**Detailed marking instructions for each question**

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|---|---|---|---|---|---|
| **1.** | | | •[1] choose appropriate equation of motion and substitute to find acceleration. | •[1] $v^2 = u^2 + 2as$<br>$14^2 = 10^2 + 2 \times 1200 \times a$<br>$a = \dfrac{14^2 - 10^2}{2400} = \dfrac{96}{2400} = 0 \cdot 04\text{ms}^{-2}$ | **5** |
| | | | •[2] find final velocity before deceleration. | •[2] $v = u + at = 14 + 0 \cdot 04 \times 120$<br>$= 18 \cdot 8\text{ms}^{-1}$ | |
| | | | •[3] substitution to find further distance travelled. | •[3] $s = ut + \frac{1}{2}at^2$<br>$14 \times 120 + \frac{1}{2} \times 0 \cdot 04 \times 120^2 = 1968\text{m}$ | |
| | | | •[4] find stopping distance | •[4] $v^2 = u^2 + 2as$<br>$0 = 18 \cdot 8^2 - 2 \times 0 \cdot 04 \times s$<br>$s = 4418$ | |
| | | | •[5] calculate total distance | •[5] $\text{total} = 1200 + 1968 + 4418$<br>$= 7586\text{m} \quad [7 \cdot 59\text{km}]$ | |

**Notes:**
1. accept distance answers in metres or kilometres

**Commonly Observed Responses:**

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|---|---|---|---|---|---|
| **2.** | (a) | | •[1] use correct form of partial fractions<br><br>•[2] equate numerators<br><br>•[3] find one constant<br><br>•[4] find remaining constants and state the partial fractions | •[1] $\frac{13+6x+5x^2}{(1+x)(2-x)(3+x)} = \frac{A}{1+x}+\frac{B}{2-x}+\frac{C}{3+x}$<br><br>•[2] $13+6x+5x^2 = A(2-x)(3+x) + B(1+x)(3+x) + C(2-x)(1+x)$<br><br>•[3] $A=2$ or $B=3$ or $C=-4$<br><br>•[4] $A=2,\ B=3,\ C=-4$ $\frac{2}{1+x}+\frac{3}{2-x}-\frac{4}{3+x}$ | **4** |
| **Notes:** | | | | | |
| **Commonly Observed Responses:** | | | | | |
| | (b) | | •[5] rewrite integral and integrate one term correctly<br><br>•[6] complete integration<br><br>•[7] substitute and simplify to correct form | •[5] $\int_0^1 \frac{2}{1+x}+\frac{3}{2-x}-\frac{4}{3+x}\,dx$ $=2\ln\lvert 1+x\rvert\ldots\ldots$<br><br>•[6] $\ldots-3\ln\lvert 2-x\rvert-4\ln\lvert 3+x\rvert$<br><br>•[7] $(2\ln 2-3\ln 1-4\ln 4)$ $-(2\ln 1-3\ln 2-4\ln 3)$ $=\ln\frac{81}{8}$ | **3** |
| **Notes:** | | | | | |
| **Commonly Observed Responses:** | | | | | |

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|---|---|---|---|---|---|
| 3. | | | •¹ use Newton's second law with frictional force<br><br>•² calculate the deceleration<br><br>•³ calculate speed immediately before the collision<br><br>•⁴ know to use conservation of momentum and start substitution<br><br>•⁵ calculate $v$ | •¹ $ma = -\mu R$<br>$ma = -\mu mg$<br><br>•² $a = -\mu g$<br>$a = -2\cdot45\text{ms}^{-2}$ $\left[\dfrac{-g}{4}\text{ms}^{-2}\right]$<br><br>•³ $v^2 = 12^2 + 2\times -2\cdot45\times20$<br>$v = 6\cdot78\text{ms}^{-1}$<br><br>•⁴ $m_1u_1 + m_2u_2 = (m_1+m_2)v$<br><br>•⁵ $10+6\cdot78+5\times0 = 15v$<br>$v = 4\cdot52\text{ms}^{-1}$ $\left[\dfrac{2\sqrt{46}}{3}\right]$ | 5 |

