Marking instructions for each question

Question			Generic scheme	Illustrative scheme	Max mark
1.	(a)		•[1] evidence of product rule with one term correct [1,4] •[2] complete differentiation [1,2,3]	•[1] $6x^5\cot 5x \pm x^6(\ldots)$ OR $-5x^6\text{cosec}^2 5x + (\ldots)\cot 5x$ •[2] $6x^5\cot 5x - 5x^6\text{cosec}^2 5x$	**2**

Notes:

1. For candidates who produce a single term only, •[1] and •[2] are not available.
2. Award •[2] for final answers such as: $6x^5\cot 5x + x^6\left(-5\text{cosec}^2 5x\right)$, $6x^5\cot 5x - x^6 5\text{cosec}^2 5x$ and $6x^5\cot 5x - 5\text{cosec}^2 5x . x^6$.
3. Do not award •[2] for final answers such as: $6x^5\cot 5x + -5x^6\text{cosec}^2 5x$, $6x^5\cot 5x + x^6 - 5\text{cosec}^2 5x$ and $6x^5\cot 5x - 5\text{cosec}^2 5x\, x^6$.
4. Where a candidate equates $f(x)$ to $f'(x)$, •[1] is not available (see **COR A.**)

Commonly Observed Responses:

A. $f(x) = x^6\cot 5x$

$= 6x^5\cot 5x - 5x^6\text{cosec}^2 5x$ Award •[2] only

B. $x^6\cot 5x = x^6\tan^{-1}(5x)$

$f'(x) = 6x^5\tan^{-1}(5x) + \dfrac{5x^6}{1+(5x)^2}$ Award •[2] only

C. $f(x) = \dfrac{x^6}{\tan 5x}$

$f'(x) = \dfrac{6x^5\tan 5x - 5x^6\sec^2 5x}{(\tan 5x)^2}$ Award •[1] and •[2]

D. $f(x) = x^6(\tan 5x)^{-1}$

$f'(x) = 6x^5(\tan 5x)^{-1} - x^6(\tan 5x)^{-2}\, 5\sec^2 5x$ Award •[1] and •[2]

Question			Generic scheme	Illustrative scheme	Max mark
1.	(b)		•3 evidence use of quotient rule with denominator and one term of numerator correct •4 complete differentiation •5 simplify [1,2]	•3 $\dfrac{6x^2\left(x^3-4\right)-\ldots}{\left(x^3-4\right)^2}$ **OR** $\dfrac{\ldots-\left(2x^3+1\right)\left(3x^2\right)}{\left(x^3-4\right)^2}$ •4 $\dfrac{6x^2\left(x^3-4\right)-\left(2x^3+1\right)\left(3x^2\right)}{\left(x^3-4\right)^2}$ •5 $\dfrac{-27x^2}{\left(x^3-4\right)^2}$	3

Notes:

1. •5 is available only where candidates have multiplied out brackets and collected like terms in the numerator.
2. •5 is not available where a candidate produces further incorrect simplification subsequent to a correct answer.

Commonly Observed Responses:

A. Candidates who rewrite function as $y=2+\dfrac{9}{x^3-4}$:

- •3 $y=2+9\left(x^3-4\right)^{-1}$ stated (or implied at •4)
- •4 $-9\left(x^3-4\right)^{-2}\ldots$
- •5 $-27x^2\left(x^3-4\right)^{-2}$

B. Candidates who use product rule:

- •3 $6x^2\left(x^3-4\right)^{-1}+\left(2x^3+1\right)\ldots$ or $\ldots\left(x^3-4\right)^{-1}-3x^2\left(2x^3+1\right)\left(x^3-4\right)^{-2}$
- •4 $6x^2\left(x^3-4\right)^{-1}-3x^2\left(2x^3+1\right)\left(x^3-4\right)^{-2}$
- •5 $-27x^2\left(x^3-4\right)^{-2}$

Question			Generic scheme	Illustrative scheme	Max mark
1.	(c)		$\bullet^6$ start differentiation [1]	$\bullet^6$ $\frac{-1}{\sqrt{1-(2x)^2}}$	**3**
			$\bullet^7$ complete differentiation [1]	$\bullet^7$ $\frac{-1}{\sqrt{1-(2x)^2}}\times 2$	
			$\bullet^8$ evaluate [2,3]	$\bullet^8$ -4	

Notes:

1. At $\bullet^6$ do not accept $\frac{-1}{\sqrt{1-2x^2}}$ unless either $\frac{\cdots}{\sqrt{1-(2x)^2}}$ or $\frac{\cdots}{\sqrt{1-4x^2}}$ appears at $\bullet^7$.
2. $\bullet^8$ is available only where a candidate's answer is consistent with their stated derivative.
3. Where a candidate produces an incorrect, rounded answer; at least 2 significant figures are required for the award of $\bullet^8$.

Commonly Observed Responses:

<table>
<tr><th colspan="3">Question</th><th>Generic scheme</th><th>Illustrative scheme</th><th>Max mark</th></tr>
<tr><td>2.</td><td>(a)</td><td></td><td>$\bullet^1$ begin process [1]

$\bullet^2$ find determinant [1,2]

$\bullet^3$ equate to 3 and find p [1]</td><td>$\bullet^1$ eg $2\begin{vmatrix} p & 2 \\ -2 & 5 \end{vmatrix} - 1\begin{vmatrix} -3 & 2 \\ -1 & 5 \end{vmatrix} + 4\begin{vmatrix} -3 & p \\ -1 & -2 \end{vmatrix}$

$\bullet^2$ $14p+45$

$\bullet^3$ -3</td><td>3</td></tr>
<tr><td colspan="6">Notes:
1. Where a candidate interchanges any 2 rows, $\bullet^1$ is available only where the determinant is equated to -3. $\bullet^2$ and $\bullet^3$ are still available.
2. At $\bullet^2$ accept $2(5p+4)-1(-13)+4(6+p)$.</td></tr>
<tr><td colspan="6">Commonly Observed Responses:</td></tr>
<tr><td></td><td>(b)</td><td></td><td>$\bullet^4$ any two simplified entries [1,2]

