Part Two: Marking Instructions for each Question

Section A

Question			Expected Answer(s)	Max Mark	Additional Guidance
A	**1**		$I = Ft = mv - mu$ $3(4) = 2v - 0$ $v = 6\text{ms}^{-1}$ Conservation of Linear Momentum: $m u_1 + m_2 u_2 = (m_1 + m_2)v$ $2(6) + m(0) = (2 + m)(3 \cdot 75)$ $m = \frac{12}{3 \cdot 75} - 2 = 1 \cdot 2\text{kg}$ **<u>ALTERNATIVE SOLUTION</u>** $F = ma$ $3 = 2a$ $a = \frac{3}{2}\text{m s}^{-2}$ u = 0 t = 4 a = $\frac{3}{2}$ v = u + at $= 0 + \frac{3}{2}(4) = 6\text{ m s}^{-1}$ Conservation of Linear Momentum: $m u_1 + m_2 u_2 = (m_1 + m_2)v$ $2(6) + m(0) = (2 + m)(3 \cdot 75)$ $m = \frac{12}{3 \cdot 75} - 2 = 1 \cdot 2\text{kg}$	**4**	**M1**: Use of impulse to calculate velocity of impact **E1:** Value for velocity of impact **M1:** Conservation of Linear Momentum **E1:** Value of *m* **M1**: Use of Newton's 2nd Law to calculate acceleration **E1:** Stuva to calculate velocity of *P* before impact **M1:** Conservation of Linear Momentum **E1:** Value of *m*

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A	**2**		$T=\dfrac{2\pi}{\omega}\Rightarrow\dfrac{2\pi}{\omega}=\dfrac{14\pi}{5}\Rightarrow\omega=\dfrac{5}{7}$ $v^2=\omega^2(a^2-x^2)$ $2\cdot5^2=\dfrac{25}{49}(a^2-1\cdot2^2)$ $\dfrac{2\cdot5^2\times49}{25}+1\cdot2^2=a^2$ $a=3\cdot7$ metres $x=A\sin\omega t$ $1\cdot2=3\cdot7\sin\left(\dfrac{5t}{7}\right)$ $t=0\cdot46$ seconds	**4**	**E1:** Value of ω **M1:** Correct formula for velocity and amplitude and correct substitution **E1:** Value for amplitude **M1:** Use of formula to find displacement and answer

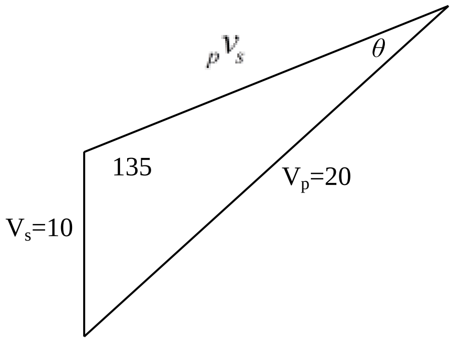
Question			Expected Answer(s)	Max Mark	Additional Guidance
A	**3**	**(i)** **(ii)**	$W = \int_0^{200} (F - 500)dx$ $= \int_0^{200} (3000 - 15x - 500)dx$ $= \left[2500x - \frac{15x^2}{2}\right]_0^{200}$ $= 200000 \text{ J} = 200 \text{ kJ}$ Work- Energy Principle: $W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$ $200000 = \frac{1}{2} \times 700 v^2$ $v = 23 \cdot 9 \text{ m s}^{-1}$ ***Alternative solution for (ii)*** $F = ma \Rightarrow a = \frac{F}{m}$ $v\frac{dv}{dx} = \frac{1}{700}\int (2500 - 15x)dx$ $F = ma$ $v\frac{dv}{dx} = \frac{1}{700}(2500 - 15x)$ $\int vdv = \frac{1}{700}\int (2500 - 15x)dx$ $\frac{v^2}{2} = \frac{1}{700}\left[2500 - \frac{15x^2}{2}\right]_0^{200}$ $v = 23 \cdot 9 \text{ ms}^{-1}$	**2** **2**	**M1**: Method of finding work done $\int F dx$ (with limits or calculation of constant included later) **E1**: Correct answer **M1**: Use of Work – Energy Principle and substitution **E1**: Correct calculation of speed **M1**: Use of correct differential equation and substitution **E1**: Correct calculation of speed

