**Marking Instructions for each question**

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|---|---|---|---|---|---|
| **1.** | (a) | | •[1] calculate impulse<br><br>•[2] calculate speed | •[1] 78<br><br>•[2] 9.75 | **2** |
| **Notes:**<br>1. Do not award •[2] for a negative answer | | | | | |
| | | | **Alternative Solution**<br>•[1] calculate acceleration<br><br>•[2] calculate speed | <br>•[1] 8.125<br><br>•[2] 9.75 | |
| | (b) | | •[3] state velocity after impact<br><br>•[4] calculate impulse on object | •[3] −9.75<br><br>•[4] 156 | **2** |
| **Notes:**<br><br>1. •[3] can be implied in •[4]<br>2. Do not award •[4] for a negative answer. However treat negative answers for both •[2] and •[4] as a repeated error. | | | | | |
| **Commonly Observed Responses:** | | | | | |

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|---|---|---|---|---|---|
| 2. | | | •[1] state expression | •[1] $\frac{A}{1+x}+\frac{B}{1-x}+\frac{C}{(1-x)^2}$ | 3 |
| | | | •[2] form equation and find one unknown | •[2] $2-3x-x^2=A(1-x)^2$ $+B(1-x)(1+x)+C(1+x)$ **and one from** $A=1, B=2, C=-1$ | |
| | | | •[3] obtain remaining unknowns | •[3] remaining from $A=1, B=2, C=-1$ | |

**Notes:**

1. Evidence for •[1] may appear at •[3]

**Commonly Observed Responses:**

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|---|---|---|---|---|---|
| **3.** | (a) | | •[1] substitute values into equation of motion<br><br>•[2] calculate maximum height | •[1] $0=\left(25\sin 30\right)^2-2gs$<br><br>•[2] 7.97 | **2** |
| | | | **Alternative Solution**<br>•[1] calculate time to maximum height<br><br>•[2] calculate speed | <br>•[1] 1.28<br><br>•[2] 9.75 | |
| **Notes:** | | | | | |
| | (b) | | •[3] substitute values into equation of motion<br><br>•[4] solve quadratic for $t$<br><br>•[5] calculate the horizontal distance | •[3] $1=(25\sin 30)t-\frac{1}{2}gt^2$<br><br>•[4] 2.47<br><br>•[5] 53.4 | **3** |
| **Notes:** | | | | | |
| **Commonly Observed Responses:** | | | | | |

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|---|---|---|---|---|---|
| 4. | (a) | | •[1] state the period of the motion | •[1] 16 | **1** |
| | (b) | | •[2] calculate value of $\omega$<br>•[3] calculate amplitude | •[2] $\frac{\pi}{8}$<br>•[3] 15.3 | **2** |

**Notes:**

**Commonly Observed Responses:**

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|---|---|---|---|---|---|
| 5. | | | •[1] solve auxiliary equation<br>•[2] state general solution<br>•[3] differentiate<br>•[4] form equations and solve for one constant<br>•[5] find second constant and state particular solution | •[1] $m=2$ , $m=-3$<br>•[2] $x=Ae^{2t}+Be^{-3t}$<br>•[3] $\frac{dx}{dt}=2Ae^{2t}-3Be^{-3t}$<br>•[4] $A=1$ or $B=-1$<br>•[5] $x=e^{2t}-e^{-3t}$ | 5 |

**Notes:**

1. •[1] may be implied by •[2]