$\bullet^5$ complete multiplication [2]</td><td>$\bullet^{4,\,5}$ $\begin{pmatrix} q+16 & 5 \\ -3q+8 & -12 \\ -2q+20 & -7 \end{pmatrix}$</td><td>2</td></tr>
<tr><td colspan="6">Notes:
1. If the order of the resultant matrix is not 3×2 award 0/2.
2. For the award of $\bullet^4$ and $\bullet^5$, accept $\begin{pmatrix} q+16 & 5 \\ pq+8 & -3+3p \\ -2q+20 & -7 \end{pmatrix}$.</td></tr>
<tr><td colspan="6">Commonly Observed Responses:</td></tr>
</table>

Question			Generic scheme	Illustrative scheme	Max mark
2.	(c)		•[6] explain [1,2]	•[6] AB is not a square matrix AND A general statement about square matrices	**1**

Notes:

1. A general statement about square matrices could take the following form:
    - Only square matrices have an inverse
    - Only square matrices have a determinant
    - Only square matrices have an identity or unit matrix
2. Where the answer contains incorrect information (before, between or after correct information), •[6] is not available.

Commonly Observed Responses:

A. Acceptable explanations:

"It's not a square matrix and inverses are only defined for square matrices".
"Since an identity matrix only exists for square matrices an inverse cannot be found. AB is not a square matrix".
"You can only find an inverse if you can find a determinant. Only 2×2 or 3×3 matrices have a determinant. Since AB is not 2×2 or 3×3, you cannot find a determinant so it has no inverse".

B. Insufficient/Unacceptable explanations

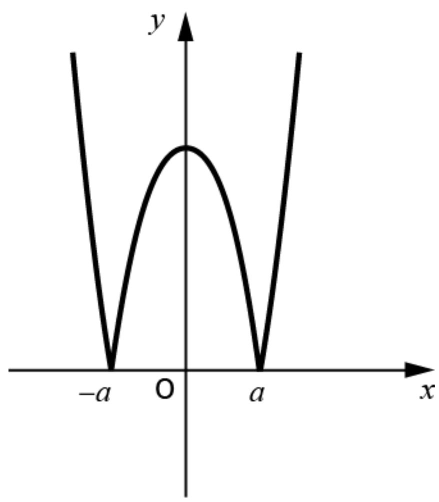
"It's not a square matrix so no inverse exists" (*restates already given information*)
" AB is not a square matrix. Only square matrices have an inverse. The determinant of AB is 0 ".
"It's not a square matrix so it has no identity matrix to invert it with".
(*meaning of second part of the statement is unclear)*
"It's not a 2×2 or a 3×3 matrix so the determinant cannot be found"
(*no general comment linking determinant and square matrices*)

Question			Generic scheme	Illustrative scheme	Max mark
3.	(a)		$\bullet^1$ state why function is even [1,2,3,4,5,6]	$\bullet^1$ graph is symmetrical about the y-axis $\therefore$ even OR $f(-x)=(-x)^2-a^2=x^2-a^2=f(x)\therefore$ even	**1**

Notes:

1. Do not accept use of the word ‘reflected’.
2. Accept phrases such as ‘symmetrical in the y-axis’, ‘symmetrical around the y-axis’ etc.
3. For justification using the graph, explicit mention of the y-axis or the line $x=0$ must be made.
4. $\bullet^1$ is not available for only stating ‘ $f(-x)=f(x)\therefore$even’ or ‘ $f(-a)=f(a)\therefore$even’.
5. $\bullet^1$ is not available for ‘ $f(-x)=-x^2-a^2=x^2-a^2=f(x)\therefore$ even’.
6. Where the answer contains incorrect information (before, between or after correct information), $\bullet^1$ is not available.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		$\bullet^2$ sketch graph [1,2,3,4]	$\bullet^2$ 	**1**

Notes:

1. The (local) maximum turning point must be on the y-axis and the graph must exhibit line symmetry.
2. Do not award $\bullet^2$ if the x intercepts are not labelled.
3. Graph must not be ‘smooth’ at x intercepts.
4. A candidate must make a reasonable attempt at reproduction when $x<-a$ and $x>a$.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
4.	(a)		•[1] complete algebraic division and express in required form	•[1] $3+\dfrac{4x+19}{x^2-x-12}$	**1**
Notes:					
Commonly Observed Responses:					
	(b)		•[2] state expression [1]	•[2] $\dfrac{A}{x+3}+\dfrac{B}{x-4}$	**3**
			•[3] form linear equation and obtain one constant	•[3] $4x+19=B(x+3)+A(x-4)$ $B=5$ or $A=-1$	
			•[4] obtain final constant and state full expression [2]	•[4] $3-\dfrac{1}{x+3}+\dfrac{5}{x-4}$	

Notes:

1. Where a candidate incorrectly factorises, •[2] is not available but •[3] and •[4] may still be awarded, including the situations illustrated in the Commonly Observed Responses.
2. Do not accept $3+-\dfrac{1}{x+3}+\dfrac{5}{x-4}$ at •[4]. Accept $3+\dfrac{-1}{x+3}+\dfrac{5}{x-4}$.

