Advanced Higher Applied 2003: Section G Solutions and marks

G1.

We are given that $\frac{d^2x}{dt^2} = 12 - 3t^2, v(0) = 0, s(0) = 0$

$$\Rightarrow \quad v(t) = 12t - t^3 \qquad \mathbf{1}$$

$$\Rightarrow \quad s(t) = 6t^2 - \frac{1}{4}t^4. \qquad \mathbf{1}$$

When the particle comes to rest

$$v(t) = 0 \quad \Rightarrow \quad 12t - t^3 = 0$$
$$\Rightarrow \quad t^2 = 0 \text{ or } t^2 = 12$$
$$\Rightarrow \quad t = 2\sqrt{3} \text{ (since } t > 0). \qquad \mathbf{1}$$

The position at this time is

$$s(2\sqrt{3}) = 6 \times 12 - \frac{1}{4} \times 12^2 = 72 - 36 = 36 \text{ m} \qquad \mathbf{1}$$

G2.

(a) Given $\mathbf{a}_A = -2\mathbf{j}$; $\mathbf{v}_A(0) = \mathbf{i}$; $\mathbf{r}_A(0) = -\mathbf{i}$

$$\mathbf{v}_A(t) = -2t\mathbf{j} + \mathbf{c} = \mathbf{i} - 2t\mathbf{j} \qquad \mathbf{1}$$

$$\Rightarrow \quad \mathbf{r}_A(t) = t\mathbf{i} - t^2\mathbf{j} - \mathbf{i} = (t-1)\mathbf{i} - t^2\mathbf{j} \qquad \mathbf{1}$$

(b) (i)

$$_A\mathbf{r}_B = \mathbf{r}_A - \mathbf{r}_B = (2-t)\mathbf{i} - \mathbf{j} \qquad \mathbf{1}$$

(ii) The square of the distance between A and B is

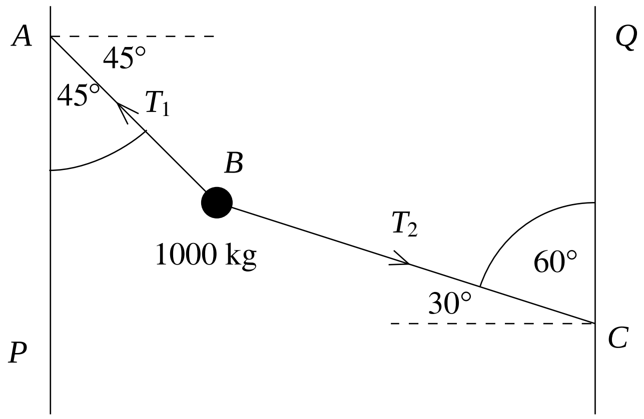
$$|_A\mathbf{r}_B|^2 = (2-t)^2 + 1. \qquad \mathbf{1}$$

This has minimum when $t = 2$, **1**

and the minimum distance is 1 metre. **1**

(Alternatively: **1** for differentiating and getting $t = 2$ and **1** for min. distance.)

G3.



(a) Resolving forces horizontally

$$T_1 \cos 45° = T_2 \cos 30° \qquad \mathbf{1}$$

$$\frac{T_1}{\sqrt{2}} = \frac{\sqrt{3}}{2}T_2$$

$$T_1 = \frac{\sqrt{3}}{\sqrt{2}}T_2 \qquad \mathbf{1}$$

(b) Resolving vertically

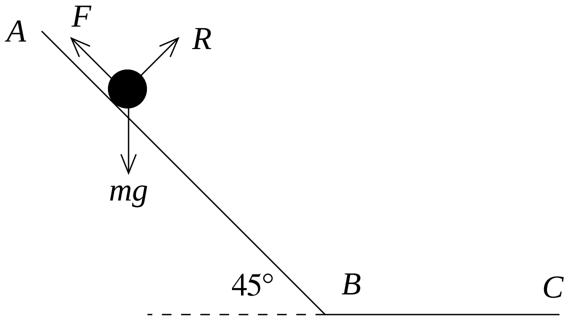
$$T_1 \sin 45° = 1000g + T_2 \sin 30° \qquad \mathbf{1}$$

$$\frac{1}{\sqrt{2}}T_1 - \frac{1}{2}T_2 = 1000g$$

$$\frac{1}{2}\left(\sqrt{3} - 1\right)T_2 = 1000g \qquad \mathbf{1}$$

$$T_2 = \frac{2000g}{\sqrt{3} - 1} \approx 26774 \text{ N} \qquad \mathbf{1}$$

G4.