Question			Expected Answer(s)	Max Mark	Additional Guidance
A	**4**		↑ Equilibrium $2T\cos 30° + T\cos 50° = 2g$ $2T\cos 30° + T\cos 50° = 2g$ $T = \dfrac{2g}{2\cos 30° + \cos 50°} = 8\cdot 25N$ $2T = 16\cdot 5N$ $\rightarrow\ F = \dfrac{mv^2}{r}$ $2T\sin 30° + T\ \sin 50° = \dfrac{2v^2}{r}$ $\tan 50° = \dfrac{r}{0\cdot 3}\quad r = 0\cdot 358$ $2T\sin 30° + T\sin 50° = \dfrac{2v^2}{0\cdot 358}$ $2v^2 = 0\cdot 358(16\cdot 5\times\dfrac{1}{2} + 8\cdot 25\times 0\cdot 766)$ $v = 1\cdot 61\text{ m s}^{-1}$	**6**	**M1**: Consider equilibrium involving both tensions and weight **E1**: Correct substitution of components **E1**: Using conditions to find tension **M1**: Horizontal use of $F = \dfrac{mv^2}{r}$ (Consistent with M1 above) **E1**: Calculation of radius of circle **E1:** Algebraic manipulation to find v

Note:
If angular speed used can achieve 3/4.

Question			Expected Answer(s)	Max Mark	Additional Guidance
A	**5**		Perpendicular to slope: $R = Mg\cos\theta = \frac{4Mg}{5}$ Along slope: $F = ma$ $-\mu R - Mg\sin\theta = Ma$ $\frac{-4Mg}{4\times 5} - \frac{3Mg}{5} = Ma$ $a = \frac{-4g}{5}$ $(= -7\cdot 84)$ Motion under constant acceleration up slope to rest: $v^2 = u^2 + 2as$ $0 = u^2 - \frac{8gs}{5}$ $s = \frac{5u^2}{8g}$ Consider motion down slope: $F = ma$ $Mg\sin\theta - \frac{Mg\cos\theta}{4} = Ma$ $a = \frac{2g}{5}$ $(= 3\cdot 92)$ Constant acceleration down slope: $v^2 = u^2 + 2as$ $4u^2 = \frac{4gs}{5}$ $s = \frac{5u^2}{g}$ Distance AC = $\frac{5u^2}{g} - \frac{5u^2}{8g} = \frac{35u^2}{8g}$	**6**	**M1**: Equilibrium perpendicular to slope with equation **M1**: $F = ma$ along slope with equation **E1**: Calculation of acceleration **E1**: Calculation of displacement up slope to rest **E1**: Calculation of acceleration down slope **E1**: Calculation of displacement down slope and distance AC

Question			Expected Answer(s)	Max Mark	Additional Guidance
A	6		**Method 1:**	6	
			$r_P = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ $r_S = \begin{pmatrix} 15\cos 45° \\ 15\sin 45° \end{pmatrix}$ $v_P = \begin{pmatrix} 20\cos\theta \\ 20\sin\theta \end{pmatrix}$ $v_s = \begin{pmatrix} 0 \\ 10 \end{pmatrix}$		**M1**: Statements of displacements and velocity vectors at 3pm
			After time t $r_P = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + t\begin{pmatrix} 20\cos\theta \\ 20\sin\theta \end{pmatrix} = \begin{pmatrix} 20t\cos\theta \\ 20t\sin\theta \end{pmatrix}$ $\mathrm{r}_s = \begin{pmatrix} 15\cos 45 \\ 15\sin 45 \end{pmatrix} + \mathrm{t}\begin{pmatrix} 0 \\ 10 \end{pmatrix} = \begin{pmatrix} 10\cdot 6 \\ 10\cdot 6 + 10t \end{pmatrix}$		**E1**: Statements of displacements after t hours
			At interception $r_P = r_s$ $20t\cos\theta = 10\cdot 6$ and $20t\sin\theta = 10\cdot 6 + 10t$ $t = \dfrac{10\cdot 6}{20\cos\theta}$ $t = \dfrac{10\cdot 6}{20\sin\theta - 10}$		**M1**: Equate components
			$20\sin\theta - 10 = 20\cos\theta$ $\sin\theta - \cos\theta = \dfrac{1}{2}$ $\sqrt{2}\sin(\theta - 45)° = \dfrac{1}{2}$		**E1**: Algebraic manipulation
			$\theta = 65\cdot 7° \Rightarrow$ patrol vessel should steer on bearing $024\cdot 3°$		**E1**: Interpret answer to state bearing of interception
			$t = \dfrac{10\cdot 6}{20\cos 65\cdot 7} = 1\cdot 28$ hours = 1 hour 17 min Interception occurs at 4:17pm		**E1**: Calculation of time