Question	Generic scheme	Illustrative scheme	Max mark

Commonly Observed Responses:

1. $3+\dfrac{4x+19}{x^2-x-12}=\dfrac{A}{x+3}+\dfrac{B}{x-4}$ — Award $\bullet^2$

$4x+19=A(x-4)+B(x+3)$

$A=-1$ **or** $B=5$ — Award $\bullet^3$

leading to a final answer of $3-\dfrac{1}{x+3}+\dfrac{5}{x-4}$ — Award $\bullet^4$

2. $3+\dfrac{4x+19}{x^2-x-12}=\dfrac{A}{x+3}+\dfrac{B}{x-4}$ — Award $\bullet^2$

$4x+19=A(x-4)+B(x+3)$

$A=-1$ or $B=5$ — Award $\bullet^3$

leading to a final answer of $-\dfrac{1}{x+3}+\dfrac{5}{x-4}$ — Do not award $\bullet^4$

3. $\dfrac{4x+19}{x^2-x-12}=\dfrac{A}{x+3}+\dfrac{Bx+C}{x-4}$ — Award $\bullet^2$

$4x+19=A(x-4)+(Bx+C)(x+3)$

$A=-1$ or $B=0$ or $C=5$ — Award $\bullet^3$

leading to $3+\dfrac{5}{x-4}-\dfrac{1}{x+3}$ — Award $\bullet^4$ (Award 2/3 if $B\neq 0$)

4. $\dfrac{3x^2+x-17}{x^2-x-12}=\dfrac{A}{x+3}+\dfrac{B}{x-4}$ — Do not award $\bullet^2$

$3x^2+x-17=A(x-4)+B(x+3)$

$A=-1$ or $B=5$ — Award $\bullet^3$ but $\bullet^4$ is not available

5. $\dfrac{3x^2+x-17}{x^2-x-12}=\dfrac{A}{x+3}+\dfrac{Bx+C}{x-4}$ — Do not award $\bullet^2$

$3x^2+x-17=A(x-4)+(Bx+C)(x+3)$

$A=-1$ or $B=3$ or $C=-7$ — Award $\bullet^3$ but $\bullet^4$ is not available

6. $\dfrac{3x^2+x-17}{x^2-x-12}=\dfrac{A}{x+3}+\dfrac{Bx+C}{x-4}$

$3x^2+x-17=A(x-4)+(Bx+C)(x+3)$

$A=-1$ or $B=3$ or $C=-7$ — Award $\bullet^2$

$\dfrac{3x-7}{x-4}=3+\dfrac{5}{x-4}$ leading to $3-\dfrac{1}{x+3}+\dfrac{5}{x-4}$ — Award $\bullet^3$ and $\bullet^4$

Question			Generic scheme	Illustrative scheme	Max mark
5.	(a)		$\bullet^1$ find $\frac{dx}{dt}$ $\bullet^2$ find $\frac{dy}{dx}$ 1	$\bullet^1$ $\frac{2}{2t+7}$ $\bullet^2$ $2t^2+7t$	**2**

Notes:

1. For $\bullet^2$ do not accept $\dfrac{t}{\dfrac{1}{2t+7}}$.

Commonly Observed Responses:

Candidates who express y explicitly as a function of x:

$\bullet^1$ $\quad y=\frac{1}{4}\left(e^x-7\right)^2$

$\bullet^2$ $\quad \frac{dy}{dx}=\frac{1}{2}\left(e^x-7\right)e^x$

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		$\bullet^3$ differentiate $\frac{dy}{dx}$ w.r.t. t and evidence of strategy 1 $\bullet^4$ find $\frac{d^2y}{dx^2}$ 1,2	$\bullet^3$ $(4t+7)\times\ldots$ $\bullet^4$ $\frac{1}{2}(2t+7)(4t+7)$	**2**

Notes:

1. $\bullet^3$ and $\bullet^4$ are not available to candidates who only differentiate $\frac{dy}{dx}$ w.r.t. t. Evidence of multiplication or division by a function of t - other than $\ln(2t+7)$ or t^2 - must be present.
2. At $\bullet^4$, accept $\frac{1}{2}\left(8t^2+42t+49\right)$.

Commonly Observed Responses:

1. Candidates who express y explicitly as a function of x.

$\frac{1}{2}\left(e^x-7\right)e^x+\frac{1}{2}e^x\left(e^x\right)$ — Award $\bullet^3$ for one term correct

$e^{2x}-\frac{7}{2}e^x$ — Award $\bullet^4$

2. Candidates who take a formula approach

$\dfrac{2\dfrac{2}{2t+7}-\ldots}{\left(\dfrac{2}{2t+7}\right)^3}$ or $\dfrac{\ldots-2t\times\left(-4(2t+7)^{-2}\right)}{\left(\dfrac{2}{2t+7}\right)^3}$ — Award $\bullet^3$

$\frac{1}{2}(2t+7)(4t+7)$ — Award $\bullet^4$

Question			Generic scheme	Illustrative scheme	Max mark
6.			•¹ evidence of relationship •² substitute [2] •³ evaluate [1,2]	•¹ $\frac{dV}{dr} = 4\pi r^2$ AND $\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$ OR $\frac{dr}{dt} = \frac{dV}{dt} \times \frac{dr}{dV}$ •² $-60 = 4\pi(3)^2 \frac{dr}{dt}$ OR $\frac{dr}{dt} = \frac{-60}{4\pi(3)^2}$ •³ $-\frac{5}{3\pi}\text{cms}^{-1}$	3

Notes:

1. At •³ units are required. Accept decimal equivalent to at least 2 significant figures ($-0\cdot53\ \text{cms}^{-1}$).
2. •² may be implied at •³.

Commonly Observed Responses:

A. Candidate attaches units to an exact value but omits them from a final answer (correctly rounded or otherwise).

$-\frac{5}{3\pi}\ \text{cms}^{-1}$ Award •³

$= -0\cdot5$

B. Candidate attaches units to an incorrect decimal approximation and not to the exact value (or appropriately rounded decimal approximation).

$-\frac{5}{3\pi}$ or $-0\cdot53$

$= -0\cdot5\ \text{cms}^{-1}$ Do not award •³

Question			Generic scheme	Illustrative scheme	Max mark
7.	(a)		•¹ find expression [1,2]	•¹ $3n^2+16n$	**1**

Notes:

1. At •¹ accept $6\times\frac{n(n+1)}{2}+13\times n$.

2. At •¹ accept $\frac{1}{2}n\left[38+6(n-1)\right]$ obtained via an arithmetic series.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		•² substitute 20 and evidence of subtraction from this term [1,2] •³ substitute for p and find expression [3]	•² $\left(3\times20^2+16\times20\right)-\ldots$ •³ $1520-3p^2-16p$	**2**

Notes:

1. Where a candidate produces further incorrect simplification, subsequent to •¹ being awarded, •² is not available.
2. Award •² for $\sum_{1}^{20}(6r+13)-\sum_{1}^{p}(6r+13)$ only where the substitution is not carried out. Disregard errors in sigma notation provided a candidate produces an answer consistent with their response to (a).
3. Do not award •³ for incorrect working subsequent to a correct answer.