**Commonly Observed Responses:**

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|---|---|---|---|---|---|
| **6.** | (a) | | •[1] integrate $a$ with respect to $t$ and include constant of integration<br><br>•[2] use initial conditions and complete | •[1] $v = \int a\,dt = at + c$<br><br>•[2] $t = 0, v = u \Rightarrow c = u$<br>$v = u + at$ | **2** |
| | | | **Alternative Solution**<br><br>•[1] set up integral and include limits<br><br>•[2] integrate and complete | •[1] $\int_u^v dv = \int_0^t a\,dt$<br><br>•[2] $v - u = at - 0$<br>$v = u + at$ | |
| **Notes:** | | | | | |
| | (b) | | •[3] Integrate expression from part a<br><br>•[4] find constant and state expression | •[3] $\int v\,dt = ut + \frac{1}{2}at^2 + k$<br><br>•[4] $t = 0 \;\; s = 0 \Rightarrow k = 0$<br>$s = ut + \frac{1}{2}at^2$ | **2** |
| **Notes:**<br>• Do not penalise the omission of a constant for •[3], however •[4] is then unavailable<br>• Accept use of the same letter for the constant in parts (a) and (b) | | | | | |
| | | | **Alternative Solution**<br><br>•[1] set up integral and include limits<br><br>•[2] integrate and complete | •[1] $\int_0^s ds = \int_0^t (u + at)\,dt$<br><br>•[2] $s = ut + \frac{1}{2}at^2$ | |

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|---|---|---|---|---|---|
| 7. | | | •[1] choose functions to differentiate and integrate and start to integrate<br><br>•[2] continue to integrate<br><br>•[3] complete integration, simplify and include constant of integration | •[1] $18x\left(-\frac{1}{3}\cos 3x\right)-\int\ldots$<br><br>•[2] $\ldots-\int\left(18\times\left(-\frac{1}{3}\cos 3x\right)\right)dx$<br><br>•[3] $-6x\cos 3x+2\sin 3x+c$ | 3 |

**Notes:**

Do not withhold •[3] if constant of integration is omitted

**Commonly Observed Responses:**

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|---|---|---|---|---|---|
| **8.** | | | •1 resolve forces vertically<br><br>•2 apply Newton's second law horizontally<br><br>•3 combine equations<br><br>•4 convert angular speed to radians per second<br><br>•5 substitute values and calculate | •1 $R = mg$<br><br>•2 $\mu R = mr\omega^2$<br><br>•3 $\mu mg = mr\omega^2$<br><br>•4 $3\pi$<br><br>•5 0.634 | **5** |

**Notes:**

1. Accept the use of $\frac{v^2}{r}$ instead of $r\omega^2$ at •2 and •3

**Commonly Observed Responses:**

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|---|---|---|---|---|---|
| **9.** | | | •¹ know that volume $=\int \pi y^2 dx$ and begin to substitute | •¹ $\int \pi y^2 dx = \int \pi \frac{\ldots}{\left(3x^3-1\right)} dx$ | **6** |
| | | | •² complete substitution and introduce limits | •² $\int_1^2 \pi \frac{36x^2}{\left(3x^3-1\right)} dx$ | |
| | | | •³ differentiate | •³ $du = 9x^2 dx$ **or** $\frac{du}{dx} = 9x^2$ | |
| | | | •⁴ determ`ine limits | •⁴ $\int_2^{23} \ldots du$ | |
| | | | •⁵ complete integral | •⁵ $4\pi \int_2^{23} \frac{1}{u^2} du$ | |
| | | | •⁶ integrate and evaluate | •⁶ $\frac{42\pi}{23}$ | |

**Notes:**

(see next page)

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|---|---|---|---|---|---|
| 9. | | | **Alternative Solution (without calculating limits for $u$)** | | |
| | | | •1 know that volume $=\int \pi y^2 dx$ and begin to substitute | •1 $\int \pi y^2 dx = \int \pi \frac{\cdots}{\left(3x^3-1\right)} dx$ | |
| | | | •2 complete substitution and introduce limits | •2 $\int_1^2 \pi \frac{36x^2}{(3x^3-1)^2} dx$ | |
| | | | •3 differentiate | •3 $du = 9x^2 dx$ **or** $\frac{du}{dx} = 9x^2$ | |
| | | | •4 state integral | •4 $4\pi \int_{\dots}^{\dots} \frac{1}{u^2} du$ | |
| | | | •5 integrate and include limits | •5 $4\pi \left[\frac{-1}{3x^3-1}\right]_1^2$ | |
| | | | •6 integrate and evaluate | •6 $\frac{42\pi}{23}$ | |

