Marks

Section C (Mechanics 1 and 2)

ONLY candidates doing the course Mechanics 1 and 2 and one unit chosen from Mathematics 1 (Section D), Statistics 1 (Section E) and Numerical Analysis 1 (Section F) should attempt this Section.

Answer all the questions.

Answer these questions in a separate answer book, showing clearly the section chosen.

Candidates should observe that g m s^{-2} denotes the magnitude of the acceleration due to gravity.
Where appropriate, take its numerical value to be 9·8 m s^{-2}.

C1. The position of a power sledge on a frozen lake at time t seconds, relative to a rectangular coordinate system, is

$$\mathbf{r}(t) = (2t^2 - t)\mathbf{i} - (3t + 1)\mathbf{j},$$

where $\mathbf{i}$, $\mathbf{j}$ are unit vectors in the x, y directions respectively and distances are measured in metres.

Calculate the time at which the speed is 5 m s^{-1}. **4**

C2. At 2 pm, a ferry leaves port O travelling at $25\sqrt{2}$ km/h in a north-easterly direction. At the same time, a liner is 10 km east of O and travelling due north at 20 km/h. Both velocities remain constant.

(*a*) By choosing an appropriate rectangular coordinate system with origin O, find the position of the ferry relative to the liner at time t, measured in hours from 2 pm. **4**

(*b*) Calculate the distance between the ferry and the liner at 3 pm. **2**

C3. A piston connected to a water wheel oscillates about a point O with simple harmonic motion of period 8π seconds and maximum acceleration 0·25 m s^{-2}.

(*a*) Calculate the amplitude of the motion. **3**

(*b*) Calculate the positions, relative to O, of the piston when it is moving with half its maximum speed. **4**

C4. A ramp consists of a rough plane inclined at angle θ to the horizontal, where $\tan\theta = \frac{5}{12}$. A box of mass m kg is given a push up the line of greatest slope of the ramp, which gives the box an initial speed of $\sqrt{gL}$ m s^{-1}, where L metres is the distance travelled before the box comes to rest.

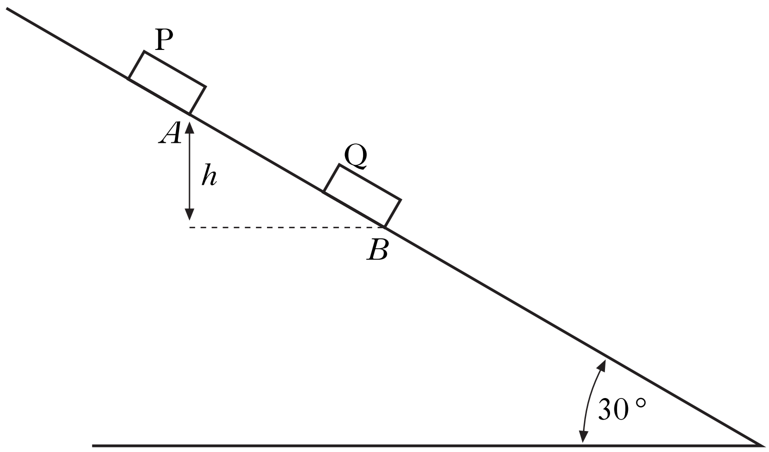
Calculate the value of the coefficient of friction between the box and the surface of the ramp. **7**

Marks

C5. An unladen helicopter of mass M kilograms can hover at a constant height above the ground when the engine exerts a lift force of P newtons.

The helicopter is loaded with cargo which increases its mass by 1%. When airborne, the engine now exerts a lift force 5% greater than P to accelerate the helicopter vertically upwards. Calculate this vertical acceleration. **5**

C6. The diagram shows a ramp, inclined at $30°$ to the horizontal, which has a smooth section above B and a rough section below B. Identical blocks, P and Q, each has weight W newtons. Block Q is stationary at B, held by friction, and block P is held at rest at A. Block P is a vertical height of h metres above block Q (where the dimensions of the blocks should be ignored).



When block P is released, it slides down the ramp colliding and coupling with block Q. The combined blocks then move down the rough section of the ramp, coming to rest at a vertical height $\frac{1}{2}h$ metres below B.

(i) Find, in terms of g and h, the speed of the combined block immediately after the collision. **3**

(ii) Using the work/energy principle, show that the constant frictional force acting on the combined block whilst it is moving has magnitude $\frac{3}{2}W$ newtons. **4**

C7. A football is kicked from a point O on a horizontal plane, giving the ball an initial speed $V\,\text{m s}^{-1}$ at an angle α to the horizontal. Assuming that gravity is the only force acting on the ball:

(*a*) Show that the maximum height, H metres, attained by the football is given by

$$H = \frac{V^2}{2g}\sin^2\alpha.$$ **3**

(*b*) A second identical football is kicked from O with the same initial speed $V\,\text{m s}^{-1}$ but at angle of projection 2α to the horizontal ($2\alpha < \frac{1}{2}\pi$). The maximum height attained by this football is h metres.

