

Homework 5 solutions

①

$$1) \quad a = (12 - 3t^2)\underline{i}$$
$$v = \left(12t - \frac{3t^3}{3}\right)\underline{i} + C \quad \text{at } t=0 \quad v=0 \Rightarrow C=0$$

$$\text{so } v = (12t - t^3)\underline{i} \checkmark$$

comes to rest when $v=0$

$$12t - t^3 = 0 \checkmark$$

$$t(12 - t^2) = 0$$

$$t=0 \quad t^2=12$$

$$\underline{t = \sqrt{12}} \checkmark$$

$$v = (12t - t^3)\underline{i}$$

$$s = \left(\frac{12t^2}{2} - \frac{t^4}{4}\right)\underline{i} + C \quad \text{at } t=0 \quad s=0 \Rightarrow C=0$$

$$s = \left(6t^2 - \frac{1}{4}t^4\right)\underline{i}$$

$$\text{at } t = \sqrt{12} \quad \underline{s = 36i \text{ metres}} \checkmark$$

$$2) \text{ a) } a_A = -2\mathbf{j}$$

$$v_A = -2t\mathbf{j} + C \quad \text{at } t=0 \quad v = \mathbf{i}$$

$$\mathbf{i} = C$$

$$v_A = \mathbf{i} - 2t\mathbf{j} \quad \checkmark$$

$$s_A = t\mathbf{i} - \frac{2t^2}{2}\mathbf{j} + C \quad \text{at } t=0 \quad s_A = -\mathbf{i}$$

$$-\mathbf{i} = C$$

$$\underline{s_A = (t-1)\mathbf{i} - t^2\mathbf{j}} \quad \checkmark$$

$$\text{b) i) } \Delta s_B = s_A - s_B$$

$$= (t-1)\mathbf{i} - t^2\mathbf{j} - [(2t-3)\mathbf{i} + (1-t^2)\mathbf{j}]$$

$$= \underline{(2-t)\mathbf{i} - \mathbf{j}} \quad \checkmark$$

$$\text{ii) } |\Delta s_B|^2 = (2-t)^2 + 1^2$$

$$= t^2 - 4t + 4 + 1$$

$$= t^2 - 4t + 5 \quad \checkmark$$

$$\frac{d|\Delta s_B|^2}{dt} = 2t - 4$$

$$\text{at min distance } \frac{d|\Delta s_B|^2}{dt} = 0 \Rightarrow 2t - 4 = 0 \quad \checkmark$$

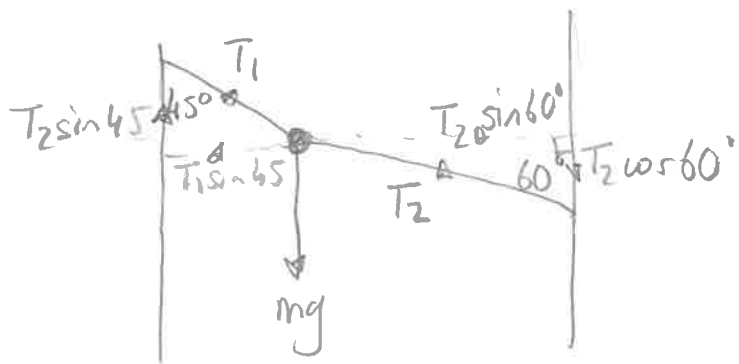
$$\underline{t = 2}$$

$$|\Delta s_B|^2 = t^2 - 2t + 5$$

$$= 4 - 4 + 5 = 5$$

$$\text{so } |\Delta s_B| = \underline{d. \text{ metres}} \quad \checkmark$$

3)



a) resolve horizontally

$$T_1 \sin 45^\circ = T_2 \sin 60^\circ$$

$$T_1 \times \frac{1}{\sqrt{2}} = T_2 \times \frac{\sqrt{3}}{2}$$

$$\underline{T_1 = \frac{\sqrt{6}}{2} T_2}$$

b) resolve vertically

$$T_1 \cos 45^\circ = T_2 \cos 60^\circ + mg$$

$$\frac{\sqrt{6}}{2} T_2 \cos 45^\circ - T_2 \cos 60^\circ = mg$$

$$0.366 T_2 = mg$$

$$\underline{T_2 = 2677.4 N}$$

4) a)

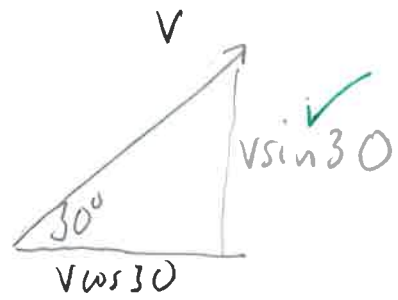
$$S = ut + \frac{1}{2} at^2$$

(4)

so $y = (v \sin 30)t - \frac{1}{2} gt^2$ ✓

$$y = \frac{1}{2} vt - \frac{1}{2} gt^2$$
 ✓

$$\underline{y = \frac{1}{2} t (v - gt)}$$



b) max height when vertical velocity = 0

$$u = v \sin 30$$

$$v = 0$$

$$a = -g$$

$$t = ?$$

$$v = u + at$$

$$0 = v \sin 30 - gt$$

$$t = \frac{v \sin 30}{g}$$

$$\underline{t = \frac{v}{2g}}$$
 ✓

so max height

$$H = \frac{1}{2} \left(\frac{v}{2g} \right) \left(v - g \left(\frac{v}{2g} \right) \right)$$

$$H = \frac{v}{4g} \left(\frac{v}{2} \right)$$

$$\underline{\underline{H = \frac{v^2}{8g}}} \quad \checkmark$$

(5)

$$a) \quad y = \frac{1}{2}t(V - gt)$$

$$\text{at } y = \frac{1}{4}H = \frac{v^2}{32g} \text{ find values of } t$$

$$\frac{v^2}{32g} = \frac{1}{2}t(V - gt) \times 32g$$

$$v^2 = 16gt(V - gt)$$

$$16g^2t^2 - 16gVt + v^2 = 0$$

$$t = \frac{16gV \pm \sqrt{(-16gV)^2 - 4 \times 16g^2 \times v^2}}{32g^2}$$

$$t = \frac{16gV \pm \sqrt{192g^2v^2}}{32g^2}$$

$$t = \frac{16gV \pm 8\sqrt{3}gV}{32g^2}$$

$$t = \frac{2V \pm \sqrt{3}V}{4g}$$

Total time $\gg \frac{1}{4}H$ when

$$T = \frac{2V + \sqrt{3}V}{4g} - \left(\frac{2V - \sqrt{3}V}{4g} \right)$$

$$T = \frac{2\sqrt{3}V}{4g}$$

$$T = \frac{\sqrt{3}V}{2g}$$

5)

$$y = \frac{\cos x}{1 - \sin x}$$

$$\frac{dy}{dx} = \frac{-\sin x (1 - \sin x) - \cos x \cdot -\cos x}{(1 - \sin x)^2} \checkmark$$

$$\frac{dy}{dx} = \frac{-\sin x + \sin^2 x + \cos^2 x}{(1 - \sin x)^2} \checkmark \quad \sin^2 x + \cos^2 x = 1$$

$$\frac{dy}{dx} = \frac{1 - \sin x}{(1 - \sin x)^2}$$

$$\frac{dy}{dx} = \frac{1}{1 - \sin x} \checkmark$$