Question			Expected Answer(s)	Max Mark	Additional Guidance
A	6		**(cont)** **Method 2:** ${}_p v_s$ must be in the direction *PS* for interception  $\frac{20}{\sin 135} = \frac{10}{\sin\theta}$ $\theta = \sin^{-1}\frac{10\sin 135}{20}$ = 20·7° Patrol vessel must sail (180 − 155·7) = 24·3º $v^2 = 10^2 + 20^2 - 2 \times 10 \times 20 \times \cos 24{\cdot}3$ v = 11·6 $t = \frac{15}{11{\cdot}6}$ = 1 hour 17 mins Interception occurs at 4:17pm		**M1**: For interception, relative velocity vector in direction PS **M1**: Correct diagram annotated **E1**: Use of trig **E1**: Interpret answer to state direction of interception **E1**: Find relative velocity **E1**: Calculation of time

Question			Expected Answer(s)	Max Mark	Additional Guidance
A	**6**		**(cont)** **Method 3:** $v_P = \begin{pmatrix} 20\cos\theta \\ 20\sin\theta \end{pmatrix} \quad v_s = \begin{pmatrix} 0 \\ 10 \end{pmatrix}$ ${}_p v_s = v_P - v_s = \begin{pmatrix} 20\cos\theta \\ 20\sin\theta - 10 \end{pmatrix}$ ${}_p v_s$ must be in the direction PS for interception $\begin{pmatrix} 20\cos\theta \\ 20\sin\theta - 10 \end{pmatrix} = k\begin{pmatrix} \cos 45° \\ \sin 45° \end{pmatrix}$ $k\cos 45 = 20\cos\theta$ $k\sin 45 = 20\sin\theta - 10$ $1 = \frac{20\sin\theta - 10}{20\cos\theta} \Rightarrow 20\cos\theta = 20\sin\theta - 10$ $\sin\theta - \cos\theta = \frac{1}{2}$ $\sqrt{2}\sin(\theta - 45)° = \frac{1}{2}$ $\theta = 65\cdot7° \Rightarrow$ patrol vessel should steer on bearing $024\cdot3°$ $t = \frac{10\cdot6}{20\cos 65\cdot7} = 1\cdot28$ hours = 1 hour 17 min Interception occurs at 4:17pm		**M1**: Statement of condition for interception **E1**: Expression for relative velocity vector **M1**: Equate components **E1**: Algebraic manipulation **E1**: Interpret answer to state bearing of interception **E1**: Calculation of time

