Section C (Mechanics 1 and 2)

Marks

ONLY candidates doing the course Mechanics 1 and 2 and one unit chosen from Mathematics 1 (Section D), Statistics 1 (Section E) and Numerical Analysis 1 (Section F) should attempt this Section.

Answer all the questions.

Answer these questions in a separate answer book, showing clearly the section chosen.

Where appropriate, candidates should take the magnitude of the acceleration due to gravity as $9{\cdot}8\,\mathrm{m\,s^{-2}}$.

C1. A motorcyclist moves from rest along a straight, horizontal road, with acceleration $2t\mathbf{i}\,\mathrm{m\,s^{-2}}$, where $\mathbf{i}$ is the unit vector in the direction of motion, and t is the time in seconds from the start of the motion.

Calculate the distance travelled by the motorcyclist in the time taken for the speed to increase from $1\,\mathrm{m\,s^{-1}}$ to $9\,\mathrm{m\,s^{-1}}$. **4**

C2. In the workshop, components produced by one machine have to be processed on a second machine. The components from the first machine are put into a box which is then put into a channel and pushed along to the second machine. The channel, which is straight and horizontal, runs from the point A, with position vector $\mathbf{i} + 2\mathbf{j}$, to the point B, with position vector $3\mathbf{i} + 12\mathbf{j}$, where $\mathbf{i}$ and $\mathbf{j}$ are perpendicular unit vectors. Deborah pushes a box from A to B at a constant velocity of $\frac{1}{4}(\mathbf{i} + 5\mathbf{j})$ metres per second by exerting a constant force of $2\mathbf{i} + 3\mathbf{j}$ newtons.

Calculate

(*a*) the work done by Deborah; **3**

(*b*) the rate at which this work is done. **2**

C3. A car of mass m kilograms is travelling in a straight line along a horizontal road at constant speed U metres per second when the driver applies the brakes. The brakes cause a constant retarding force R newtons which brings the car to rest in a distance of D metres.

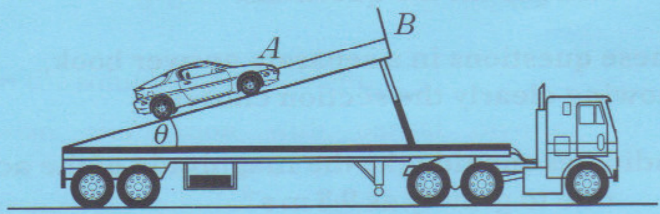
Find an expression for the stopping distance D in terms of m, U and R. **3**

Comment on how the stopping distance depends on the mass of the car. **1**

[Turn over

Marks

C4. The diagram below shows a car of mass m kilograms which is held in equilibrium on the back of a stationary lorry by means of a light inextensible chain AB which runs parallel to the sloping surface. This surface is inclined at an angle of θ to the horizontal and the coefficient of friction between the car and the surface is μ.



When $\theta = 30°$ the tension required in the chain AB to prevent the car slipping down the slope is T newtons. When θ is increased to $45°$ the tension required in AB becomes $2T$ newtons.

(*a*) When $\theta = 30°$ show that

$$T = \tfrac{1}{2}\left(1-\sqrt{3}\mu\right)mg,$$

where $g\ \mathrm{m\,s^{-2}}$ is the magnitude of the acceleration due to gravity. **3**

(*b*) Find an expression for T in terms of m, μ, and g when $\theta = 45°$. **2**

(*c*) Find the value of μ. **2**

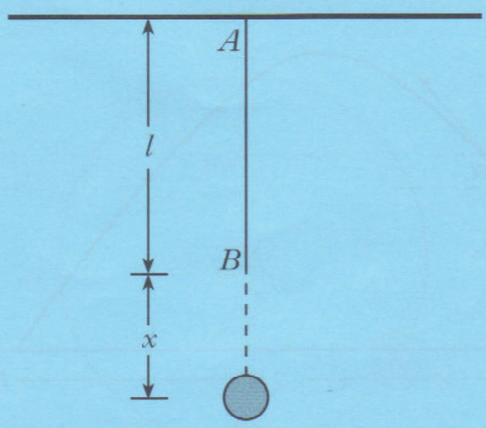
C5. A planet of radius R kilometres has two moons, A and B. Moon A is in a circular orbit round the planet at a height of $3R$ kilometres above the surface of the planet. Moon B, which has the same mass as A, is also in a circular orbit at a height of $5R$ kilometres above the planet's surface.

Assume that the only force acting is that due to the inverse square law of gravitation between each moon and the planet and ignore any interaction between the moons. Representing the angular speeds of A and B by ω_A, ω_B radians per second respectively, show that

$$8\omega_A^2 = 27\omega_B^2.$$ **5**

Marks

C6. One end of an elastic string, with natural length l metres and modulus of elasticity λ newtons, is attached to a fixed horizontal beam at A and a mass m kilograms is attached to its other end B, where $AB = l$ metres, as shown below.



