National Qualifications 2024

2024 Mathematics of Mechanics

Advanced Higher

Question Paper Finalised Marking Instructions

These marking instructions have been prepared by examination teams for use by SQA appointed markers when marking external course assessments.

Marking Instructions for each question

Question			Generic scheme	Illustrative scheme	Max mark
1.			•[1] resolve vertically •[2] resolve horizontally •[3] calculate θ •[4] calculate T	•[1] $mg = 26\cos 50° + T\cos\theta$ •[2] $26\sin 50° = T\sin\theta$ •[3] $17.9°$ •[4] 64.8 N	**4**
Notes:					
Commonly Observed Responses:					
2.	(a)		•[1] state expression •[2] form linear equation and obtain one constant •[3] find remaining constant and state full expression	•[1] $\frac{A}{2x-1}+\frac{B}{x+1}$ •[2] $7-2x = A(x+1)+B(2x-1)$ $A = 4$ or $B = -3$ •[3] $\frac{4}{2x-1}-\frac{3}{x+1}$	**3**
Notes: Do not accept $\frac{4}{2x-1}+\frac{-3}{x+1}$					
Commonly Observed Responses:					
	(b)		•[4] integrate	•[4] $2\ln\lvert 2x-1\rvert - 3\ln\lvert x+1\rvert + c$	**1**
Notes: Do not penalise the omission of the constant of integration					
Commonly Observed Responses:					

Question			Generic scheme	Illustrative scheme	Max mark
3.			•[1] find expression for momentum after collision •[2] apply conservation of momentum •[3] calculate mass	•[1] $\frac{-30u}{3}+\frac{m_B u}{2}$ •[2] $30u=\frac{-30u}{3}+\frac{m_B u}{2}$ •[3] 80 grams	**3**
Notes: 1. •[1]may be implied by •[2].					
Commonly Observed Responses:					

Question			Generic scheme	Illustrative scheme	Max mark
4.			$\bullet^1$ start differentiation with evidence of use of quotient rule with denominator and one term of numerator correct $\bullet^2$ complete differentiation $\bullet^3$ simplify answer	$\bullet^1$ $\dfrac{3\left(1+x^2\right)-\ldots}{\left(1+x^2\right)^2}$ or $\dfrac{\ldots-3x\left(2x\right)}{(1+x^2)^2}$ $\bullet^2$ $\dfrac{3\left(1+x^2\right)-3x\left(2x\right)}{\left(1+x^2\right)^2}$ $\bullet^3$ $\dfrac{3-3x^2}{\left(1+x^2\right)^2}$	**3**
Notes: Do not award $\bullet^3$ if there is incorrect working after a correct answer e.g. erroneous simplification of the algebraic fraction					
Commonly Observed Responses:					

Question			Generic scheme	Illustrative scheme	Max mark
5.			•[1] create equation for maximum speed or acceleration •[2] create second equation and divide to find ω •[3] calculate amplitude •[4] calculate speed	•[1] $a\omega^2 = 20$ **or** $a\omega = 10$ •[2] $a\omega = 10$ **or** $a\omega^2 = 20$ **and** $\omega = 2\,\text{rad}\,\text{s}^{-1}$ •[3] 5 metres •[4] $\sqrt{96}$ or $9.80\,\text{ms}^{-1}$	**4**
Notes:					
Commonly Observed Responses:					

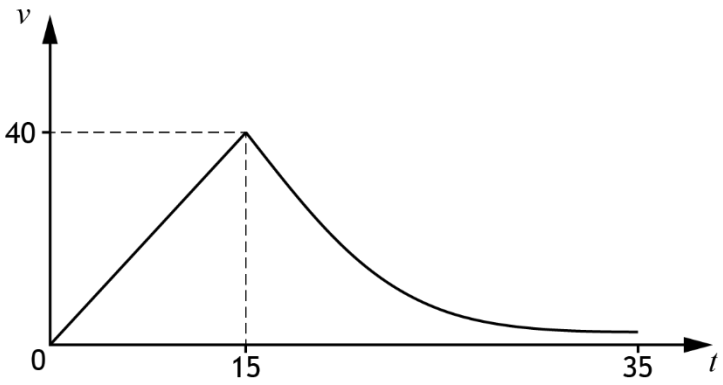
Question			Generic scheme	Illustrative scheme	Max mark
6.			•[1] start to use chain rule •[2] complete chain rule •[3] evaluate	•[1] $2\text{cosec}(3x)\times\ldots$ •[2] $2\text{cosec}(3x)\times(-\text{cosec}(3x)\cot(3x))\times 3$ •[3] 12	**3**
Notes:					
Commonly Observed Responses:					
7.			•[1] take moments about any point •[2] equate to moments in opposite direction •[3] calculate reaction force at Q	•[1] eg $22g\times 1\cdot 5+45g\times 2$ or 3R_Q •[2] eg $22g\times 1\cdot 5+45g\times 2=3\text{R}_Q$ •[3] $41g$ or 401.8 N	**3**
Notes: •[3] is unavailable if *g* is absent by •[2] stage					
Commonly Observed Responses:					
Alternative method					
			•[1] take moments about any point •[2] resolve forces vertically •[3] calculate reaction force at Q	•[1] eg $22g\times 2\cdot 5+45g\times 3=\text{R}_P+4\text{R}_Q$ •[2] eg $\text{R}_P+\text{R}_Q=22g+45g$ •[3] $41g$ or 401.8 N	**3**
Notes:					
Commonly Observed Responses:					