Question			Expected Answer(s)	Max Mark	Additional Guidance
A	**7**		$a_L = \frac{1}{9}g \quad a_B = g$ ↓ $_B a_L = g - \frac{1}{9}g = \frac{8g}{9}$ $_B v_L = \int {_B a_L}\, dt = \frac{8g}{9}t + c$ $t = 0, v = -3\cdot5 \quad \Rightarrow v = \frac{8g}{9}t - 3\cdot5$ $_B r_L = \int {_B v_L}\, dt = \frac{4g}{9}t^2 - 3\cdot5t + k$ $t = 0, r = -1 \quad \Rightarrow {_B r_L} = \frac{4g}{9}t^2 - 3\cdot5t - 1$ $_B r_L(t) = 0$ when ball hits floor $\frac{4g}{9}t^2 - 3\cdot5t - 1 = 0$ $4gt^2 - 31\cdot5t - 9 = 0$ $39\cdot2t^2 - 31\cdot5t - 9 = 0$ $t = \frac{31\cdot5 \pm \sqrt{(-31\cdot5)^2 - 4\times39\cdot2\times-9}}{78\cdot4}$ $t = 1\cdot03$ or $t = -0\cdot22$ $t = 1\cdot03$ seconds (reject negative answer)	**7**	**M1:** Find relative acceleration **M1**: Use of calculus to find relative velocity **E1:** Correct expression for relative velocity **E1:** Correct expression for relative displacement **M1:** Statement for conditions when ball hits floor of lift **E1:** Process of calculating time **E1**: Correct answer for time

Question			Expected Answer(s)	Max Mark	Additional Guidance
A	**7**		**(cont)**		
			Second solution using relative acceleration and stuva		
			$a_L = \frac{1}{9}g \quad a_B = g$		**M1** Show understanding of motion under constant relative acceleration
			$_Ba_L = g - \frac{1}{9}g = \frac{8g}{9}$		**M1:** Find relative acceleration
			$s = 1 \quad t = \quad u = -3\cdot5 \quad v = \quad a = \frac{8g}{9}$		**M1 :**Consider motion ↓under constant relative acceleration
			$s = ut + \frac{1}{2}at^2 \qquad 1 = -3\cdot5t + \frac{4g}{9}t^2$		**E1:** *stuva* and substitution
			$4gt^2 - 31\cdot5t - 9 = 0 \qquad 39\cdot2t^2 - 31\cdot5t - 9 = 0$		**E1:** Correct quadratic equation
			$t = \frac{31\cdot5 \pm \sqrt{(-31\cdot5)^2 - 4 \times 39\cdot2 \times -9}}{78\cdot4}$		**E1:** Process of calculating time
			$t = 1\cdot03$ or $t = -0\cdot22$		
			$t = 1\cdot03$ seconds (reject negative answer)		**E1**: Correct answer for time
			Alternative solution		
			Ball $a = -g$ $v = -gt + c$ $t = 0 \; v = 3\cdot5 \Rightarrow c = 3\cdot5$ Lift $a = \frac{-g}{9}$ $v = \frac{-gt}{9} + k$ $t = 0 \; v = 0 \Rightarrow k = 0$		**M1** Vertical motion of ball and lift separately
			Ball $r = \frac{-gt^2}{2} + 3\cdot5 + c_2$ $t = 0 \; r = 0 \Rightarrow c_2 = 0$ $r = \frac{-gt^2}{2} + 3\cdot5$		**E1:** Correct expression for displacement of lift/ball after *t* secs
			Lift $r = \frac{-gt^2}{18} + k_2$ $t = 0 \; r = -1 \Rightarrow k_2 = -1$ $r = \frac{-gt^2}{18} - 1$		**E1 :** Correct expression for displacement of otherafter *t* secs
			$r_1 = r_2$ $\frac{-gt^2}{2} + 3\cdot5t = \frac{-gt^2}{18} - 1 \Rightarrow 8gt^2 - 63t - 18 = 0$		**M1:** Equating displacements
			$4gt^2 - 31\cdot5t - 9 = 0 \qquad 39\cdot2t^2 - 31\cdot5t - 9 = 0$		**E1:** Correct quadratic equation
			$t = \frac{31\cdot5 \pm \sqrt{(-31\cdot5)^2 - 4 \times 39\cdot2 \times -9}}{78\cdot4}$		**E1:** Process of calculating time
			$t = 1\cdot03$ or $t = -0\cdot22 \Rightarrow$ $t = 1\cdot03$ seconds (reject negative answer)		**E1**: Correct answer for time