**Notes:**

1. •⁴ initial or final momentum should begin to be calculated

**Commonly Observed Responses:**

•³ $a = +2\cdot45\text{ms}^{-2}$ leading to •⁵ $v = 10\cdot4\text{ms}^{-1}$

| Question | | | | Generic scheme | Illustrative scheme | Max mark |
|---|---|---|---|---|---|---|
| **Alternative solution (work/energy principle)** | | | | | | |
| 3. | | | | •[1] consider energy at start and immediately before collision | •[1] start $E_K = \frac{1}{2}mv^2 = \frac{1}{2}\times 10\times 12^2 = 720J$<br>before collision $E_K = \frac{1}{2}mv^2$ | 5 |
| | | | | •[2] calculate work done by friction | •[2] Friction $= \mu mg = 0\cdot 25\times 10\times 9\cdot 8 = 24\cdot 5N$<br>work $= Fs = 24\cdot 5\times 20 = 490J$ | |
| | | | | •[3] use conservation of energy to calculate speed just before collision | •[3] $720 - \frac{1}{2}mv^2 = 490 \Rightarrow \frac{1}{2}\times 10\times v^2 = 230J$<br>$v = \sqrt{\frac{230\times 2}{10}} = \sqrt{46} = 6\cdot 78ms^{-1}$ | |
| | | | | •[4] know to use conservation of momentum and start substitution | •[4] $m_1u_1 + m_2u_2 = (m_1 + m_2)v$ | |
| | | | | •[5] calculate $v$ | •[5] $10 + 6\cdot 78 + 5 \times 0 = 15v$<br>$v = 4\cdot 52ms^{-1}$ | |

**Notes:**

1. •[4] initial or final momentum should begin to be calculated

**Commonly Observed Responses:**

| Question | | | Generic Scheme | Illustrative Scheme | Max Mark |
|---|---|---|---|---|---|
| 4. | | | •[1] start to use chain rule to find derivative<br><br>•[2] complete the differentiation<br><br>•[3] substitute $x = \frac{\pi}{4}$ | •[1] $f'(x) = e^{\sec^2 x} \times \frac{d}{dx}\sec^2 x$<br><br>•[2] $2\sec^2 x \tan x\, e^{\sec^2 x}$<br><br>•[3] $\sec\frac{\pi}{4} = \sqrt{2}$ $\sec^2\frac{\pi}{4} = 2$ $\tan\frac{\pi}{4} = 1$<br>$f'\left(\frac{\pi}{4}\right) = 2\times 2\times 1\times e^2$ | 3 |

**Notes:**

1. •[1] clear evidence to show multiplication by the ***derivative*** of $\sec^2 x$.

**Commonly Observed Responses:**

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|---|---|---|---|---|---|
| 5. | | | •[1] denotes quantities appropriately (via diagram or otherwise) and resolve vertically | •[1] $T\cos\theta = mg$ | 5 |
| | | | •[2] use Newton's 2nd law horizontally with circular motion | •[2] $T\sin\theta = mr\omega^2$ | |
| | | | •[3] eliminate $T$ and $m$ | •[3] $\tan\theta = \frac{r\omega^2}{g}$ | |
| | | | •[4] use $l = 2r$ to find a value for $\tan\theta$ or evaluate $\theta$ | •[4] $\tan\theta = \frac{1}{\sqrt{3}}$ $\left[\sin\theta = \frac{1}{2} \quad \theta = 30^\circ\right]$ <br> $\frac{1}{\sqrt{3}} = \frac{l\omega^2}{2g}$ | |
| | | | •[5] complete proof | •[5] $\sqrt{3}l\omega^2 = 2g$ <br> $\omega^2 = \frac{2g}{\sqrt{3}l}$ | |
| **Notes:** | | | | | |
| **Commonly Observed Responses:** | | | | | |

| Question | | | Generic Scheme | Illustrative Scheme | Max Mark |
|---|---|---|---|---|---|
| 6. | | | •1 express volume as an integral<br><br>•2 use integral with limits substitute for $y^2$<br><br>•3 integrate<br><br>•4 evaluate | •1 $V = \pi\int y^2 dx$<br><br>•2 $V = \pi\int_{-2}^{3}(9 - x^2)\,dx$<br><br>•3 $V = \pi\left[9x - \frac{1}{3}x^3\right]_{-2}^{3}$<br><br>•4 $\frac{100\pi}{3}$ [105] | 4 |
| **Notes:** | | | | | |
| **Commonly Observed Responses:** | | | | | |