Question			Generic scheme	Illustrative scheme	Max mark
8.	(a)		•[1] find expression for time •[2] find expression for height •[3] substitute expression for time and simplify to required form	•[1] $t = \dfrac{x}{u\cos\theta}$ •[2] $u\sin\theta \times t - \dfrac{1}{2}gt^2$ •[3] $u\sin\theta \times \dfrac{x}{u\cos\theta} - \dfrac{1}{2}g\dfrac{x^2}{u^2\cos^2\theta}$ leading to $y = x\tan\theta - \dfrac{gx^2}{2u^2\cos^2\theta}$	**3**
Notes:					
Commonly Observed Responses:					
	(b)		•[4] substitute into trajectory equation •[5] set up quadratic equation •[6] solve for $\tan\theta$ •[7] give range of angles	•[4] $9 = 30\tan\theta - \dfrac{9.8\times 30^2}{2\times 20^2}\left(1+\tan^2\theta\right)a$ •[5] eg $11.025\tan^2\theta - 30\tan\theta + 20.025 = 0$ •[6] $\tan\theta = 1.174$ or 1.546 •[7] $49.6^\circ < \theta < 57.1^\circ$	**4**
Notes:					
Commonly Observed Responses:					

Question	Generic scheme	Illustrative scheme	Max mark
9.	•[1] differentiate implicitly with respect to t	•[1] $3\frac{dv}{dt}+\ldots$	**5**
	•[2] start to differentiate using product rule	•[2] $2te^{v}+\ldots$	
	•[3] complete differentiation	•[3] $3\frac{dv}{dt}+2te^{v}+t^{2}e^{v}\frac{dv}{dt}=0$	
	•[4] determine value of t when $v=0$	•[4] $t=3$	
	•[5] evaluate instantaneous acceleration	•[5] $-\frac{1}{2}\ \text{ms}^{-2}$	
Notes:			
Commonly Observed Responses:			
Alternative method			
	•[1] start to differentiate implicitly with respect to v using product rule	•[1] $2t\frac{dt}{dv}e^{v}$	**5**
	•[2] complete product rule	•[2] $t^{2}e^{v}$	
	•[3] complete differentiation	•[3] $3+2t\frac{dt}{dv}e^{v}+t^{2}e^{v}=0$	
	•[4] determine value of t when $v=0$	•[4] $t=3$	
	•[5] evaluate instantaneous acceleration	•[5] $-\frac{1}{2}\ \text{ms}^{-2}$	
Notes:			
Commonly Observed Responses:			

Question			Generic scheme	Illustrative scheme	Max mark
10.			•[1] consider energy at top •[2] consider energy at bottom and use conservation of energy. •[3] determine the angle •[4] determine the speed	•[1] $mgr(1-\cos\theta)$ •[2] $\frac{1}{2}mv^2 = mgr(1-\cos\theta)$ •[3] 80.2° (1.4 radians) •[4] $4.9\,\text{ms}^{-1}$	**4**
Notes:					
Commonly Observed Responses:					

Question			Generic scheme	Illustrative scheme	Max mark
11.			$\bullet^1$ find integrating factor $\bullet^2$ multiply by integrating factor and state equation $\bullet^3$ integrate and include constant of integration $\bullet^4$ find constant and state particular solution in correct form	$\bullet^1$ e^{-2x} $\bullet^2$ $\frac{d}{dx}\left(e^{-2x}y\right)=3$ $\bullet^3$ $e^{-2x}y=3x+c$ $\bullet^4$ $y=3xe^{2x}+5e^{2x}$ or equivalent	**4**
Notes: 1. Only $\bullet^1$and $\bullet^2$ available if c is omitted.					
Commonly Observed Responses:					

