SECTION E (Mechanics 1)

E1. (a) From the equation of motion for the vertical motion

$$\dot{y} = V \sin 45° - gt = \frac{1}{\sqrt{2}}V - gt. \qquad \mathbf{1}$$

The shell attains its maximum height when

$$\dot{y} = 0 \Rightarrow V = \sqrt{2}\,gt = 69.3 \text{ m s}^{-1}. \qquad \mathbf{1}$$

(b) The shell hits the ground again after 10 seconds. From the equation of motion for horizontal motion

$$x = V \cos 45° t = \frac{1}{\sqrt{2}} Vt. \qquad \mathbf{1}$$

The range is

$$R = \frac{1}{\sqrt{2}} Vt \approx 490 \text{ m}. \qquad \mathbf{1}$$

E2. (a) The position of the car is

$$x_C = \tfrac{1}{2} at^2, \qquad \mathbf{1}$$

and the position of the lorry

$$x_L = Ut + \tfrac{1}{4} at^2. \qquad \mathbf{1}$$

When the car and the lorry draw level

$$x_C = x_L \qquad \mathbf{1}$$

$$\Leftrightarrow t\left(\tfrac{1}{4}at - U\right) = 0$$

$$\Leftrightarrow t = 0 \text{ or } t = \frac{4U}{a}$$

and as $t > 0$ we take $t = \dfrac{4U}{a}$. **1**

(b) When the car draws level with the lorry it has travelled

$$x_C = \frac{1}{2} a\left(\frac{4U}{a}\right)^2 = \frac{8U^2}{a}. \qquad \mathbf{1}$$

E3. (a) Resolving perpendicular to the plane

$$N + P \sin 30° = mg \cos 30° \qquad \mathbf{1}$$

$$\Rightarrow N = \sqrt{3}\,g - \tfrac{1}{2}P$$

$$= \tfrac{1}{2}\left(2\sqrt{3}\,g - P\right). \qquad \mathbf{1}$$

The frictional force is

$$F = \mu N = \tfrac{1}{4}\left(2\sqrt{3}\,g - P\right). \qquad \mathbf{1}$$

(b) Resolving parallel to the plane and using Newton II

$$P \cos 30° = mg \sin 30° + F \qquad \mathbf{1}$$

$$\Leftrightarrow \frac{\sqrt{3}}{2}P = g + \frac{1}{4}\left(2\sqrt{3}\,g - P\right)$$

$$\Leftrightarrow \tfrac{1}{2}\left(\sqrt{3} + \tfrac{1}{2}\right)P = \left(1 + \tfrac{1}{2}\sqrt{3}\right)g \qquad \mathbf{1}$$

$$\Leftrightarrow P = \frac{2\left(2 + \sqrt{3}\right)g}{\left(2\sqrt{3} + 1\right)} \approx 16{\cdot}4 \text{ N}. \qquad \mathbf{1}$$

E4. (a) Resolving forces horizontally gives

$$T_1 \cos 30° = T_2 \cos 60° \qquad \mathbf{1}$$

$$\Rightarrow \frac{\sqrt{3}}{2} T_1 = \frac{1}{2} T_2$$

$$\Rightarrow T_2 = \sqrt{3}\, T_1 > T_1. \qquad \mathbf{1}$$

(b) Resolving forces vertically and using Newton II

$$ma = T_1 \sin 30° + T_2 \sin 60° - mg \qquad \mathbf{1}$$

$$\Rightarrow \frac{1}{2} T_1 + \frac{\sqrt{3}}{2} T_2 = m(a + g) \qquad \mathbf{1}$$

$$\frac{1}{2} \frac{1}{\sqrt{3}} T_2 + \frac{\sqrt{3}}{2} T_2 = m(a + g) \qquad \mathbf{1}$$

$$\frac{1}{2}\left(\frac{1}{\sqrt{3}} + \sqrt{3}\right) T_2 = m(a + g)$$

$$\Rightarrow T_2 = \frac{\sqrt{3}}{2} m(a + g) \qquad \mathbf{1}$$

E5. (a) Since $\mathbf{a}_A = -\frac{2}{5}t\mathbf{i}$, $\mathbf{v}_A(t) = -\frac{1}{5}t^2\mathbf{i} + \mathbf{c}$.

Since $\mathbf{v}_A(0) = 10\mathbf{i}$, we have $\mathbf{c} = 10\mathbf{i}$ so

$$\mathbf{v}_A(t) = \left(10 - \tfrac{1}{5}t^2\right)\mathbf{i} \qquad \mathbf{1}$$

Integrating again gives

$$\mathbf{r}_A(t) = \left(10t - \tfrac{1}{15}t^3\right)\mathbf{i} + \mathbf{c}_2$$

but since $\mathbf{r}(0) = \mathbf{0}$ then $\mathbf{c}_2 = \mathbf{0}$ and

$$\mathbf{r}_A(t) = \frac{t}{15}\left(150 - t^2\right)\mathbf{i} \qquad \mathbf{1}$$

(b)(i)

$$\dot{\mathbf{r}}_B = \tfrac{1}{15}\left\{75 - 3t^2\right\}\mathbf{i} = \mathbf{0} \text{ when} \qquad \mathbf{1}$$

$$3t^2 = 75. \qquad \mathbf{1}$$

$$t = 5$$

When $t = 5$ $\quad \mathbf{r}_B = \tfrac{1}{15}\left\{45 + 375 - 125\right\}\mathbf{i} + 4\mathbf{j}$

$$= \tfrac{59}{3}\mathbf{i} + 4\mathbf{j}. \qquad \mathbf{1}$$

So the distance from the origin = $\sqrt{\left(\frac{59}{3}\right)^2 + 4^2} \approx 20.1$ m $\qquad \mathbf{1}$

(ii)

$$\mathbf{r}_A - \mathbf{r}_B = \tfrac{1}{15}t\left(150 - t^2\right)\mathbf{i} - \tfrac{1}{15}\left(45 + 75t - t^3\right)\mathbf{j} - 4\mathbf{j}$$

$$= \tfrac{1}{15}\left(75t - 45\right)\mathbf{i} - 4\mathbf{j} = \left(5t - 3\right)\mathbf{i} - 4\mathbf{j} \qquad \mathbf{1}$$

$$|\mathbf{r}_A - \mathbf{r}_B|^2 = (5t - 3)^2 + 16 \qquad \mathbf{1}$$

To find the minimum value

$$\frac{d}{dt}\left(|\mathbf{r}_A - \mathbf{r}_B|^2\right) = 2(5t - 3) \times 5 = 0 \qquad \mathbf{1}$$

so the minimum occurs when $t = \frac{3}{5}$. $\qquad \mathbf{1}$

The minimum distance is then $\sqrt{16} = 4$ m. $\qquad \mathbf{1}$

[END OF MARKING INSTRUCTIONS]