**Notes:**

1. For •1 $dx$ must appear prior to •3
2. For •2 to be awarded, correct limits must be present. Evidence of this may appear elsewhere
3. •1 may also be awarded for $\int_1^2 \pi y^2 dx$
4. •6 is unavailable if the limits 1 and 2 are substituted for $u$
5. Treat the appearance of the limits 1 and 2 at •4 in 1st method as bad form if it is later corrected

**Commonly Observed Responses:**

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|---|---|---|---|---|---|
| **10.** | (a) | | •[1] consider total energy at A<br><br>•[2] use conservation of energy to find speed at B | •[1] $= 0.1 \times 9.8 \times 0.6 + \frac{1}{2} \times 0.1 \times 1.2^2$<br><br>•[2] 3.63 | **2** |
| **Notes:** | | | | | |
| **Commonly Observed Responses:** | | | | | |
| | (b) | | •[3] state the force equation when rod is at A<br><br>•[4] calculate tension<br><br>•[5] identify as thrust/compression | •[3] $T - mg\cos 180° = \frac{mv^2}{r}$<br><br>•[4] −0.5<br><br>•[5] rod is in thrust/compression | **3** |
| **Notes:**<br>1. If a positive answer is awarded •[4] as a follow through, do not award •[5] for "rod is in tension" | | | | | |
| **Commonly Observed Responses:** | | | | | |
| | (c) | | •[6] consider forces in equilibrium when tension is zero.<br><br>•[7] consider potential energy at this point.<br><br>•[8] use conservation of energy to find kinetic energy at angle $\theta$<br><br>•[9] combine equations to eliminate $v$<br><br>•[10] solve for $\theta$ | •[6] $\frac{mv^2}{r} + mg\cos\theta = 0$<br><br>•[7] $E_P = mgr(1 - \cos\theta)$<br><br>•[8] $\frac{1}{2}mv^2 = \frac{1}{2}mv_B{}^2 - E_P$<br><br>•[9] $\frac{m}{r}\left(v_B{}^2 - 2gr(1 - \cos\theta)\right) + mg\cos\theta = 0$<br><br>•[10] $146°$ | **5** |
| **Notes:**<br>1. Evidence for •[6] may appear later in the solution<br>2. For •[10] accept 2.55 radians | | | | | |
| **Commonly Observed Responses:** | | | | | |

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|---|---|---|---|---|---|
| 11. | | | •[1] identify integral form of integrating factor | •[1] $e^{\int -\frac{1}{x}dx}$ | 6 |
| | | | •[2] determine integrating factor | •[2] $\frac{1}{x}$ | |
| | | | •[3] write as integral equation | •[3] $\frac{1}{x}y=\int e^{2x}dx$ | |
| | | | •[4] integrate | •[4] $\frac{1}{x}y=\frac{1}{2}e^{2x}+c$ | |
| | | | •[5] evaluate constant | •[5] $c=e^2$ | |
| | | | •[6] form particular solution | •[6] $y=\frac{1}{2}xe^{2x}+e^2x$ | |

**Notes:**

1. If constant of integration is omitted at •[4], award •[4] but •[5] and •[6] are unavailable

**Commonly Observed Responses:**

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|---|---|---|---|---|---|
| **12.** | (a) | | •[1] resolve forces perpendicular to the plane<br><br>•[2] resolve forces parallel to the plane for $\mu R$ acting up the plane<br><br>•[3] strategy to eliminate $R$ and substitute for F<br><br>•[4] simplify by eliminating $mg$ and fractions<br><br>•[5] use $\tan\theta = \frac{\sin\theta}{\cos\theta}$ and rearrange to required answer | •[1] $R + F\sin\theta = mg\cos\theta$<br><br>•[2] $\mu R + F\cos\theta = mg\sin\theta$<br><br>•[3] $\mu\left(mg\cos\theta - \frac{1}{2}mg\sin\theta\right) + \frac{1}{2}mg\cos\theta = mg\sin\theta$<br><br>•[4] $2\mu\cos\theta - \mu\sin\theta + \cos\theta = 2\sin\theta$<br><br>•[5] working legitimately leading to $\mu = \frac{2\tan\theta - 1}{2 - \tan\theta}$ | **5** |