(*a*) Show that t seconds after the mass is released the extension, x metres, of the string beyond its natural length satisfies

$$\frac{d^2x}{dt^2} + \omega^2 x = g,$$

where $g\,\mathrm{m\,s^{-2}}$ is the magnitude of the acceleration due to gravity and ω is a constant. **2**

Express ω^2 in terms of m, l and λ. **1**

(*b*) Find the equilibrium extension x_e in terms of g and ω. **2**

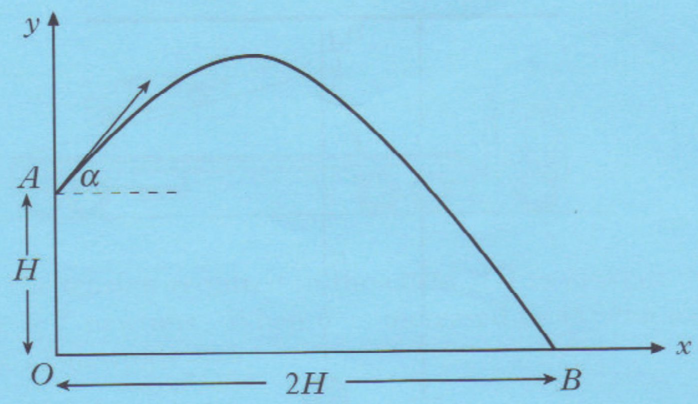
C7. A light aircraft is travelling due north at a constant altitude of 1 km with constant speed $100\sqrt{2}$ km/h. At 1 pm a helicopter is $50\sqrt{2}$ km due west of the aircraft, and travelling in a north-easterly direction at a constant altitude of 2 km with constant speed 100 km/h.

Taking the position of the aircraft at 1 pm as the origin, and defining an appropriate set of unit vectors, find the position of the helicopter relative to the aircraft in terms of time t hours after 1 pm. **6**

[Turn over

Marks

C8. An artillery shell is launched from the point A, which is H metres vertically above the point O on level ground, as shown below. The shell is projected at an angle α above the horizontal, where $0 < \alpha < \frac{\pi}{2}$, with speed $\sqrt{2gH}$ metres per second, where $g\,\text{m s}^{-2}$ is the magnitude of the acceleration due to gravity.



(*a*) Show that, referred to the axes shown, the equation of the trajectory of the shell is

$$y = H + x\tan\alpha - \frac{(1+\tan^2\alpha)}{4H}x^2.$$ **4**

[Note that $\frac{1}{\cos^2\alpha} = 1 + \tan^2\alpha$.]

(*b*) The shell lands at the point B on the ground, a horizontal distance of $2H$ metres from O, as shown.

Show that $\tan\alpha = 2$. **3**

(*c*) Show further that the maximum height above the ground attained by the shell is $\frac{9}{5}H$ metres. **4**

C9. A racing car of mass m kilograms accelerates from rest and moves in a straight line along a horizontal racetrack. The engine supplies a constant power P watts and the car experiences an air resistance force of mkv newtons, where k is a constant and v is the speed of the car in metres per second.

(*a*) Show that the speed of the car satisfies the differential equation

$$m\frac{dv}{dt} = \frac{P - mkv^2}{v},$$

where t seconds is the time from the start of the motion. **3**

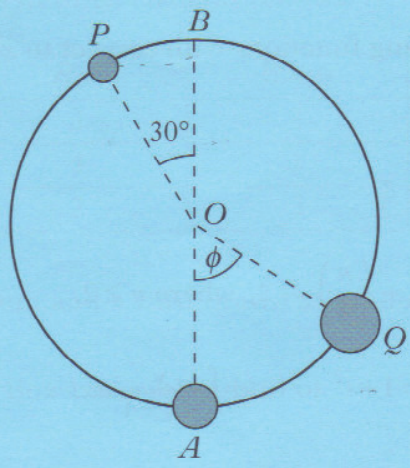
(*b*) By making the substitution $w = P - mkv^2$, or otherwise, solve the differential equation in (*a*) and show that

$$v^2 = \frac{P}{mk}\left(1 - e^{-2kt}\right).$$ **6**

(*c*) What happens to the speed of the car as t gets large? **1**

Marks

C10. Two plastic beads have been threaded onto a frictionless circular wire of radius R metres and centre O, set in a vertical plane, as shown below. The diameter AOB is vertical. At the point P, a bead of mass m kilograms is held at rest with angle POB equal to $30°$. A second bead, of mass $2m$ kilograms, is stationary at A. When the bead at P is released, it travels along the wire and adheres to the bead at A.



