Homework 2

1)

The position of a power sledge on a frozen lake at time t seconds, relative to a rectangular coordinate system, is

$$\mathbf{r}(t) = (2t^2 - t)\mathbf{i} - (3t + 1)\mathbf{j},$$

where $\mathbf{i}$, $\mathbf{j}$ are unit vectors in the x, y directions respectively and distances are measured in metres.

Calculate the time at which the speed is $5\,\mathrm{m\,s^{-1}}$. 4

2)

The velocity of an ice skater relative to a rectangular coordinate system with origin O, is given by

$$\mathbf{v} = 3(t^2 - 4t + 2)\mathbf{i} + 4\mathbf{j},$$

where $\mathbf{i}$, $\mathbf{j}$ are unit vectors in the Ox and Oy directions, t seconds is the time and the speed is measured in $\mathrm{m\,s^{-1}}$. Initially the skater has position vector $-4\mathbf{j}$.

(*a*) Find the time at which the acceleration is instantaneously equal to zero. 2

(*b*) Calculate the distance of the skater from O when the acceleration is instantaneously equal to zero. 4

3)

Relative to a rectangular coordinate system, the position of an ice skater at time t seconds is

$$\mathbf{r}(t) = \left(\frac{1}{3}t^3 - 4t^2\right)\mathbf{i} - (2t^2 - 1)\mathbf{j},$$

where $\mathbf{i}$, $\mathbf{j}$ are the unit vectors in the x, y directions respectively and distances are measured in metres.

Find the speed of the ice skater at the instant when the acceleration is parallel to the y-axis. 5

4)

The position of a remote controlled model boat on a pond, relative to a rectangular coordinate system with origin O, is given by

$$\mathbf{r} = (3t^2 - 12t + 5)\mathbf{i} + (4t - t^2)\mathbf{j},$$

where $\mathbf{i}$, $\mathbf{j}$ are unit vectors in the Ox and Oy directions respectively, t is time measured in seconds and distances are measured in metres.
Calculate the distance of the boat from the origin O when it comes to instantaneous rest. 4

5)

A particle has velocity $3t(2 - t)\mathbf{j}$ where $\mathbf{j}$ is the unit vector in the direction of motion. The time t is measured in seconds from the start of the motion and the displacement is measured in metres. Initially the particle is at the point with position vector $3\mathbf{j}$ relative to the origin O. Calculate the distance of the particle from O when the velocity is a maximum. 4

6)

An aircraft capable of flying at 290km/h in still air is to be flown from airport A to airport B. Airport B is situated 2000km from airport A on a bearing of 120°.

If there is a wind blowing at 60km/h in a north easterly direction find the course the pilot must set to reach B.
Find the total time taken to the nearest minute. 5