Commonly Observed Responses:

A. $6\times\frac{n(n+1)}{2}+13$ incorrect expression from (a)

leading to:

$\left(3\times20^2+3\times20+13\right)-\ldots$ Award •²

$1260-3p^2-3p$ Award •³

Question			Generic scheme	Illustrative scheme	Max mark
8.			•[1] solve auxiliary equation •[2] state general solution [1] •[3] differentiate [2] •[4] form equations and solve for a constant •[5] find second constant and state particular solution [3]	•[1] $m=-4,-7$ •[2] $y=Ae^{-4x}+Be^{-7x}$ •[3] $\frac{dy}{dx}=-4Ae^{-4x}-7Be^{-7x}$ stated or implied at •[4] •[4] $A=3$ or $B=-3$ •[5] $y=3e^{-4x}-3e^{-7x}$	5

Notes:

1. Do not withhold •[2] for the omission of '$y=$'.
2. Do not withhold •[3] for the omission of '$\frac{dy}{dx}=$'.
3. To award •[5], '$y=$' must be present.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
9.	(a)		•¹ state general term [1,2,3] •² simplify powers of x or coefficients [2] •³ state simplified general term (complete simplification) [2,4,5]	•¹ $\binom{7}{r}\left(2x^2\right)^{7-r}\left(\frac{-d}{x^3}\right)^r$ •² x^{14-5r} or $2^{7-r}(-d)^r$ •³ $\binom{7}{r}2^{7-r}(-d)^r x^{14-5r}$	3

Notes:

1. Candidates may also start with a general term of $\binom{7}{r}\left(2x^2\right)^r\left(\frac{-d}{x^3}\right)^{7-r}$ to obtain a simplified general term of $\binom{7}{r}2^r(-d)^{7-r}x^{-21+5r}$.
2. Where candidates write out a full binomial expansion, •¹, •² and •³ are not available unless the general term is identifiable in (b).
3. Candidates who write down $\binom{7}{r}2^{7-r}(-d)^r x^{14-5r}$ with no working receive full marks.
4. •³ is unavailable to candidates who, in (a), produce further incorrect simplification subsequent to a correct answer eg $(-2d)^{7-2r}$.
5. Where 2^{7-r} and x^{14-5r} do not appear within a single term, •³ is not available

Commonly Observed Responses:

1. General term has not been isolated.

$$\sum_{r=0}^{7}\binom{7}{r}\left(2x^2\right)^{7-r}\left(\frac{-d}{x^3}\right)^r$$

$$=\sum_{r=0}^{7}\binom{7}{r}2^{7-r}(-d)^r x^{14-5r}$$

Do not award •¹. Award •² and •³ .

2. General term has been isolated.

$$\sum_{r=0}^{7}\binom{7}{r}\left(2x^2\right)^{7-r}\left(\frac{-d}{x^3}\right)^r$$

$$=\binom{7}{r}2^{7-r}(-d)^r x^{14-5r}$$

Disregard the incorrect use of the final equals sign. Award •¹, •² and •³ .

3. Binomial expression has been equated to general term.

$$\left(2x^2-\frac{d}{x^3}\right)^7 = \binom{7}{r}\left(2x^2\right)^{7-r}\left(\frac{-d}{x^3}\right)^r$$

Disregard the incorrect use of the equals sign. Award •¹.

4. Negative sign omitted.

$\binom{7}{r}\left(2x^2\right)^{7-r}\left(\frac{d}{x^3}\right)^r$ Do not award •¹ but •² and •³ are still available.

5. Brackets omitted around $-d$

$\binom{7}{r}2^{7-r}-d^r x^{14-5r}$ Do not award •³.

Question			Generic scheme	Illustrative scheme	Max mark
9.	(b)		•⁴ obtain value of r [1,2] •⁵ find value of d [3]	•⁴ $r=3$ •⁵ $d=5$	**2**

Notes:

1. The alternative expansion leads to $r=4$.
2. Where a candidate writes out a full expansion •⁴ may be awarded only where this is complete and correct at least as far as the required term (in either direction).
3. Where a candidate obtains an incorrect binomial expansion, •⁵ will be available only where the evaluation of a root is required.

Commonly Observed Responses:

Binomial expansion:

$$128x^{14}-448dx^{9}+672d^{2}x^{4}-560d^{3}x^{-1}+280d^{4}x^{-6}-84d^{5}x^{-11}+14d^{6}x^{-16}-d^{7}x^{-21}$$

Question			Generic scheme	Illustrative scheme	Max mark
10.	(a)		•1 apply chain or product rule •2 complete differentiation •3 express $\frac{dy}{dx}$ in terms of x and y 1	•1 $2y\frac{dy}{dx}$ or $y+x\frac{dy}{dx}$ •2 $2x+2y\frac{dy}{dx}=y+x\frac{dy}{dx}$ •3 $\frac{dy}{dx}=\frac{y-2x}{2y-x}$	**3**

Notes:

1. •3 is available only where $\frac{dy}{dx}$ appears more than once, after the candidate has completed their differentiation.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		•4 equate denominator of $\frac{dy}{dx}$ to zero 1 •5 calculate values of k 1,2	•4 $2y-x=0$ •5 $k=\pm4$	**2**

Notes:

1. At •5, accept $x=\pm4$.
2. Where a candidate equates the numerator to zero, •4 and •5 are not available.

Commonly Observed Responses:

Intersection method.