(*a*) Show that the speed of the resulting combined bead immediately after the collision is

$$\tfrac{1}{3}\sqrt{(2+\sqrt{3})\,gR},$$

where $g\,\text{m}\,\text{s}^{-2}$ is the magnitude of the constant acceleration due to gravity. **4**

Determine the fraction of the kinetic energy that is lost in the collision. **3**

(*b*) The combined bead continues to move along the wire until it is instantaneously at rest at the point Q, where angle $AOQ = \phi$, $0 < \phi < 90°$.

Show that

$$\cos\phi = \frac{16-\sqrt{3}}{18}.$$ **4**

[*END OF SECTION C*]

All candidates who have attempted Section C (Mechanics 1 and 2) should now attempt ONE of the following

Section D (Mathematics 1) on *Page fourteen*

Section E (Statistics 1) on *Pages fifteen* and *sixteen*

Section F (Numerical Analysis 1) on *Pages seventeen* and *eighteen*.

[Turn over

Section G (Mechanics 1)

Marks

Answer all the questions.

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Where appropriate, candidates should take the magnitude of the acceleration due to gravity as $9{\cdot}8\,\mathrm{m\,s^{-2}}$.

G1. A motorcyclist moves from rest along a straight, horizontal road, with acceleration $2t\mathbf{i}\,\mathrm{m\,s^{-2}}$, where $\mathbf{i}$ is the unit vector in the direction of motion, and t is the time in seconds from the start of the motion.

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G2. A car of mass m kilograms is travelling in a straight line along a horizontal road at constant speed U metres per second when the driver applies the brakes. The brakes cause a constant retarding force R newtons which brings the car to rest in a distance of D metres.

Find an expression for the stopping distance D in terms of m, U and R. **3**

Comment on how the stopping distance depends on the mass of the car. **1**

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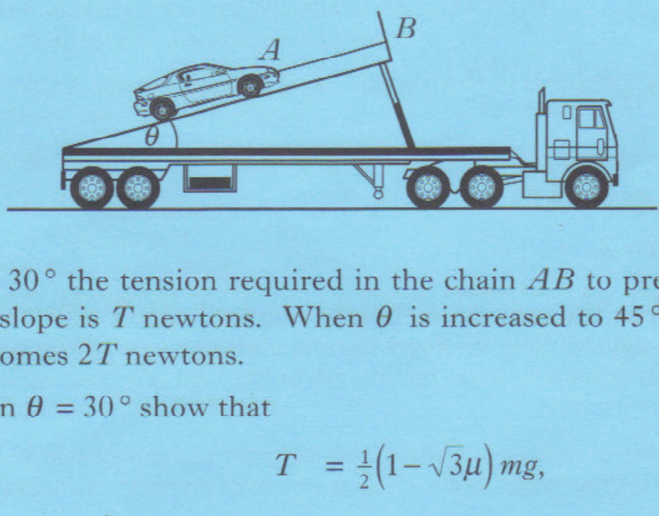

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$$T = \tfrac{1}{2}\left(1-\sqrt{3}\mu\right)mg,$$

where $g\,\mathrm{m\,s^{-2}}$ is the magnitude of the acceleration due to gravity. **3**

(*b*) Find an expression for T in terms of m, μ, and g when $\theta = 45°$. **2**

(*c*) Find the value of μ. **2**

[Turn over for Questions G4 and G5 on *Page twenty*

Marks

G4. A light aircraft is travelling due north at a constant altitude of 1 km with constant speed $100\sqrt{2}$ km/h. At 1 pm a helicopter is $50\sqrt{2}$ km due west of the aircraft, and travelling in a north easterly direction at a constant altitude of 2 km with constant speed 100 km/h.

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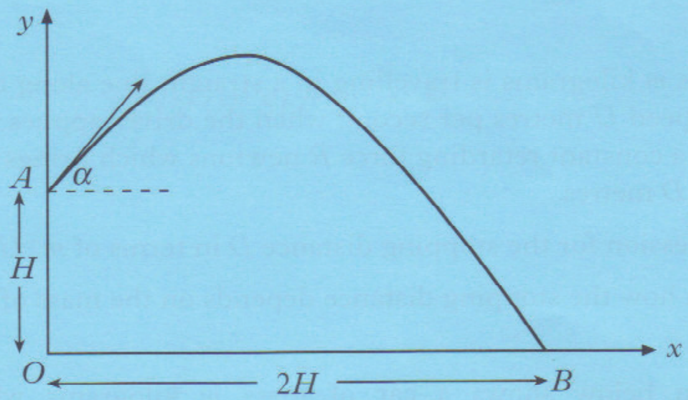

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(*b*) The shell lands at the point B on the ground, a horizontal distance of $2H$ metres from O, as shown.

Show that $\tan\alpha = 2$. **3**

(*c*) Show further that the maximum height above the ground attained by the shell is $\frac{9}{5}H$ metres. **4**

[*END OF SECTION G*]

[*END OF QUESTION PAPER*]