| Question | | | Generic Scheme | Illustrative Scheme | Max Mark |
|---|---|---|---|---|---|
| 7. | (a) | | •[1] calculate $\omega$<br><br>•[2] state equation for position and start to solve<br><br>•[3+4] obtain values for $t$ | •[1] $\omega = \frac{2\pi}{10}$ $\quad \omega = \frac{\pi}{5}$<br><br>•[2] $x = 6\sin\frac{\pi}{5}t \quad 6\sin\frac{\pi}{5}t = 4$<br><br>•[3+4] $\frac{\pi}{5}t = 0\cdot730,\ 2\cdot41$<br><br>$t = 1\cdot16,\ 3\cdot84$ | 4 |
| **Notes:**<br>**1.** •[3+4] Horizontal and vertical marking. | | | | | |
| **Commonly Observed Responses:** | | | | | |
| | (b) | | **Method 1**<br><br>•[5] use second value of $t$ to find $v$<br><br>•[6] evaluate and interpret solution | **Method 1**<br><br>$v = a\omega\cos\omega t$<br>•[5] $v = \frac{6\pi}{5}\cos(\frac{\pi}{5}\times 3\cdot84)$<br><br>•[6] $v = -2{\cdot}81\,\text{ms}^{-1}$<br>so particle will be travelling back towards A with speed of $2{\cdot}81\text{ms}^{-1}$ | 2 |
| **Notes:**<br>•[6] only available where $v$ is negative. | | | | | |
| **Commonly Observed Responses:** | | | | | |
| | (b) | | **Method 2**<br><br>•[5] use second value of $t$ to find $v$<br><br>•[6] evaluate and interpret solution. | **Method 2**<br><br>$v^2 = \omega^2(a^2 - x^2)$<br>•[5] $v^2 = \left(\frac{\pi}{5}\right)^2(6^2 - 4^2)$<br><br>•[6] $v = -2{\cdot}81\,\text{ms}^{-1}$<br>so for second time particle will be travelling back towards $A$ with a speed of $2{\cdot}81\,\text{ms}^{-1}$. | 2 |
| **Notes:** | | | | | |
| **Commonly Observed Responses:** | | | | | |

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|---|---|---|---|---|---|
| 8. | | | •¹ find $\frac{dx}{dt}$<br><br>•² find $\frac{dy}{dt}$<br><br>•³ evaluate derivatives when $t = 3$<br><br>•⁴ substitute into appropriate formula and calculate speed | •¹ $\frac{dx}{dt} = 2t + 4$<br><br>•² $\frac{dy}{dt} = (1-t)^3 - 3t(1-t)^2$<br><br>•³ $\frac{dx}{dt}(t=3) = 10$ and $\frac{dy}{dt}(t=3) = -44$<br><br>•⁴ $\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$<br>$\sqrt{10^2 + (-44)^2} = \sqrt{2036} = 45 \cdot 1$ | 4 |
| **Notes:** | | | | | |
| **Commonly Observed Responses:** | | | | | |

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|---|---|---|---|---|---|
| 9. | (a) | | **Method 1**<br>•[1] use appropriate formula for time of half flight with substitution<br>•[2] find expression for total time of flight<br>•[3] find expression for range using total time of flight<br>•[4] simplify using double angle formula | **Method 1**<br>•[1] $v = u + at \Rightarrow 0 = v\sin\theta - gt$<br>•[2] $t = \frac{v\sin\theta}{g} \Rightarrow 2t = \frac{2v\sin\theta}{g}$<br>•[3] $R = v\cos\theta \times 2t = \frac{v\cos\theta \times 2v\sin\theta}{g}$<br>•[4] $R = \frac{v^2 \times 2\sin\theta\cos\theta}{g} = \frac{v^2\sin 2\theta}{g}$ | 4 |
| **Notes:** | | | | | |
| **Commonly Observed Responses:** | | | | | |
| | (a) | | **Method 2**<br>•[1] state horizontal range of flight and use it to give expression for $t$<br>•[2] use appropriate formula with substitution<br>•[3] solve the equation for t<br>•[4] substitute for t to give required formula | **Method 2**<br>•[1] $R = v\cos\theta \times t$<br>•[2] $s = ut + \frac{1}{2}at^2$<br>$0 = v\sin\theta \times t - \frac{1}{2}gt^2$<br>•[3] $0 = t\left(v\sin\theta - \frac{1}{2}gt\right)$<br>$[t = 0]$ or $t = \frac{2v\sin\theta}{g}$<br>•[4] $R = \frac{v\cos\theta \times 2v\sin\theta}{g} = \frac{v^2\sin 2\theta}{g}$ | 4 |
| **Notes:**<br>•[3] Do not penalise omission of $t = 0$ | | | | | |
| **Commonly Observed Responses:** | | | | | |