Question			Expected Answer(s)	Max Mark	Additional Guidance
A	8	(a) (b)	 At Q: total energy = $\frac{1}{2}mv^2 = 1\cdot 5u^2$ At top of circle total energy: $mgh + \frac{1}{2}mv^2$ $= 3g \times 1\cdot 8 + \frac{3}{2}v^2 = 5\cdot 4g + \frac{3}{2}v^2$ For complete circles $v > 0$: $1\cdot 5u^2 > 5\cdot 4g$ $u > \sqrt{\frac{18g}{5}}$ ms^{-1} Height at any time: $0\cdot 9(1-\cos\theta)$ At rest (maximum height): Energy = $mgh = 3g \times 0\cdot 9(1-\cos\theta)$ If u = 4 : Energy at Q = $\frac{1}{2} \times 3 \times 4^2 = 24$ $24 = 3g \times 0\cdot 9(1-\cos\theta)$ $\cos\theta = 0\cdot 093$ $\theta = 84\cdot 7°$ Angle of oscillation = 169·4° Maximum tension when $\theta = 0$ $T - 3g = \frac{mv^2}{r}$ ↑ $T = 3g + \frac{3 \times 4^2}{0\cdot 9} = 82\cdot 7\text{N}$	3 7	 **M1**: Consideration of conservation of energy. **E1**: Correct statements of energy at bottom and top of circle **M1**: For complete circles $v > 0$ and find u **M1:** General expression for height at any time **M1**: Energy when rod is at rest **E1**: Equate this with energy vertically below P **E1**: Solve trig equation to find angle of oscillation **M1**: Understanding of maximum tension (stated or implied) **M1**: Use of $F = \frac{mv^2}{r}$ **E1**: Calculation of Tension

Question			Expected Answer(s)	Max Mark	Additional Guidance
A	9	(a)	**(i)** **(ii)** $v=\int a\,dt=\int 13\left(\frac{3}{8}-\frac{t}{16}\right)dt=13\left(\frac{3}{8}t-\frac{t^2}{32}\right)+c$ $t=0, 0=0 \quad \Rightarrow c=0$ $v=13\left(\frac{3}{8}t-\frac{t^2}{32}\right)$ $t=\frac{5}{2}: v=13\left(\frac{3}{8}\times\frac{5}{2}--\frac{\left(\frac{5}{2}\right)^2}{32}\right)=9\cdot 65\text{ ms}^{-1}$ $\rightarrow: R=9\cdot 65\cos 25°\times t$ $\uparrow: s=ut+\frac{1}{2}at^2$ $0=9\cdot 65\sin 25°\times t-\frac{1}{2}gt^2$ $t(4\cdot 08-4\cdot 9t)=0$ $t=0$ or $t=0\cdot 83$ $\rightarrow: R=9\cdot 65\cos 25°\times 0\cdot 83=7\cdot 26$ metres	**3** **3**	**M1:** Integration to find expression for velocity **E1:** Substitution and correct expression **E1:** Substitute for *t* and correct answer for speed **M1:** Consider motion horizontally and vertically with substitution **E1:** Value of *t* **E1:** Value of *R*
A	9	(b)	**(i)** **(ii)** $\rightarrow 7\cdot 51=10\cdot 2\cos\theta\times t \quad t=\frac{7\cdot 51}{10\cdot 2\cos\theta}$ $\uparrow: s=ut+\frac{1}{2}t^2$ $0=\frac{10\cdot 2\sin\theta\times 7\cdot 51}{10\cdot 2\cos\theta}-\frac{g}{2}\left(\frac{7\cdot 51}{10\cdot 2\cos\theta}\right)^2$ $7\cdot 51\tan\theta-2\cdot 656....\sec^2\theta=0$ $7\cdot 51\tan\theta-2\cdot 656....\tan^2\theta-2\cdot 656....=0$ $\tan\theta=2\cdot 41$ or $\tan\theta=0\cdot 41$ $\theta=67\cdot 2°$ $\quad\theta=22\cdot 3°$ $\uparrow: v^2=u^2+2as$ $s=4\cdot 51$ or $s=0\cdot 76m$ Athlete cannot jump 4·51m vertically ⇒ Take-off angle ≈ 22·3°	**3** **2**	**M1:** Consider motion → and expression for *t* **M1:** Consider motion vertically with this value of *t* and substitution **E1:** Solution of trig equation to give 2 angles of projection **E1:** Find two possible heights **E1:** Explanation of answer