Question			Generic scheme	Illustrative scheme	Max mark
12.	(a)		•[1] correct shape of graph •[2] all correct annotations  		**2**
Notes:					
Commonly Observed Responses:					
	(b)		•[3] set up integral •[4] integrate •[5] evaluate integral •[6] hence determine total distance travelled	•[3] $\int_{15}^{35} 905e^{-0.20793t}\,dt$ •[4] $\left[-4352.4e^{-0.20793t}\right]_{15}^{35}$ •[5] 189 •[6] distance $= \frac{1}{2}\times 15\times 40 + 189$ $= 489\,\text{m}$	**4**
Notes: At •[3] do not penalise the omission of *dt*					
Commonly Observed Responses:					

Question			Generic scheme	Illustrative scheme	Max mark
13.			$\bullet^1$ separate the variables $\bullet^2$ integrate $\bullet^3$ determine the value of the constant of integration $\bullet^4$ calculate velocity	$\bullet^1$ $\int \frac{1}{v}dv = \int \frac{2}{1+t}dt$ $\bullet^2$ $\ln v = 2\ln(1+t) + c$ $\bullet^3$ $\ln 2$ $\bullet^4$ $32\,\text{ms}^{-1}$	**4**

Notes:

1. If constant of integration is omitted at $\bullet^2$, marks $\bullet^3$ and $\bullet^4$ are unavailable.
2. Do not award $\bullet^1$ if either dv or dt or both are omitted
3. Where a candidate attempts to integrate an expression involving v with respect to t, or vice versa, award 0/4

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
14.	(a)		$\bullet^1$ state integral	$\bullet^1$ $\sec 3x + c$	**1**
	(b)		$\bullet^2$ start integration by parts $\bullet^3$ complete integration	$\bullet^2$ $\sin^2 3x \sec 3x$ $\bullet^3$ $\sin^2 3x \sec 3x + 2\cos 3x + c$	**2**
Notes: 1. Do not withhold $\bullet^1$ or $\bullet^3$ for the omission of the constant of integration.					
Commonly Observed Responses:					

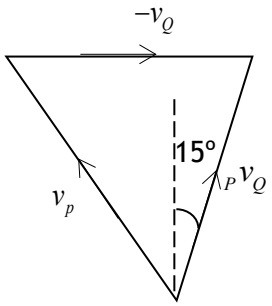
Question			Generic scheme	Illustrative scheme	Max mark
15.	(a)		$\bullet^1$ calculate distance	$\bullet^1$ 10 metres	**1**
	(b)		$\bullet^2$ calculate velocity of car after 5 seconds	$\bullet^2$ $4\,\text{ms}^{-1}$	**4**
			$\bullet^3$ obtain expression for displacement of car or motorbike	$\bullet^3$ $0.9t^2$ or $0.4t^2+4t+10$	
			$\bullet^4$ obtain second expression and equate	$\bullet^4$ $0.9t^2=0.4t^2+4t+10$	
			$\bullet^5$ calculate time	$\bullet^5$ 10 seconds	
Notes:					
Commonly Observed Responses:					
Alternative method 1					
			$\bullet^2$ obtain expression for displacement of car or motorbike	$\bullet^2$ $0.9t^2$ or $0.4(t+5)^2$	**4**
			$\bullet^3$ obtain second expression for displacement	$\bullet^3$ $0.4(t+5)^2$ or $0.9t^2$	
			$\bullet^4$ equate expressions and start to solve	$\bullet^4$ $0.9t^2=0.4t^2+4t+10$	
			$\bullet^5$ calculate time	$\bullet^5$ 10 seconds	
Notes:					
Commonly Observed Responses:					
Alternative method 2					
			$\bullet^2$ obtain expression for displacement of car or motorbike	$\bullet^2$ $0.4t^2$ or $0.9(t-5)^2$	**4**
			$\bullet^3$ obtain second expression for displacement	$\bullet^3$ $0.9(t-5)^2$ or $0.4t^2$	
			$\bullet^4$ equate expressions and start to solve	$\bullet^4$ $0.4t^2=0.9t^2-9t+22.5$	
			$\bullet^5$ calculate time	$\bullet^5$ $t=15$ leading to 10 seconds	

