2t - \frac{2}{3}t^{3/2}$$

$$s = t^2 - \frac{4}{15}t^{5/2} + C$$

when $t = 0$ $s = 0 \Rightarrow$ $s = t^2 - \frac{4}{15}t^{5/2}$

$$t^2 - \frac{4}{15}t^{5/2} = 0$$

$$t^2\left(1 - \frac{4}{15}t^{1/2}\right) = 0$$

$$t = 0, \quad \frac{4}{15}t^{1/2} = 1$$

$$t^{1/2} = 15/4$$

$$t = \frac{225}{16}$$

or $\underline{t = 14.1 \text{ secs}}$

3) <u>car</u>

$u = 0$
$a = a$

$s = ut + \frac{1}{2}at^2$

$s_c = \frac{1}{2}at^2$

<u>lorry</u>

$a = 0$
$u = U$

$s = ut + \frac{1}{2}at^2$

$s_L = Ut$

will draw level when $s_c = s_L$

$$\frac{1}{2}at^2 = Ut$$

$$at^2 - 2Ut = 0$$

$$t(at - 2U) = 0 \Rightarrow t = \frac{2U}{a}$$

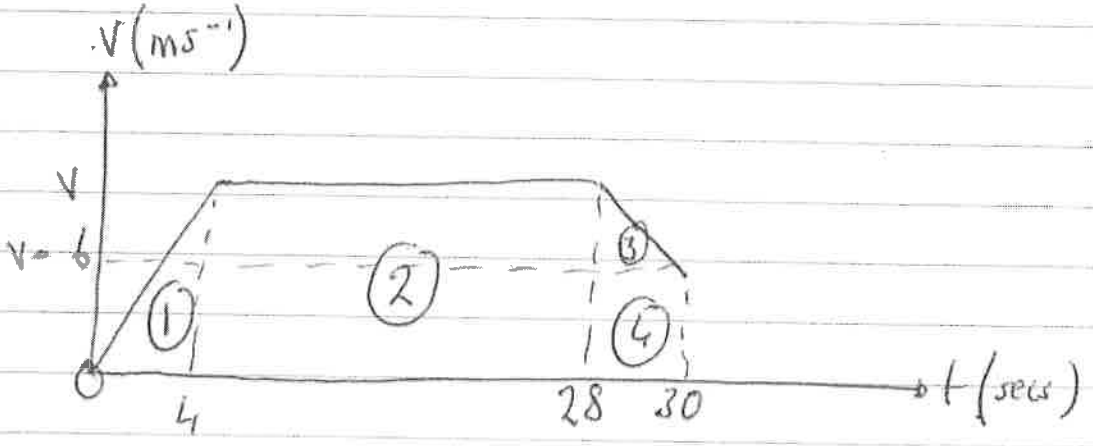
substitute $t = \frac{2U}{a}$ into either S_C or S_L

$$S_L = Ut$$

$$S_L = U\left(\frac{2U}{a}\right)$$

$$S_L = \frac{2U^2}{a}$$

4)



Area ① $= \frac{4v}{2} = 2v$

Area ② $= 24v$

Area ③ $= \frac{1}{2} \times 2 \times 6 = 6$

Area ④ $= 2(v-6)$

$2v + 24v + 6 + 2v - 12 = 200$

$28v = 206$

$v = 7.36 ms^{-1}$