Question			Expected Answer(s)	Max Mark	Additional Guidance
A	9	(b)	**(cont)**		
			Method 2:		
			$\rightarrow 7\cdot51 = 10\cdot2\cos\theta \times t \quad t = \frac{7\cdot51}{10\cdot2\cos\theta}$		**M1:** Consider motion → and expression for *t*
			$\uparrow: s = ut + \frac{1}{2}t^2$		
			$0 = \frac{10\cdot2\sin\theta \times 7\cdot51}{10\cdot2\cos\theta} - \frac{g}{2}\left(\frac{7\cdot51}{10\cdot2\cos\theta}\right)^2$		**M1:** Consider motion vertically with this value of *t* and substitution
			$\frac{7\cdot51\sin\theta}{\cos\theta} - \frac{2\cdot656....}{\cos^2\theta} = 0 \qquad [\times\cos^2\theta]$		
			$7\cdot51\sin\theta\cos\theta - 2\cdot656.... = 0$		
			$3\cdot755\sin2\theta - 2\cdot656.... = 0$		**E1:** Solution of trig equation to give 2 angles of projection
			$\sin2\theta = 0\cdot707....$		
			$2\theta = 45\cdot02.... \qquad 2\theta = 134\cdot976....$		
			$\theta = 22\cdot5° \qquad \theta = 67\cdot5°$		
			$\uparrow: v^2 = u^2 + 2as$		**E1:** Find two possible heights
			$s = 4\cdot51$ or $s = 0\cdot76m$		
			Athlete cannot jump 4·51m vertically ⇒ Take-off angle ≈ 22·5°		**E1:** Explanation of answer
			Method 3 (equation of a trajectory):		
			$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$		**M1:** Consider equation of trajectory.
			$0 = 7\cdot51\tan\theta - \frac{g(7\cdot51^2)}{2(10\cdot2^2)\cos^2\theta}$		**E1:** When $y = 0$ $x = 7\cdot51$ and $u = 10\cdot2$ arrange in suitable form and prepare to solve
			$7\cdot51\tan\theta - 2\cdot656....\sec^2\theta = 0$		
			$7\cdot51\tan\theta - 2\cdot656....\tan^2\theta - 2\cdot656.... = 0$		
			$\tan\theta = 2\cdot41$ or $\tan\theta = 0\cdot41$		**E1:** Solution of trig equation to give 2 angles of projection
			$\theta = 67\cdot2° \qquad \theta = 22\cdot3°$		
			$\uparrow: v^2 = u^2 + 2as$		**E1:** Find two possible heights
			$s = 4\cdot51$ or $s = 0\cdot76m$		
			Athlete cannot jump 4·51m vertically ⇒ Take-off angle ≈ 22·3°		**E1:** Explanation of answer

Question			Expected Answer(s)	Max Mark	Additional Guidance
A	10		$F=\frac{kmg}{v}$ $\frac{kmg}{v}-mg=mv\frac{dv}{dx}$ $v^2\frac{dv}{dx}=g(k-v)$ $\int\frac{v^2}{k-v}dv=\int gdx$ $\frac{-v^2}{2}-kv-k^2\ln\lvert k-v\rvert=gx+C$ $x=0\quad v=0:-\frac{1}{2}(0^2)-k(0)-k^2\ln\lvert k-0\rvert=0+C$ $\Rightarrow C=-k^2\ln\lvert k\rvert$ $gx=k^2\ln\left\lvert\frac{k}{k-v}\right\rvert-kv-\frac{1}{2}v^2$ At height h: $v=u\Rightarrow gh=k^2\ln\left\lvert\frac{k}{k-u}\right\rvert-ku-\frac{1}{2}u^2$ $mgh+\frac{1}{2}mu^2=mk^2\ln\left\lvert\frac{k}{k-u}\right\rvert-mku-\frac{mu^2}{2}+\frac{mu^2}{2}$ $=m\left[k^2\ln\left\lvert\frac{k}{k-u}\right\rvert-ku\right]$ $\frac{m\left[k^2\ln\left\lvert\frac{k}{k-u}\right\rvert-ku\right]}{mkg}=\frac{k}{g}\ln\left\lvert\frac{k}{k-u}\right\rvert-\frac{u}{g}$	2 8	**M1**: $F=\frac{P}{v}$ Connect power and force and substitution. **E1**: $F=ma$ and using $a=v\frac{dv}{dx}$ **M1**: Integration to find displacement and separation of variables **E1**: Process of Integration **E1**: Substitution and simplification **E1:** Final substitution and expression processed **M1**: Work done = Change of energy **E1**: Algebraic manipulation **M1**: Time = $\frac{\text{Work}}{\text{Power}}$ **E1**: Final manipulation

