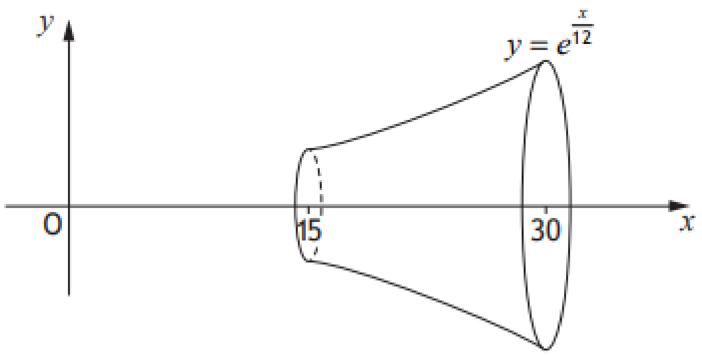
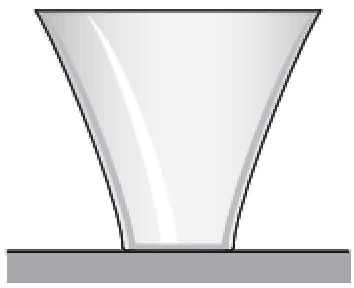
Grade Booster 2: Pure Maths	
(a) If $f(x)=\frac{\ln x}{2x^2}$, $x\neq 0$, find $f'(x)$. Fully simplify your answer. (b) If $y=\operatorname{cosec}^2 3x$, show that $$\frac{dy}{dx}+6y\cot 3x=0.$$	3 2
The velocity of a particle after t seconds of travel can be expressed as $\mathbf{v}=(3\sin 2t)\mathbf{i}+(\cos 2t-3)\mathbf{j}\,\text{ms}^{-1}$ where $\mathbf{i}$ and $\mathbf{j}$ are unit vectors in horizontal and vertical directions respectively. Find the magnitude of the acceleration of the particle when $t=\frac{\pi}{6}$ seconds.	4
Find the equation of the tangent to the curve $y=x\ln x$ at the point where $x=e$.	3
(a) Show that $\frac{3x^3+8x^2-11}{(x+1)(x+3)(x-2)}$ can be written as $3+\frac{2x^2+15x+7}{x^3+2x^2-5x-6}$. (b) Hence express $\frac{3x^3+8x^2-11}{(x+1)(x+3)(x-2)}$ in partial fractions.	3 4
A glass bowl is modelled by rotating the curve $y=e^{\frac{x}{12}}$ between $x=15$ and $x=30$ through 2π radians about the x-axis as shown in the diagram.   (a) Find the volume of the bowl. (b) A line is to be put on the bowl to indicate when it is half full. How far above the base of the bowl should this line be marked?	 3 3

Given that $y = e^{5x}\tan 2x$, find $\frac{dy}{dx}$.	3
A curve is defined by $y = \frac{\sin x}{2-\cos x}$ for $0 \le x \le \pi$. Find the exact values of the coordinates of the stationary point of this curve.	5
Express $\frac{3x^2+4x+17}{(x-3)(x^2+5)}$ as a sum of partial fractions.	4
Use integration by parts to obtain $\int x^2 \sin 5x \, dx$.	5
A curve is defined by $3y^2 - x^2 y = 4$, $x \ge 0$, $y \ge \frac{2}{\sqrt{3}}$. Use implicit differentiation to find the gradient of the tangent when $x = 2$.	5
A mass of 0·25 kg is attached to a horizontal spring of natural length 1 metre and modulus of elasticity 20 newtons. The spring is stretched and then released. It experiences a resistive force of magnitude $6v$ newtons, where v is the velocity of the mass. (a) Show that the subsequent motion satisfies the second order differential equation $\frac{d^2x}{dt^2} + 24\frac{dx}{dt} + 80x = 0.$	2
(b) Solve this second order differential equation given that the mass is released from rest with an extension in the spring of 0·2 m.	6
(c) Show that the acceleration is equal to zero when $t = \frac{1}{16}\ln 5$ seconds and find the displacement at this time.	3
Find the general solution, in the form $y = f(x)$, of the differential equation $\frac{1}{\cos x}\frac{dy}{dx} + y\tan x = \tan x$, $0 < x < \frac{\pi}{2}$	6

(*a*) Express $\dfrac{1}{1-y^2}$ in partial fractions.	3
(*b*) Use the substitution $u = \sqrt{1-x}$ to obtain $\displaystyle\int \frac{dx}{x\sqrt{1-x}}$, $0 < x < 1$.	6

Qu	Solutions
1	$\dfrac{1-2\ln x}{2x^3}$ $\dfrac{dy}{dx} + 6y\cot 3x = 0$
2	$\lvert \mathbf{a} \rvert = \sqrt{12} = 2\sqrt{3}\,\text{ms}^{-2}$ $[3\cdot 46\,\text{ms}^{-2}]$
3	$y - e = 2(x - e)$
4	$3 + \dfrac{2x^2 + 15x + 7}{x^3 + 2x^2 - 5x - 6}$ $3 + \dfrac{2x^2 + 15x + 7}{(x+1)(x+3)(x-2)}$
5	$V = 6\pi e^5 - 6\pi e^{2\cdot 5}$ (2570cm^3) Hence line should be positioned $10\cdot 1\text{cm}$ up the side of the bowl.
6	$= e^{5x}.2\sec^2 2x + \tan 2x.5e^{5x}$ $= e^{5x}\left(2\sec^2 2x + 5\tan 2x\right)$

7	For a S.P., $\frac{dy}{dx}=0 \Leftrightarrow \frac{2\cos x-1}{(2-\cos x)^2}=0$ $\Leftrightarrow 2\cos x-1=0$ $\Leftrightarrow \cos x=\frac{1}{2}$ $x=\frac{\pi}{3}$ when $x=\frac{\pi}{3}$, $y=\frac{\sin\frac{\pi}{3}}{\left(2-\cos\frac{\pi}{3}\right)}=\frac{\sqrt{3}}{3}$
8	$\frac{4}{x-3}+\frac{1-x}{x^2+5}$
9	$I=\frac{-x^2}{2}\cos 5x+\frac{2}{5}\left(\frac{x}{5}\sin 5x+\frac{1}{25}\cos 5x\right)+c$ $=\left[\left(\frac{2}{125}-\frac{x^2}{5}\right)\cos 5x+\frac{2x}{25}\sin 5x+c\right]$
10	$\frac{dy}{dx}=\frac{2\cdot 22}{6\cdot 2-2^2}=1$
11	$\frac{d^2x}{dt^2}+24\frac{dx}{dt}+80x=0$ $A=0\cdot 25,\ B=-0\cdot 05$ $x=0\cdot 25e^{-4t}-0\cdot 05e^{-20t}$
12	General Solution $y=1+Ce^{\cos x}$ **ALTERNATIVE SOLUTION** $y=1-Be^{\cos x}$

13	$\frac{1}{1-y^2}=\frac{1}{2}\left(\frac{1}{1+y}+\frac{1}{1-y}\right)$ $\ell n\left\|\frac{1-\sqrt{1-x}}{1+\sqrt{1-x}}\right\|+C$