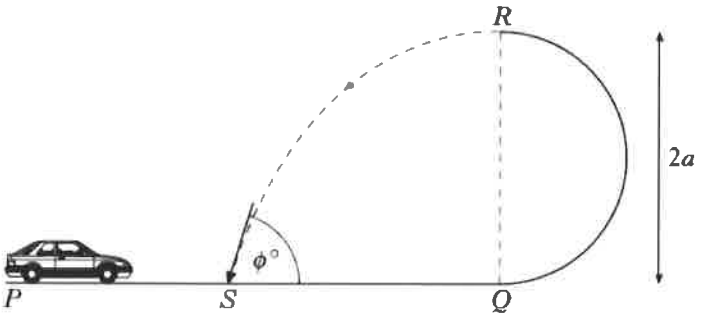
Homework 14

1) The diagram below shows a smooth plastic track. The section PQ is horizontal and the section QR is semi-circular and in the same plane as PQ. The diameter QR is vertical and has length $2a$ metres.



A toy car is projected along PQ with speed $3\sqrt{ga}$ m s^{-1}. The car travels around the track to R, where it leaves the track horizontally, landing on PQ at the point S, where the angle between the car's trajectory and the line SQ is $\phi°$.

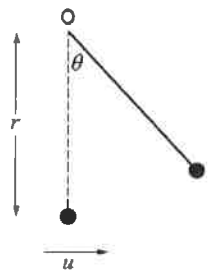
(a) Find the speed of the car at R, expressing your answer in the form $\sqrt{kga}$, where k is a constant. 3

(b) Show that at R the car is in contact with the track. 2

(c) Show that $SQ = 2\sqrt{5}a$ metres. 3

2) A light inextensible string of length r metres has one end attached to a fixed point O and the other end is attached to a particle of mass m kilograms.

From its equilibrium position, the particle is given a horizontal velocity u m s^{-1}, as shown in the diagram.



(a) (i) Show that the tension, T, in the string can be expressed as

$$T = \frac{mu^2}{r} + mg(3\cos\theta - 2)$$

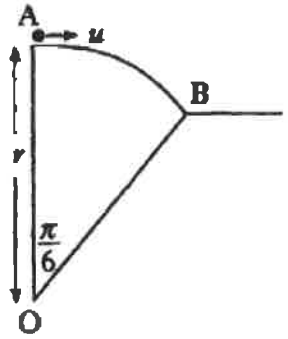
where θ is the angle between the string and the downward vertical through O. 4

(ii) Determine a condition for u in terms of r and g, so that the particle executes a complete circle. 2

(b) Given that the value of u is $2\sqrt{rg}$, find an expression in terms of r for the height of the particle above its starting position when the string goes slack. 3

3) Consider a car crossing a hump-backed bridge.

Let the car be modelled by a particle, of mass m, which has speed u at the highest point on the bridge. Assume that the effects of friction and air resistance are negligible. Let the downhill side of the bridge be modelled by the arc of a sector of a circle of radius r, subtending an angle of $\frac{\pi}{6}$ radians at the centre, as in the diagram below.



(a) Show that the greatest speed the car can have at the top of the bridge, without leaving the road surface on the downhill side of the bridge, is given by

$$u = \sqrt{\frac{gr}{2}(3\sqrt{3} - 4)},$$

where g is the magnitude of the acceleration due to gravity. **6**

(b) (i) Given that $u = \sqrt{\frac{3gr}{4}}$ and that the car leaves the surface of the bridge at the point C on arc AB, calculate the angle AOC. **3**