Question			Expected Answer(s)	Max Mark	Additional Guidance
A	10		**(cont)** ***Alternative for last 2 marks:*** Work done = $\int_0^T Fvdt = \int_0^T \frac{kmg}{v} \times vdt = \int_0^T kmgdt = kmgT$ $kmgT = \frac{1}{2}mu^2 + mgh$ $= \frac{1}{2}mu^2 + m(k^2 \ln\left\|\frac{k}{k-u}\right\| - ku - \frac{1}{2}u^2)$ $T = \frac{k}{g}\ln\left\|\frac{k}{k-u}\right\| - \frac{u}{g}$		

[*END OF SECTION A*]

Section B (Mathematics for Applied Mathematics)

Question			Expected Answer(s)	Max Mark	Additional Guidance
B	**1**		$y = 2x\sqrt{x-1}$ $\frac{dy}{dx} = 2x.\frac{d}{dx}\left(\sqrt{x-1}\right) + \sqrt{x-1} \times \frac{d}{dx}(2x)$ $= 2x.\frac{1}{2}(x-1)^{-\frac{1}{2}} + \sqrt{x-1} \times 2$ Gradient given by $\frac{dy}{dx}$ when x = 10, Gradient = $10.(9)^{-\frac{1}{2}} + \sqrt{9} \times 2$ $= \frac{28}{3}$	**4**	**1** product rule **1** first correct term **1** second correct term **1** evaluation (accept decimal equivalent to minimum of 3 sf)
B	**2**	**(a)**	$A + B = \begin{pmatrix} 4 & -7 & 6 \\ k-3 & 9 & -1 \\ 5 & 1 & 1 \end{pmatrix}$	**1**	**1** evaluation
B	**2**	**(b)**	$\det A = 1\begin{vmatrix} 0 & -1 \\ 3 & 0 \end{vmatrix} - 3\begin{vmatrix} k & -1 \\ 5 & 0 \end{vmatrix} + 4\begin{vmatrix} k & 0 \\ 5 & 3 \end{vmatrix}$ = 1(0 + 3) −3(0 + 5) + 4(3k – 0) = 12k – 12	**2**	**1** form of determinant 1 evaluation
B	**2**	**(c)**	$BC = \begin{pmatrix} 3 & -10 & 2 \\ -3 & 9 & 0 \\ 0 & -2 & 1 \end{pmatrix}\begin{pmatrix} 3 & 2 & -6 \\ 1 & 1 & -2 \\ 2 & 2 & -1 \end{pmatrix}$ $= \begin{pmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix}$	**1**	**1** evaluation
B	**2**	**(d)**	$BC = 3I$. $B = 3C^{-1}$ or $C = 3B^{-1}$	**2**	**1** identity matrix connection or mention of inverse **1** relationship correct

Question			Expected Answer(s)	Max Mark	Additional Guidance
B	**3**		$I = \int x \sin 3x \, dx$	**5**	
			$u = x \qquad dv = \sin 3x$ $du = 1 \qquad v = \int \sin 3x dx$		**1** evidence of integration by parts
			$= \frac{-1}{3} \cos 3x$		**1** correct choice of *u*, *dv*
			$I = x.\frac{-1}{3}\cos 3x - \int 1.\frac{-1}{3}\cos 3x \, dx$		**1** correct substitution
			$= \frac{-x}{3}\cos 3x + \frac{1}{3}\int \cos 3x \, dx$		
			$= \frac{-x}{3}\cos 3x + \frac{1}{9}\sin 3x$		**1** final integration correct
			$I_0^{2\pi} = \left[\frac{-x}{3}\cos 3x + \frac{1}{9}\sin 3x\right]_0^{2\pi}$		
			$= \left[\frac{-2\pi}{3}\cos 6\pi + \frac{1}{9}\sin 6\pi\right] - \left[0 + \frac{1}{9}\sin 0\right]$		
			$= \frac{-2\pi}{3}$		1 evaluation