	$y^2-ky+\left(k^2-12\right)=0$	Substitute for x and express in general form
•4	$\left(-k\right)^2-4\left(k^2-12\right)=0$	Communicate condition for equal roots
•5	$k=\pm4$	

Question			Generic scheme	Illustrative scheme	Max mark
11.	(a)		•¹ state counterexample [1,2]	•¹ eg when $n=4,\ n^2+n+1=21$ which is not prime	**1**

Notes:

1. A candidate must demonstrate a value of n, evaluate n^2+n+1 and communicate that this value is not prime.
2. Where the answer contains incorrect information (before, between or after correct information), •¹ is not available.

Commonly Observed Responses:

$4^2+4+1=21$, which is not prime. Award •¹
(value of n has been demonstrated)

Question			Generic scheme	Illustrative scheme	Max mark
	(b)	(i)	•² write down contrapositive statement [1,2,8]	•² If n is even then n^2-2n+7 is odd	**1**
		(ii)	•³ write down appropriate form for n **AND** substitute [1,3,4,5,9]	•³ $n=2k,\ k\in\mathbb{N}$ and $(2k)^2-2(2k)+7$	**3**
			•⁴ show n^2-2n+7 is odd [1,6,7,9]	•⁴ eg $2\left(2k^2-2k+3\right)+1$ which is odd since $2k^2-2k+3\in\mathbb{N}$	
			•⁵ communicate [1,8,9]	•⁵ contrapositive statement is true **AND** therefore original statement is true	

Notes:

1. Marks •², •³, •⁴ and •⁵ are not available to a candidate whose statement of the contrapositive begins "If n^2-2n+7…".
2. Award •² for 'If n is not odd then n^2-2n+7 is not even'.
3. At •³ accept $k\in\mathbb{Z}^+$ but do not accept $k\in\mathbb{Z}$.
4. At •³ do not accept $n=2n$.
5. At •³ the form of n must be consistent with the candidate's response to b(i).
6. Do not withhold •⁴ for the omission of $2k^2-2k+3\in\mathbb{N}$.
7. At •⁴ accept any valid expression of the form $ab+c$, where a is even, b is an integer and c is odd.
8. •⁵ is available only where a candidate's conclusion states that the contrapositive is true and links to the original statement.
9. Where a candidate's response mentions contradiction, •³, •⁴ and •⁵ are not available.

Question	Generic scheme	Illustrative scheme	Max mark

Commonly Observed Responses:

Refer to note 3 when considering any of the responses below. Where a candidate uses $n=2k+1$ *then* k *must be suitably defined eg "*k *is a whole number".*

A.	If n is odd then n^2-2n+7 is even	Do not award $\bullet^2$
	$n=2k-1,\ \ k\in\mathbb{N}$	
	$(2k-1)^2-2(2k-1)+7$	Award $\bullet^3$
	$2(2k^2-4k+5)$ which is even	Award $\bullet^4$
	The contrapositive statement is true so the original statement is true.	Award $\bullet^5$
B.	If n is odd then n^2-2n+7 is odd	Do not award $\bullet^2$
	$n=2k-1,\ \ k\in\mathbb{N}$	
	$(2k-1)^2-2(2k-1)+7$	Award $\bullet^3$
	$2(2k^2-4k+5)$ which is not odd	Do not award $\bullet^4$. $\bullet^5$ is not available.
C.	If n is even then n^2-2n+7 is even	Do not award $\bullet^2$
	$n=2k,\ \ k\in\mathbb{N}$	
	$(2k)^2-2(2k)+7$	Award $\bullet^3$
	$2(2k^2-2k+3)+1$ which is odd	Do not award $\bullet^4$. $\bullet^5$ is not available.

Question			Generic scheme	Illustrative scheme	Max mark
12.			$\bullet^1$ convert to base 10 [1]	$\bullet^1$ 276	**3**
			$\bullet^2$ method leading to a quotient of 0 or equivalent	$\bullet^2$ eg $\begin{aligned} 276 &= 7\times 39+3 \\ 39 &= 7\times 5+4 \\ 5 &= 7\times 0+5 \end{aligned}$	
			$\bullet^3$ express in base 7 [1,2,3]	$\bullet^3$ 543_7	

Notes:

1. Where a candidate converts 231_{10} into a number in base 7, $\bullet^1$ is not available.
2. At $\bullet^3$, disregard the omission of base 7.
3. A candidate who finds three, or more, remainders and writes them in reverse order may be awarded $\bullet^3$.

Commonly Observed Responses:

1.

7^3	7^2	7^1	7^0
343	49	7	1
	5	4	3

Award $\bullet^2$ for all entries in row 2 and the '5' in row 3

leading to 543 (identified) Award $\bullet^3$.

2.

$$\begin{array}{rl} 7\,\underline{|276} & \\ 7\,\underline{|39} & r\ 3 \\ 7\,\underline{|5} & r\ 4 \\ 0 & r\ 5 \end{array}$$

Award $\bullet^2$ (compare with illustrative scheme).

leading to 543 (identified) Award $\bullet^3$.

3.

$$\begin{aligned} 231 &= 7\times 33+0 \\ 33 &= 7\times 4+5 \\ 4 &= 7\times 0+4 \end{aligned}$$

leading to a final answer of 450 Do not award $\bullet^1$. Award $\bullet^2$ and $\bullet^3$.

Question			Generic scheme	Illustrative scheme	Max mark
13.			•1 separate variables and write integral equation [1]	•1 $\int \frac{1}{12-V}\,dV = \int k\,dt$	5
			•2 integrate LHS	•2 $-\ln(12-V)$	
			•3 integrate RHS [2]	•3 $kt+c$	
			•4 evaluate constant of integration [2]	•4 $-\ln 10$	
			•5 express V in terms of k and t [2,3,4]	•5 $V = 12-10e^{-kt}$	

Notes:

1. Do not award •1 where $\int \ldots dV$ and $\int \ldots dt$ do not appear.
2. For candidates who omit the constant of integration, •3 may be awarded but •4 and •5 are unavailable.
3. •5 is unavailable to candidates who omit the negative sign at •2.
4. At •5, accept $V = 12 - \frac{10}{e^{kt}}$ or $V = \frac{12e^{kt}-10}{e^{kt}}$ but do not accept the appearance of eg $e^{-kt+\ln 10}$ in the final answer.