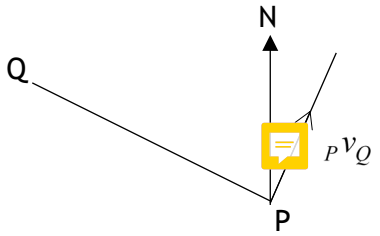
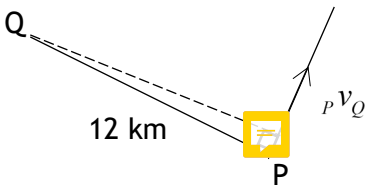
Question			Generic scheme	Illustrative scheme	Max mark
15.	(c)		$\bullet^6$ find velocity of car after 15.8 secs	$\bullet^6$ $12.64\,ms^{-1}$	**3**
			$\bullet^7$ find the distance car has to decelerate	$\bullet^7$ 190.144m	
			$\bullet^8$ calculate deceleration	$\bullet^8$ $0.42ms^{-2}$	
Notes: For $\bullet^8$ accept $a = -0.42ms^{-2}$					
Commonly Observed Responses:					

Question			Generic scheme	Illustrative scheme	Max mark
16.			•[1] find $\frac{dx}{dt}$ or $\frac{dy}{dt}$ •[2] find $\frac{dy}{dx}$ •[3] solve for t •[4] find coordinates	•[1] $\frac{dx}{dt} = 3e^{3t} - 2e^{2t}$ or $\frac{dy}{dt} = 3e^{3t} + 2e^{2t}$ •[2] $\frac{3e^{3t} + 2e^{2t}}{3e^{3t} - 2e^{2t}}$ stated or implied by •[3] $\ln 2$ •[4] (4,12)	**4**
Notes: For •[4] accept $x = 4, y = 12$					
Commonly Observed Responses:					

Question			Generic scheme	Illustrative scheme	Max mark
17.			$\bullet^1$ calculate ω	$\bullet^1$ $\frac{1}{1000}$	**5**
			$\bullet^2$ apply Newton's inverse law of gravitation at the surface of the planet	$\bullet^2$ $a = \frac{GM}{R^2}$	
			$\bullet^3$ apply Newton's inverse law of gravitation at the satellite and equate with angular acceleration	$\bullet^3$ $\frac{GM}{(pR)^2} = \omega^2 pR$	
			$\bullet^4$ combine equations and substitute value of ω	$\bullet^4$ $\frac{aR^2}{(pR)^2} = \left(\frac{1}{1000}\right)^2 pR$	
			$\bullet^5$ rearrange to the required result	$\bullet^5$ $\frac{a}{p^2} = \frac{1}{1000^2} pR$ leading to $R = \frac{1000^2 a}{p^3}$	
Notes: Accept the use of k instead of GM at $\bullet^2$ and $\bullet^3$					
Commonly Observed Responses:					

Question			Generic scheme	Illustrative scheme	Max mark
18.	(a)		$\bullet^1$ resolve forces perpendicular to plane for full box	$\bullet^1$ $R = 60g\cos 10°$	**5**
			$\bullet^2$ resolve forces parallel to plane for full box	$\bullet^2$ $P = 60g\sin 10° + \mu R$	
			$\bullet^3$ resolve forces parallel to plane for empty box	$\bullet^3$ $Q + 40g\sin 10° = \mu(40g\cos 10°)$	
			$\bullet^4$ equate P to 5Q	$\bullet^4$ $60g(\sin 10° + \mu\cos 10°) = 5(40g(\mu\cos 10° - \sin 10°))$	
			$\bullet^5$ rearrange and calculate value of μ	$\bullet^5$ $\mu = 0\cdot 327$	
Notes: 1. Appropriate working must appear after $\bullet^4$ has been awarded before the award of $\bullet^5$.					
Commonly Observed Responses:					
	(b)		$\bullet^6$ set up equation of motion for system under tension	$\bullet^6$ $300 - 60g(\mu\cos 10° + \sin 10°) = 60a$	**6**
			$\bullet^7$ calculate acceleration for system under tension	$\bullet^7$ $a = 0.142$	
			$\bullet^8$ calculate distance travelled by boxes before cable snaps	$\bullet^8$ $s = 7.1$	
			$\bullet^9$ calculate velocity of boxes at the point the cable snaps	$\bullet^9$ v=1.42	
			$\bullet^{10}$ calculate acceleration for system moving under gravity	$\bullet^{10}$ $a = -4.86$	
			$\bullet^{11}$ calculate remaining distance travelled and total distance	$\bullet^{11}$ 7.31 metres	
Notes: 1. At $\bullet^{11}$, accept any answer that rounds to 7.3.					
Commonly Observed Responses: Values with exact coefficient of friction are as follows: $a = 0.137\ldots$, $s = 6.89\ldots$, $v = 1.37\ldots$, $a = -4.86\ldots$, 7.08…					