**Notes:**

1. $W$ may be used instead of $mg$.
2. •[3] is unavailable if $R = mg$ is stated at •[1]

**Commonly Observed Responses:**

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|---|---|---|---|---|---|
| 12. | (b) | | **Method 1** | **Method 1** | 5 |
| | | | •[6] determine the value of $\mu$ | •[6] 0.109 | |
| | | | •[7] resolve forces parallel to the slope | •[7] $kmg\cos 30° = 0.109R + mg\sin 30°$ | |
| | | | •[8] resolve forces perpendicular to the slope | •[8] $R + kmg\sin 30° = mg\cos 30°$ | |
| | | | •[9] substitute for $R$ | •[9] $kmg\cos 30° = 0.109(mg\cos 30° - kmg\sin 30°) + mg\sin 30°$ | |
| | | | •[10] state magnitude of force in terms of $m$ and $g$ | •[10] $0.646mg$ | |
| | | | **Method 2** | **Method 2** | |
| | | | •[6] determine the value of $\mu$ | •[6] 0.109 | |
| | | | •[7] resolve forces parallel to the slope | •[7] $F\cos 30° = 0.109R + mg\sin 30°$ | |
| | | | •[8] resolve forces perpendicular to the slope | •[8] $R + F\sin 30° = mg\cos 30°$ | |
| | | | •[9] substitute for $R$ | •[9] $F\cos 30° = 0.109(mg\cos 30° - F\sin 30°) + mg\sin 30°$ | |
| | | | •[10] state magnitude of force | •[10] $0.646mg$ | |

**Notes:**

1. •[9] is unavailable if $R = mg$ is stated at •[8]

**Commonly Observed Responses:**

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|---|---|---|---|---|---|
| **13.** | (a) | | •¹ evidence of use of quotient rule with denominator and one term in numerator correct | •¹ $\frac{(\tan x+1)\sec x\tan x-\ldots}{(\tan x+1)^2}$ or $\frac{\ldots-\sec x\sec^2 x}{(\tan x+1)^2}$ | **3** |
| | | | •² complete differentiation | •² $\frac{(\tan x+1)\sec x\tan x-\sec x\sec^2 x}{(\tan x+1)^2}$ | |
| | | | •³ simplify and complete | $\ldots=\frac{\sec x(\tan x-1)}{(\tan x+1)^2}$ leading to **either** $f(x)\frac{\tan x-1}{\tan x+1}$ **or** $\frac{\sec x}{\tan x+1}.\frac{\tan x-1}{\tan x+1}$ | |

**Notes:**

1. For •³ to be awarded, the use of $1+\tan^2 x=\sec^2 x$ or equivalent should be obvious

| | | | | | |
|---|---|---|---|---|---|
| | (b) | | •⁴ recognise form of integral | •⁴ $\int\frac{f'(x)}{f(x)}dx$ **stated or implied by •⁵** | **2** |
| | | | •⁵ integrate | •⁵ $\ln\left\|\frac{\sec x}{\tan x+1}\right\|+c$ | |

**Notes:**

1. Accept $\ln\left(\frac{\sec x}{\tan x+1}\right)+c$
2. Do not penalise the omission of the constant of integration

**Commonly Observed Responses:**

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|---|---|---|---|---|---|
| **14.** | (a) | | •[1] state condition for maximum velocity with substitution<br><br>•[2] solve the equation for positive value of $x$. | •[1] $15+x-2x^2=0$<br><br>•[2] 3 | **2** |
| | (b) | (i) | •[3] set up integral for work done<br><br>•[4] integrate<br><br>•[5] calculate the work done | •[3] $\int\left(75+5x-10x^2\right)dx$<br><br>•[4] $\left[75x+\frac{5}{2}x-\frac{10}{3}x^2\right]_0^3$<br><br>•[5] 157.5 | **3** |
| | | (ii) | •[6] use the work-energy principle<br><br>•[7] determine value of | •[6] $\frac{1}{2}\times5\times v^2-\frac{1}{2}\times5\times0^2=157.5$<br><br>•[7] 7.94 **or** $\sqrt{63}$ | **2** |
| | | | **Alternative Solution**<br><br>•[6] replace acceleration with $v\frac{dv}{dx}$, separate variables and set up integration<br><br>•[7] integrate and complete | $\int v\,dv=\int\left(15+x-2x^2\right)dx$ **or**<br>$\int_0^v v\,dv=\int_0^3\left(15+x-2x^2\right)dx$<br><br>•[7] 7.94 **or** $\sqrt{63}$ | |