Commonly Observed Responses:

Using integrating factor.

$\frac{dV}{dt} + kV = 12k$

IF = e^{kt} Award •1

$\frac{d}{dt}\left(Ve^{kt}\right) = 12ke^{kt}$

$Ve^{kt} = \int 12ke^{kt}\,dt$ Award •2

$Ve^{kt} = 12e^{kt} + c$ Award •3

$c = -10$ Award •4

$V = 12-10e^{-kt}$ Award •5

Question			Generic scheme	Illustrative scheme	Max mark
14.			•¹ show true when $n=1$ [1]	•¹ when $n=1$ LHS $=1!\times1=1$ RHS $=(1+1)!-1=1$ so result is true when $n=1$.	**5**
			•² assume (statement) true for $n=k$ **AND** consider whether (statement) true for $n=k+1$ [2]	•² suitable statement AND $\sum_{r=1}^{k} r!r=(k+1)!-1$ AND $\sum_{r=1}^{k+1} r!r=\ldots$	
			•³ state sum to $(k+1)$ terms using inductive hypothesis [5]	•³ $(k+1)!-1+(k+1)!(k+1)$	
			•⁴ extract $(k+1)!$ as common factor [3,5]	•⁴ $(k+1)!(k+2)-1$	
			•⁵ express sum explicitly in terms of $(k+1)$ or achieve stated aim/goal **AND** communicate [4,5,6]	•⁵ $((k+1)+1)!-1$ AND If true for $n=k$ then true for $n=k+1$. Also shown true for $n=1$ therefore, by induction, true for all positive integers n.	

Question	Generic scheme	Illustrative scheme	Max mark

Notes:

1. "RHS = 1 , LHS = 1" and/or "True for $n=1$" are insufficient for the award of $\bullet^1$. A candidate must demonstrate evidence of substitution into both expressions.
   Accept $2!-1$ for RHS.
   Where a candidate does not independently evaluate the LHS and RHS, $\bullet^1$ may still be awarded.

2. For $\bullet^2$ acceptable phrases for $n=k$ contain:
   - "If true for..."; "Suppose true for…"; "Assume true for…".

   For $\bullet^2$ insufficient phrases for $n=k$ contain:
   - "Consider $n=k$ ", "assume $n=k$ " , "let $n=k$".

   For an insufficient phrase, do not award $\bullet^2$ unless an acceptable statement subsequently appears as part of the conclusion at $\bullet^5$.

   For $\bullet^2$ unacceptable phrases for $n=k$ contain:
   - "True for $n=k$", "Consider true for $n=k$"

   For an unacceptable phrase, do not award $\bullet^2$ but $\bullet^5$ may still be available.

   For $\bullet^2$ unacceptable phrases for $n=k+1$ contain:
   - "Consider true for $n=k+1$", "true for $n=k+1$" ; "$\sum_{r=1}^{k+1} r!r=(k+2)!-1$ " (with no further working)

3. At $\bullet^4$ accept $(k+1)!(1+k+1)-1$.

4. $\bullet^5$ is unavailable to candidates who have not been awarded $\bullet^4$.

5. Full marks are available to candidates who state an aim/goal earlier in the proof and who subsequently achieve the stated aim/goal, provided $((k+1)+1)!-1$ appears at some point.

6. Following the required algebra and statement of the inductive hypothesis, the minimal acceptable response for $\bullet^5$ is:
   "Then true for $n=k+1$, but since true for $n=1$, then true for all n" or equivalent.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
15.	(a)		$\bullet^1$ verify that the line lies on one plane [1] $\bullet^2$ verify for other plane and state conclusion [2] **OR** $\bullet^1$ substitute parameter for x, y or z into both equations $\bullet^2$ solve simultaneous equations leading to parametric equations [1] **OR** $\bullet^1$ use vector product to find direction vector OR substitute eg $z=0$ to find common point $\bullet^2$ find parametric equations	$\bullet^1$ eg $2(2\lambda+3)-3(\lambda-1)-\lambda=9$ $\bullet^2$ eg $2\lambda+3+\lambda-1-3\lambda=2$; therefore the line lies on both planes **OR** $\bullet^1$ eg $2x-3y-\lambda=9$ $x+y-3\lambda=2$ $\bullet^2$ $x=2\lambda+3; y=\lambda-1; z=\lambda$ **OR** $\bullet^1$ eg $10\mathbf{i}+5\mathbf{j}+5\mathbf{k}$ OR $(3, -1, 0)$ $\bullet^2$ $(3, -1, 0)$ OR $10\mathbf{i}+5\mathbf{j}+5\mathbf{k}$ AND $x=2\lambda+3; y=\lambda-1; z=\lambda$	**2**

Notes:

1. $\bullet^2$ is available only where there is supporting algebraic evidence.
2. Where a candidate elects to substitute the parametric equations for L_1 into the equations of π_1 and π_2 and concludes that "L_1 intersects π_1 and π_2", do not award $\bullet^2$.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
15.	(b)		•³ identify vectors [1] •⁴ start to calculate angle [2,3] •⁵ calculate complement [2,4]	•³ $\begin{pmatrix}2\\1\\1\end{pmatrix}, \begin{pmatrix}-2\\4\\3\end{pmatrix}$ •⁴ $\cos\theta = \left(\frac{3}{\sqrt{6}\sqrt{29}}\right)$ •⁵ any answer which rounds to 0·229 or 13°	**3**

Notes:

1. At •³, accept the appearance of the vectors within an attempt to find a scalar or vector product.
2. For a candidate who uses $\sin^{-1}\left(\frac{3}{\sqrt{6}\sqrt{29}}\right)$ as a means of obtaining the complement directly (with no further processing) •⁴ and •⁵ may be awarded.
3. For a candidate who finds $\sin^{-1}\left(\frac{3}{\sqrt{6}\sqrt{29}}\right)$ and proceeds to find its complement, •⁴ is unavailable.
4. Do not award •⁵ where the degree symbol has been omitted.