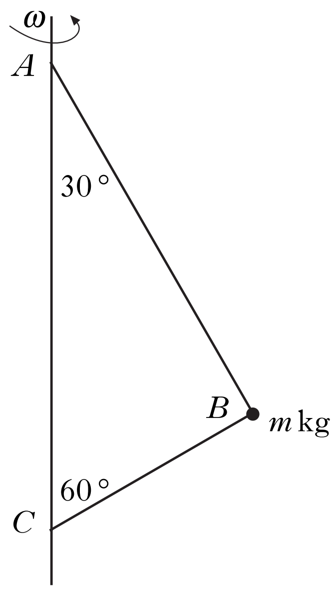
(i) Show that

$$h = 4H\left(1 - \frac{2gH}{V^2}\right).$$ **3**

[Note that $\sin 2\alpha = 2\sin\alpha\cos\alpha$.]

(ii) Given that the maximum height attained by the second football is three times that attained by the first, find the angles of projection of each of the two footballs. **4**

C8. A bead of mass m kilograms is attached to a vertical rotating column by two strings, as shown below. String AB is elastic, with natural length L metres and modulus of elasticity $2mg$ newtons. The string is attached to the column at A and to the bead at B. String BC is inextensible and has length L metres. The vertical column is rotating at $\omega\,\text{rad s}^{-1}$, such that the strings AB and BC are taut and remain in a vertical plane. Angles ACB and BAC are $60°$ and $30°$ respectively.



(*a*) Show that the tension in the string AB is $2(\sqrt{3} - 1)\,mg$ newtons. **4**

(*b*) Find, in terms of m and g, an expression for the tension in the string BC. **3**

(*c*) Given that $L = 1$, calculate ω. **4**

Marks

C9. A particle of mass m kg moves in a horizontal straight line from the origin O with initial velocity $U\mathbf{i}\,\text{m s}^{-1}$, where $\mathbf{i}$ is the unit vector in the direction of motion. A resistive force $-mkv^3\mathbf{i}$ acts on the particle, where k is a constant and $v\mathbf{i}$ is the velocity of the particle at time t seconds measured from the start of the motion.

(i) Show that the velocity of the particle satisfies the differential equation

$$\frac{dv}{dx} = -kv^2,$$

where x is the distance of the particle from O. **2**

Hence show that $v = \dfrac{U}{1 + kUx}$. **3**

(ii) Using (i), or otherwise, show that

$$kUx^2 + 2x = 2Ut.$$ **3**

(iii) Find an expression, in terms of k and U, for the time taken for the speed of the particle to reduce to half its initial value. **3**

[*END OF SECTION C*]

All candidates who have attempted Section C (Mechanics 1 and 2) should now attempt ONE of the following

Section D (Mathematics 1) on *Page fifteen*

Section E (Statistics 1) on *Pages sixteen* and *seventeen*

Section F (Numerical Analysis 1) on *Pages eighteen* and *nineteen.*

Section D (Mathematics 1)

Answer all the questions.

Answer these questions in a separate answer book, showing clearly the section chosen.

D1. Expand $(4x - 5y)^4$ simplifying as far as possible. **4**

When $y = \frac{1}{x}$, find the term independent of x. **1**

D2. For the function defined by $y = x^2 \ln x,\ x > 0$, find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. **4**

Hence show that $x\frac{d^2y}{dx^2} - \frac{dy}{dx} = kx,$ stating the value of the constant k. **2**

D3. For the following system of equations in a, b and c

$$\begin{aligned} a + b - 2c &= -6 \\ 3a - b + c &= 7 \\ 2a + b - \lambda c &= -2 \end{aligned}$$

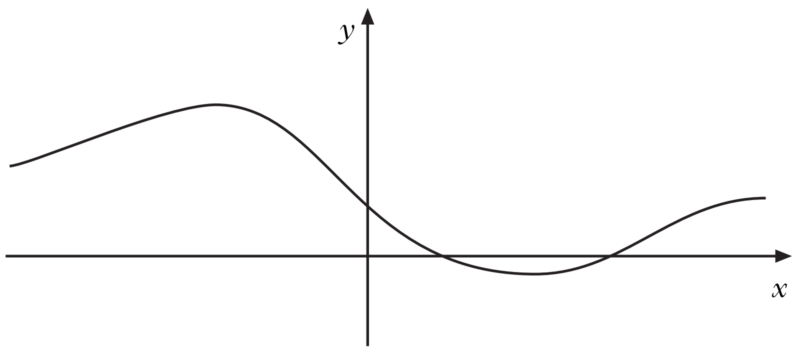
use Gaussian elimination to find

(*a*) the value of λ for which there is no solution, **3**

(*b*) the values of a, b and c when $\lambda = 1$. **2**

D4. Use the substitution $u = x + 1$ to obtain $\int \frac{x^2 + 2}{(x+1)^2}\,dx$. **5**

D5.



The diagram shows part of the graph of $y = f(x)$ where $f(x) = \frac{(x-1)(x-4)}{x^2+4}$.

(*a*) Express $f(x)$ in the form $A + \frac{Bx + C}{x^2 + 4}$ for suitable constants A, B and C. **3**

(*b*) Identify the asymptote of the curve. **1**

(*c*) Obtain the stationary points. **3**

(*d*) Evaluate the area of the finite region bounded by the curve and the x-axis. **4**

[*END OF SECTION D*]

Section G (Mechanics 1)

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Candidates should observe that g m s^{-2} denotes the magnitude of the acceleration due to gravity. Where appropriate, take its numerical value to be 9·8 m s^{-2}.

G1. The position of a power sledge on a frozen lake at time t seconds, relative to a rectangular coordinate system, is

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Marks

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[Note that $\sin 2\alpha = 2\sin\alpha\cos\alpha$.]

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[*END OF SECTION G*]

[*END OF QUESTION PAPER*]