Homework 3

1) The position of a power sledge on a frozen lake at time t seconds, relative to a rectangular coordinate system, is

$$\mathbf{r}(t) = (2t^2 - t)\mathbf{i} - (3t + 1)\mathbf{j},$$

where $\mathbf{i}$, $\mathbf{j}$ are unit vectors in the x, y directions respectively and distances are measured in metres.

Calculate the time at which the speed is $5\,\text{m}\,\text{s}^{-1}$. **4**

2) As a set of traffic lights changes to green, a car accelerates uniformly from rest along a straight horizontal road at $a\,\text{m}\,\text{s}^{-2}$. At the same instant, a lorry travelling at constant speed $U\text{m}\,\text{s}^{-1}$ overtakes the car.

Find an expression, in terms of U and a, for the distance travelled by the car when it draws level with the lorry. **4**

3) Towards the end of a long-distance race, Tessa is running at a uniform speed of $2\,\text{m}\,\text{s}^{-1}$. When she is 120 m from the finishing line she starts to increase her speed. In doing so, she accelerates uniformly at $0{\cdot}25\,\text{m}\,\text{s}^{-2}$ for T seconds until she crosses the finishing line.

Show that T satisfies the equation

$$T^2 + 16T - 960 = 0$$

and hence find her speed as she crosses the finishing line. **6**

4) . A lift is initially at rest at ground level. It begins to accelerate upwards at $\frac{1}{8}g\ \mathrm{m\,s^{-2}}$. At the same instant, a light bulb in the ceiling of the lift begins to fall towards the lift floor. The initial distance between the lift floor and the light bulb is 2 metres.

(*a*) Measuring distances in metres relative to the ground level, show that the position of the light bulb relative to the lift floor is

$$\left(2-\frac{9}{16}gt^2\right)\mathbf{j},$$

where $\mathbf{j}$ is the unit vector in the upward vertical direction, and t is the time in seconds from the start of the motion of the lift. **3**

(*b*) Calculate the distance the light bulb falls before hitting the lift floor. **3**

5) Relative to a rectangular coordinate system with origin O the position vector of a passenger aircraft is $-100\mathbf{i} + 250\mathbf{j}$, at 09.00 hours, where $\mathbf{i}$ and $\mathbf{j}$ are unit vectors in the Ox and Oy directions. The aircraft is travelling with uniform velocity $300\mathbf{i} + 400\mathbf{j}$.

Relative to the same coordinate system, a military aircraft travelling with uniform velocity $600\mathbf{i} + 500\mathbf{j}$, has position vector $-100\mathbf{i} + 400\mathbf{j}$ at 09.30 hours. In these expressions, the distances are measured in kilometres and speeds in kilometres per hour.

Show that the two aircraft are on a collision course. **5**