Question			Generic scheme	Illustrative scheme	Max mark
19.	(a)	(i)	$\bullet^1$ interpret given information	$\bullet^1$ $\mathbf{v}_Q = \begin{pmatrix} -18 \\ 0 \end{pmatrix}$ ${}_P\mathbf{v}_Q = \begin{pmatrix} 20\sin 15° \\ 20\cos 15° \end{pmatrix}$	**3**
			$\bullet^2$ find vector for velocity of P	$\bullet^2$ $\begin{pmatrix} -12.8 \\ 19.3 \end{pmatrix}$	
			$\bullet^3$ find speed of P	$\bullet^3$ $23.2\,\text{kmh}^{-1}$	
		(ii)	$\bullet^4$ find direction of P	$\bullet^4$ Bearing: 326.4°	**1**
Alternative method					
	(a)	(i)	$\bullet^1$ construct triangle with Q brought to rest	$\bullet^1$ 	**3**
			$\bullet^2$ use cosine rule to start to find magnitude of vector for velocity of P	$\bullet^2$ $\sqrt{18^2 + 20^2 - 2\times 18\times 20\times \cos\ldots}$	
			$\bullet^3$ substitute correct angle of 75° and calculate speed	$\bullet^3$ $23.2\,\text{kmh}^{-1}$	
		(ii)	$\bullet^4$ find direction of P	$\bullet^4$ Bearing: 326.4°	**1**

Question			Generic scheme	Illustrative scheme	Max mark
19.	(b)		•5 express displacement for both boats after t hours •6 state and simplify expression for relative displacement •7 use appropriate method to minimise displacement •8 differentiate and equate to zero to minimise displacement •9 interpret answer to give time at which they are closest.	•5 $\mathbf{r}_P = \begin{pmatrix} -12.8t \\ 19.3t \end{pmatrix}$, $\mathbf{r}_Q = \begin{pmatrix} -12\sin 70° - 18t \\ 12\cos 70° \end{pmatrix}$ •6 ${}_Q\mathbf{r}_P = \begin{pmatrix} -5.2t - 12\sin 70° \\ 12\cos 70° - 19.3t \end{pmatrix}$ •7 $\left\|{}_Q\mathbf{r}_P\right\|^2 = (-5.2t - 12\sin 70)^2 + (12\cos 70 - 19.3t)^2$ •8 $\frac{d}{dt}\left\|{}_v\mathbf{r}_P\right\|^2 = 799.06t - 41.15 = 0$ •9 12:03 pm	**5**
Alternative method 1					
			•5 assemble facts and know to use PQ and ${}_P v_Q$ •6 establish suitable right-angled triangle for closest approach •7 calculate angle at P •8 calculate closest approach •9 state time	•5  •6  •7 85° •8 1.05 km •9 12:03 pm	**5**

Question			Generic scheme	Illustrative scheme	Max mark
19.	(b)		**(continued)**		
Alternative method 2					
				$\mathbf{r}_P = \begin{pmatrix} -12.8t \\ 19.3t \end{pmatrix}$	**5**
			•5 express displacement for both boats after t hours	•5 $\mathbf{r}_Q = \begin{pmatrix} -12\sin 70° - 18t \\ 12\cos 70° \end{pmatrix}$	
			•6 state and simplify expression for relative displacement	•6 $_Q\mathbf{r}_P = \begin{pmatrix} -5.2t - 12\sin 70° \\ 12\cos 70° - 19.3t \end{pmatrix}$	
			•7 use appropriate method to minimise displacement	•7 eg $_Q\mathbf{r}_P \cdot {}_P\mathbf{v}_Q = 0$	
			•8 find expression for scalar product	•8 $-399.76t + 20.92 = 0$	
			•9 interpret answer to give time at which they are closest.	•9 12:03 pm	

Question			Generic scheme	Illustrative scheme	Max mark
20.	(a)		•[1] apply Newton's second law with substitution for tractive force	•[1] $\frac{P}{V} - \mu R = ma$	**2**
			•[2] substitute for normal reaction force and obtain expression	•[2] $\frac{P}{V} - \mu mg = ma$ leading to $P = mV\left(a + 0.1g\right)$	
	(b)		•[3] apply Newton's second law parallel to the slope	•[3] $\frac{3P}{V} - \mu R - mg\sin\theta = ma$	**4**
			•[4] resolve perpendicular to the slope	•[4] $R = mg\cos\theta$	
			•[5] combine equations and substitute previous expression	•[5] $\frac{3mV\left(a + \mu g\right)}{V} - \mu mg\cos\theta - mg\sin\theta = ma$	
			•[6] calculate acceleration	•[6] 1.40 ms^{-2}	
Notes:					
Commonly Observed Responses:					

[END OF MARKING INSTRUCTIONS]