Commonly Observed Responses:

Use of definition of vector product:

$\sin\theta = \frac{\sqrt{165}}{\sqrt{6}\sqrt{29}}$ Award •⁴

Question			Generic scheme	Illustrative scheme	Max mark
15.	(c)		•⁶ parametric equations for L_2 [2]	•⁶ $x=-2\mu+1;\ y=4\mu+3;$ $z=3\mu-2$	**4**
			•⁷ two equations for two parameters	•⁷ any two from $2\lambda+3=-2\mu+1;$ $\lambda-1=4\mu+3;\ \lambda=3\mu-2$	
			•⁸ solve for two possible parameters [1]	•⁸ eg $\mu=-1;\ \lambda=0$	
			•⁹ substitute into remaining equation and state conclusion [3]	•⁹ eg LHS $=0$, RHS $=-5$ so lines do not intersect.	

Notes:

1. Alternative responses:
   Equating x and z :
   $2\lambda+3=-2\mu+1$
   $\lambda=3\mu-2$
   leading to $\lambda=-\frac{5}{4},\ \mu=\frac{1}{4}$
   LHS $=-\frac{9}{4}$, RHS $=4$

   Equating y and z :
   $\lambda-1=4\mu+3$
   $\lambda=3\mu-2$
   leading to $\lambda=-20,\ \mu=-6$
   LHS $=-37$, RHS $=13$

2. Where candidates employ the same parameter twice leading to $x=-2\lambda+1; y=4\lambda+3;\ z=3\lambda-2$ only •⁶ may be awarded.

3. For a final response of "$0=-5$ so the lines do not intersect" do not award •⁹ unless the candidate subsequently communicates the inconsistency of $0=-5$.

Commonly Observed Responses:

A.

$z=0$, $z=-3-2$, lines do not intersect — Award •⁹

Question			Generic scheme	Illustrative scheme	Max mark
16.	(a)		•1 evidence of integration by parts 1	•1 $\frac{e^{4x}}{4}\left(x^2-2x+1\right)-\ldots$	5
			•2 complete first application 1	•2 $\ldots\int\left(2x-2\right)\frac{e^{4x}}{4}\,dx$	
			•3 second application of integration by parts 1	•3 $\ldots\left[\frac{e^{4x}}{16}\left(2x-2\right)-\frac{1}{8}\int e^{4x}\,dx\right]$	
			•4 complete integration and include limits 2	•4 $\left[\frac{e^{4x}}{4}\left(x^2-2x+1\right)\right]_0^1-\left[\frac{1}{16}\left(2x-2\right)e^{4x}-\frac{1}{32}e^{4x}\right]_0^1$	
			•5 evaluate 2,3	•5 $\frac{1}{32}\left(e^4-13\right)$	

Notes:

1. Disregard the omission of 'dx'.
2. Evidence of limits may not appear until •5.
3. Do not award •5 for a decimal approximation, unless preceded by the exact value.

Commonly Observed Responses:

	(b)		•6 correct form of integral 1,2,3	•6 $\pi\int_0^1 y^2\,dx$	3
			•7 find expression to integrate 4	•7 $16\pi\int_0^1\left(x^2-2x+1\right)e^{4x}\,dx$	
			•8 integrate and evaluate 5,6	•8 $\frac{\pi}{2}\left(e^4-13\right)$	

Notes:

1. For the award of •6, limits must appear at some point.
2. •6 is not available unless "dx" appears at some point.
3. At •6, accept $\pi\int_0^1\left[f\left(x\right)\right]^2\,dx$.
4. Evidence for the award of •7 must include all of the following:
   - 16
   - $\left(x^2-2x+1\right)$ or $\left(x-1\right)^2$
   - e^{4x}

   unless an exact value appears at •8.
5. Do not award •8 for a decimal approximation unless:

   preceded by an exact value
   OR
   •5 has been withheld for the same reason **AND** there is sufficient evidence for •7.
6. Do not award •8 for a negative volume (including eg $\frac{\pi}{2}\left(e^2-13\right)$).

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
17.	(a)		$\bullet^1$ substitute and calculate one ratio [1,2,3,4] $\bullet^2$ calculate second ratio and state common ratio [1,5]	$\bullet^1$ $\frac{-21}{63}=-\frac{1}{3}$ or $\frac{7}{-21}=-\frac{1}{3}$ $\bullet^2$ $\frac{7}{-21}=-\frac{1}{3}$ or $\frac{-21}{63}=-\frac{1}{3}$ So $r=-\frac{1}{3}$	**2**

Notes:

1. Where a candidate calculates the first three terms only, $\bullet^1$ and $\bullet^2$ are not available.
2. Where a candidate calculates the first three terms and simply states $r=-\frac{1}{3}$, award $\bullet^1$.
3. Where a candidate finds the first three terms followed by eg "$r=\frac{-21}{7}$, so $r=-\frac{1}{3}$", do not award $\bullet^1$.
4. Where a candidate calculates the first three terms and then substitutes one pair of numbers into the n^{th} term formula to calculate r, award $\bullet^1$ only.
5. For the award of $\bullet^2$, there must be evidence that the candidate has considered a second pair of terms.

Commonly Observed Responses:

A. First three terms found followed by:

$\frac{-21}{63}=-\frac{1}{3}$	Award $\bullet^1$
$-21\times\left(-\frac{1}{3}\right)=7$ so $r=-\frac{1}{3}$	Award $\bullet^2$

Question			Generic scheme	Illustrative scheme	Max mark
	(b)	(i)	$\bullet^3$ state condition [1,2]	$\bullet^3$ $\left\lvert -\frac{1}{3}\right\rvert<1$	**1**

Notes:

1. At $\bullet^3$, $-\frac{1}{3}$ may be replaced by a letter consistent with the candidate's answer in (a). However, in the case where a candidate obtains a value in (a) outside the open interval $(-1,1)$, $\bullet^3$ will be available only where they also acknowledge that there is no sum to infinity.