(a) Resolving perpendicular to the chute gives $R = \frac{1}{\sqrt{2}}mg$ so
$F = \frac{1}{2} \times \frac{1}{\sqrt{2}} mg = \frac{mg}{2\sqrt{2}}$ **1**

Over section AB, applying Newton II

$$ma = mg \sin 45° - \frac{1}{2\sqrt{2}}mg \qquad \mathbf{1}$$

$$\Rightarrow \qquad a = \frac{g}{2\sqrt{2}}. \qquad \mathbf{1}$$

The speed of Jill at B, v_B, is given by $v_B^2 = 2aL = \frac{gL}{\sqrt{2}} \Rightarrow v_B = \sqrt{\frac{gL}{\sqrt{2}}}$. **1**

(b) Over the section BC, applying Newton II

$$ma_{BC} = -\tfrac{1}{2}mg$$

$$a_{BC} = -\frac{1}{2}g. \qquad \mathbf{1}$$

so that at C

$$v_C^2 = \frac{gL}{\sqrt{2}} + 2\left(\frac{-g}{2}\right) \times \frac{L}{2} \qquad \mathbf{1}$$

$$= \frac{gL}{2}\left(\sqrt{2} - 1\right) \qquad \mathbf{1}$$

$$\Rightarrow \qquad v_C = \sqrt{\frac{gL}{2}\left(\sqrt{2} - 1\right)}.$$

G5. (a) $\mathbf{V} = V(\cos 30°\mathbf{i} + \sin 30°\mathbf{j}) = \frac{1}{2}V(\sqrt{3}\mathbf{i} + \mathbf{j})$ or for V_y only. **1**

The y-component of the equation of motion gives

$$\ddot{y} = -g \Rightarrow \dot{y} = \frac{V}{2} - gt \qquad \mathbf{1}$$

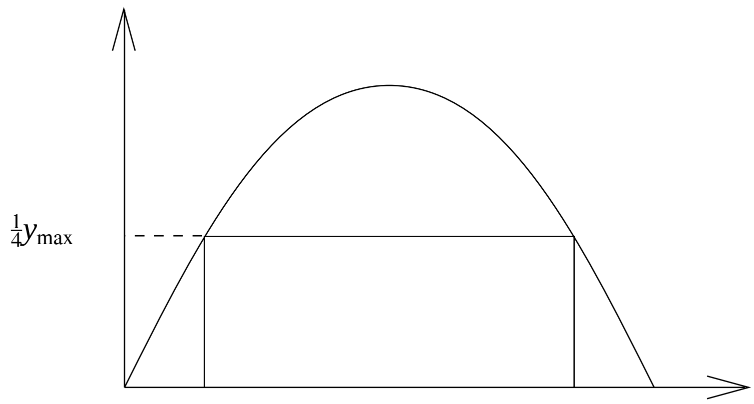
$$\Rightarrow y = \frac{Vt}{2} - \frac{1}{2}gt^2 = \frac{t}{2}(V - gt). \qquad \mathbf{1}$$

(b) Note that $\dot{y} = \frac{1}{2}V - gt$ so the maximum height occurs when $t = \frac{V}{2g}$. **1**

Hence

$$y_{\max} = \frac{V}{4g}\left(V - \frac{V}{2}\right) = \frac{V^2}{8g}. \qquad \mathbf{1}$$

(c)



We need the times when $y = \frac{1}{4}y_{\max}$.

$$\Rightarrow \frac{1}{2}Vt - \frac{1}{2}gt^2 = \frac{V^2}{32g} \qquad \mathbf{1}$$

$$\Rightarrow t^2 - \frac{V}{g}t + \frac{V^2}{16g^2} = 0 \qquad \mathbf{1}$$

$$\Rightarrow t = \frac{1}{2}\left[\frac{V}{g} \pm \left(\frac{V^2}{g^2} - \frac{V^2}{4g^2}\right)^{1/2}\right] \qquad \mathbf{1}$$

$$= \frac{V}{2g}\left[1 \pm \frac{\sqrt{3}}{2}\right] \qquad \mathbf{1}$$

The time the missile appears on the radar is

$$\frac{V}{2g}\left[1 + \frac{\sqrt{3}}{2}\right] - \frac{V}{2g}\left[1 - \frac{\sqrt{3}}{2}\right] \qquad \mathbf{1}$$

$$= \frac{\sqrt{3}V}{2g}.$$

[END OF MARKING INSTRUCTIONS]