**Notes:**

1. If an indefinite integral is used in the alternative solution, a constant of integration must be considered for •[7] to be awarded

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|---|---|---|---|---|---|
| **15.** | (a) | (i) | •1 obtain velocity vector<br><br>•2 substitute and calculate speed | •1 $\begin{pmatrix} 3000t+240 \\ 0 \\ 80t+50 \end{pmatrix}$<br><br>•2 843 | **2** |
| | | | **Alternative Solution**<br><br>substitute into equation of motion<br><br>•2 calculate speed | •1 $\begin{pmatrix} 240 \\ 0 \\ 50 \end{pmatrix}+\begin{pmatrix} 3000 \\ 0 \\ 80 \end{pmatrix}\times 0.2$<br><br>•2 843 | |
| **Notes:**<br>Do not penalise candidates who use two-dimensional vectors in (a)(i) | | | | | |
| | | (ii) | •3 obtain position vector<br><br>•4 evaluate position vector | •3 $\begin{pmatrix} 1500t^2+240t \\ 0 \\ 40t^2+50t \end{pmatrix}$<br><br>•4 $\begin{pmatrix} 108 \\ 0 \\ 11.6 \end{pmatrix}$ | **2** |
| | | | **Alternative Solution**<br><br>•1 substitute into equation of motion<br><br>•2 evaluate position vector | •1 $\begin{pmatrix} 240 \\ 0 \\ 50 \end{pmatrix}+\begin{pmatrix} 3000 \\ 0 \\ 80 \end{pmatrix}\times 0.2$<br><br>•2 $\begin{pmatrix} 108 \\ 0 \\ 11.6 \end{pmatrix}$ | |
| **Notes:**<br>Do not penalise candidates who use two-dimensional vectors in (a)(ii) | | | | | |

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|---|---|---|---|---|---|
| 15. | (b) | (i) | •5 Find resultant velocity<br><br>•6 Calculate angle | •5 $\begin{pmatrix} 832.6 \\ -50 \\ 0 \end{pmatrix}$<br><br>•6 $3.4^\circ$ | 2 |
| **Notes:** | | | | | |
| **Commonly Observed Responses:** | | | | | |
| | | (ii) | •7 Find displacement vector<br><br>•8 Find displacement after 90 mins<br><br>•9 Find horizontal component | •7 $\begin{pmatrix} 832.6t+108 \\ -50t \\ 11.6 \end{pmatrix}$<br><br>•8 $\begin{pmatrix} 1190.37 \\ -65 \\ 11.6 \end{pmatrix}$<br><br>•9 1192.1 | 3 |
| **Notes:** | | | | | |
| **Commonly Observed Responses:** | | | | | |

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|---|---|---|---|---|---|
| **16.** | (a) | | •[1] use conservation of energy with substitution<br><br>•[2] solve for $v$ | •[1] $20 = \frac{1}{2} \times 0.1 \times v^2 + 0.1 \times 9.8 \times 10$<br><br>•[2] 14.3 | **2** |
| **Notes:** | | | | | |
| **Commonly Observed Responses:** | | | | | |
| | (b) | | •[3] calculate speed<br><br>•[4] use conservation of energy to calculate height | •[3] $\sqrt{41}\,[6.40]$<br><br>•[4] 18.3 | **2** |
| **Notes:**<br>Evidence for •[3] may appear in working for •[4] | | | | | |
| **Commonly Observed Responses:** | | | | | |
| | (c) | | •[5] use horizontal velocity to calculate kinetic energy | •[5] 0.8 | **1** |
| **Notes:** | | | | | |
| **Commonly Observed Responses:** | | | | | |

**[END OF MARKING INSTRUCTIONS]**