2. Award $\bullet^3$ only for a strict inequality, whether expressed algebraically or in words.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
17.	(b)	(ii)	•4 begin to substitute [1,2,3] •5 calculate sum [1,2,3]	•4 $\dfrac{\cdots}{1-\left(-\frac{1}{3}\right)}$ •5 $\dfrac{189}{4}$ or $47\cdot25$	2

Notes:

1. Where a candidate calculates a common ratio outwith the open interval $(-1,1)$, •4 and •5 are not available.
2. Where a candidate writes $S_n = \dfrac{63\left(1-\left(-\frac{1}{3}\right)^n\right)}{1-\left(-\frac{1}{3}\right)}$, •4 will be available only where a candidate states that as $n \to \infty$ $\left(-\frac{1}{3}\right)^n \to 0$. •5 is still available.
3. For a correct answer with no working, •4 and •5 are not available.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
17.	(c)	(i)	•6 equate ratios •7 perform algebraic manipulation leading to formation of quadratic equation [1]	•6 $\dfrac{-2x+1}{5x+8} = \dfrac{x-4}{-2x+1}$ •7 $x^2 - 8x - 33 = 0$	2

Notes:

1. Evidence for the award of •7 must include the expansion of the products of two pairs of brackets.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
		(ii)	•8 calculate second value of x •9 find first three terms	•8 $x = -3$ •9 $-7,\ 7,\ -7$	2

Notes:

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
		(iii)	•10 state S_{2n} and justify [1,2]	•10 0 since eg $2n$ is even and so pairs of terms cancel each other out	1

Notes

1. For a descriptive justification, reference must be made either to an even number of terms or to the fact that $2n$ is even (and the consequence thereof).
2. At •10 accept $S_{2n} = 0$ since $\dfrac{-7\left(1-(-1)^{2n}\right)}{1-(-1)} = 0$.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
18.	(a)	(i)	$\bullet^1$ write in Cartesian form	$\bullet^1$ $a - a\sqrt{3}i$	**1**

Notes:

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
		(ii)	$\bullet^2$ calculate modulus [1,6] $\bullet^3$ calculate argument [2,3,4] $\bullet^4$ write in polar form [1,4,5,6]	$\bullet^2$ $2a$ $\bullet^3$ $-\frac{\pi}{3}$ $\bullet^4$ $2a\left(\cos\left(-\frac{\pi}{3}\right)+i\sin\left(-\frac{\pi}{3}\right)\right)$	**3**

Notes:

1. At $\bullet^2$ accept $\sqrt{4a^2}$, but it must be simplified at $\bullet^4$.
2. For $\bullet^3$, accept any answer of the form $-\frac{\pi}{3}+2k\pi$, $k \in \mathbb{Z}$.
3. Accept an argument expressed in degrees, with or without a degree symbol.
4. Evidence for $\bullet^3$ may not appear until b(i). In this case, $\bullet^4$ is not available.
5. At $\bullet^4$, accept $w = 2a\left(\cos\frac{\pi}{3} - i\sin\frac{\pi}{3}\right)$.
6. Do not withhold $\bullet^4$ for an unsimplified modulus if $\bullet^2$ has already been withheld for the same reason.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
18.	(b)	(i)	•5 begin process [1]	•5 $z_1 = 8^{\frac{1}{3}}\left(\cos\left(-\frac{\pi}{3}\right)+i\sin\left(-\frac{\pi}{3}\right)\right)^{\frac{1}{3}}$ stated or implied by •6	4
			•6 complete process [1]	•6 $z_1 = 8^{\frac{1}{3}}\left(\cos\left(-\frac{\pi}{9}\right)+i\sin\left(-\frac{\pi}{9}\right)\right)$	
			•7 state value of k [1,2]	•7 $k=2$	
			•8 state value of m [1,2]	•8 $m=-9$	

Notes:

1. Where the operations carried out on the modulus and argument are incompatible eg cubing the modulus and dividing the argument by three, do not award •5 or •6; however, •7 and •8 are still available.

2. Where a candidate obtains a non-integer value for k or m, •7 or •8 is not available.

Commonly Observed Responses:

A.	$z_1^3 = k^3\left(\cos\frac{\pi}{m}+i\sin\frac{\pi}{m}\right)^3$ stated or implied by •6	Award •5
	$z_1^3 = k^3\left(\cos\frac{3\pi}{m}+i\sin\frac{3\pi}{m}\right)$	Award •6
B.	$w^3 = 8^3\left(\cos\left(-\frac{\pi}{3}\right)+i\sin\left(-\frac{\pi}{3}\right)\right)^3$	Do not award •5
	$w^3 = 8^3\left(\cos(-\pi)+i\sin(-\pi)\right)$	Award •6
	$k=512$	Award •7
	$m=-1$	Award •8
C.	**Answers without working:**	
	1. $k=2$ and $m=-9$	Award full marks
	2. $k=2$ and $m\neq-9$	Award •7 only
	3. $k\neq2$ and $m=-9$	Award •8 only

Question			Generic scheme	Illustrative scheme	Max mark
18.	(b)	(ii)	•[9] begin to add or subtract $\frac{2\pi}{3}$ to or from argument of z_1 •[10] state roots	•[9] $\ldots\pm\frac{2\pi}{3}$ stated or implied by •[10] •[10] $z_2 = 2\left(\cos\frac{5\pi}{9} + i\sin\frac{5\pi}{9}\right)$ $z_3 = 2\left(\cos\left(-\frac{7\pi}{9}\right) + i\sin\left(-\frac{7\pi}{9}\right)\right)$	**2**

Notes:

1. The addition of other multiples of $\frac{2\pi}{3}$, leading to other forms of roots, is acceptable.
2. Where a candidate finds one further root, consistent with adding or subtracting $\frac{2\pi}{3}$ to their response to b(i) and without working, •[9] may be awarded.
3. •[10] is available only where a candidate produces exactly two roots, with consistent spacing, distinct from one another and also from z_1.

Commonly Observed Responses:

[END OF MARKING INSTRUCTIONS]