Question			Expected Answer(s)	Max Mark	Additional Guidance
B	**4**		$\sum_{r=1}^{80} 3r^2 = 3\sum_{r=1}^{80} r^2$ using $\sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6}$ * $3\sum_{r=1}^{80} r^2 = 3\left(\frac{80(81)(2\cdot 80+1)}{6}\right)$ = 521,640	**2**	**1** correct substitution into * **1** evaluation (using incorrect formula – this mark available if of equivalent difficulty eg $\sum_{r=1}^{n} r^2 = \left(\frac{n(n+1)}{2}\right)^2$
B	**5**	**(a)**	$(e^x + 2)^4$ $= 1.(e^x)^4(2)^0 + 4(e^x)^3(2)^1 + 6(e^x)^2(2)^2$ $+ 4.(e^x)^1(2)^3 + 1.(e^x)^0(2)^4$ $= e^{4x} + 8e^{3x} + 24e^{2x} + 32e^x + 16$	**3**	Accept Binomial expansion ***or*** Pascal's Triangle **1** correct coefficients **1** correct powers of e^x and 2 **1** simplification
B	**5**	**(b)**	$\int (e^x + 2)^4 dx$ $= \int (e^{4x} + 8e^{3x} + 24e^{2x} + 32e^x + 16) dx$ $= \frac{e^{4x}}{4} = \frac{8e^{3x}}{3} + \frac{24e^{2x}}{2} + 32e^x + 16x + c$	**2**	**1** correct integration of composite function (at least one correct term involving composite exponential) **1** completion of integral (+ *c* not essential)

Question			Expected Answer(s)	Max Mark	Additional Guidance
B	**6**	**(a)**	10 000 people.	**1**	
B	**6**	**(b)**	$\frac{10\,000}{N(20\,000-N)}=\frac{A}{N}+\frac{B}{20\,000-N}$ 10 000 = $A(20\,000 - N) + BN$ $A=\frac{1}{2},\quad B=\frac{1}{2}$ Using $\frac{10\,000}{N(20\,000-N)}dN=dt$ gives $\frac{1}{2}\left(\frac{1}{N}+\frac{1}{20\,000-N}\right)dN=dt$ Integrating, $\int\left(\frac{1}{N}+\frac{1}{20\,000-N}\right)dN=\int 2dt$ $\ell nN-\ell n(20\,000-N)=2t+c$ $\ell n\frac{N}{20\,000-N}=2t+c$	**5**	**1** appropriate form of partial fractions **1** correct values of A and B **1** separate variables **1** starts integration eg $\int\frac{1}{N}dN$ correct **1** completes integration (moduli signs not required)

Question			Expected Answer(s)	Max Mark	Additional Guidance
B	**6**	**(c)**	Using $\ell n\dfrac{N}{20000-N}=2t+c$ gives $\dfrac{N}{20000-N}=e^{2t+c}$ $\dfrac{N}{20000-N}=Ke^{2t}\left(\text{where } K=e^{c}\right)$	**4**	**1** accurately converts to exponential form (stating explicitly $K=e^{c}$ not required)
			When $t = 0$, $N = 100$ $\dfrac{100}{19900}=K$ $K=\dfrac{1}{199}$		**1** interprets initial condition **1** K valve
			Hence $N=\left(20000-N\right)\dfrac{e^{2t}}{199}$ $199N = (20\,000 - N)\,e^{2t}$ $N(199 + e^{2t}) = 20\,000e^{2t}$ $N=\dfrac{20000e^{2t}}{199+e^{2t}}$		**1** correctly gathers N terms

[*END OF SECTION B***]**

[*END OF QUESTION PAPER***]**