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|---|---|---|---|---|---|
| 9. | (a) | | **Method 3** | **Method 3** | 4 |
| | | | •1 consider horizontal and vertical motion | •1 $\ddot{x}=0 \Rightarrow \dot{x}=v\cos\theta \Rightarrow x=vt\cos\theta$<br>$\ddot{y}=-g \Rightarrow \dot{y}=-gt+v\sin\theta$<br>$\Rightarrow y=-\frac{1}{2}gt^2+vt\sin\theta$ | |
| | | | •2 set up equation for vertical motion at start and finish | •2 $y=-\frac{1}{2}gt^2+vt\sin\theta=0$ | |
| | | | •3 solve the equation for t | •3 $0=t\left(v\sin\theta-\frac{1}{2}gt\right)$<br>$[t=0]$ or $t=\frac{2v\sin\theta}{g}$ | |
| | | | •4 substitute for end value for t to give range formula | •4 $x=v\cos\theta\times t=\frac{v\cos\theta\times 2v\sin\theta}{g}$<br>$=\frac{v^2\sin 2\theta}{g}$ | |

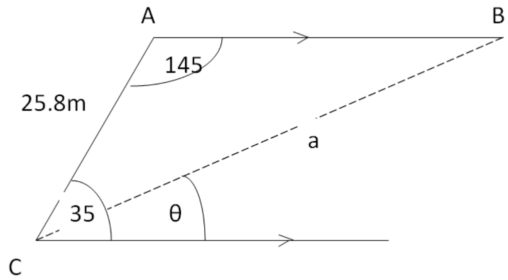
**Notes:**

•3 Do not penalise omission of $t=0$

**Commonly Observed Responses:**

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|---|---|---|---|---|---|
| 9. | (b) | (i) | •5 substitute both angles into range formula<br><br>•6 by substituting for $R$ set up equation in $v$<br><br>•7 re-arrange and solve for $v$ | •5 $R=\dfrac{v^2\sin 60°}{g}$ $R+5=\dfrac{v^2\sin 70°}{g}$<br><br>•6 $\dfrac{v^2\sin 60°}{g}+5=\dfrac{v^2\sin 70°}{g}$<br><br>•7<br>$\dfrac{v^2\left(\sin 70°-\sin 60°\right)}{g}=5$<br>$v^2=\dfrac{5g}{\sin 70°-\sin 60°}$ $[665\cdot 2]$<br>$v=25\cdot 8\text{ms}^{-1}$ | 3 |
| **Notes:** | | | | | |
| **Commonly Observed Responses:**<br>•7 not available where calculator set in radians | | | | | |
| | | (ii) | •8 calculate initial velocity when $\theta = 35°$<br><br>•9 calculate time of flight<br><br>•10 calculate range with $\theta = 35°$ | •8 $\mathbf{v}=\begin{pmatrix}25\cdot 8\cos 35°+7\\25\cdot 8\sin 35°\end{pmatrix}$ $\left[\begin{pmatrix}28\cdot 13\\14\cdot 80\end{pmatrix}\right]$<br><br>$v=u+at$<br>$0=14\cdot 8-gt$<br>•9 $t=\dfrac{14\cdot 8}{g}=1\cdot 51$<br>Total time $=3\cdot 02$<br><br>•10 $28\cdot 13\times 3\cdot 02=85\cdot 0$ metres | 3 |
| **Notes:**<br>**1.** •8 can be implied in further working and does not have to be explicitly stated<br><br>2. •10 accept 85m or 84·9m (exact values used throughout) | | | | | |
| **Commonly Observed Responses:** | | | | | |

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|---|---|---|---|---|---|
| **Alternative solution 1** | | | | | |
| | (b) | (ii) | •[8] substitute original velocity into range formula for $\theta = 35^\circ$<br><br>•[9] calculate time of flight<br><br>•[10] add on extra distance for wind assistance | •[8] $R = \frac{25 \cdot 8^2 \times \sin 70^\circ}{9 \cdot 8} = 63 \cdot 8\text{m}$<br><br>•[9] $t = \frac{2v\sin\theta}{g} = \frac{2 \times 25 \cdot 8 \times \sin 35}{9 \cdot 8} = 0 \cdot 302$<br><br>•[10] $R = 63 \cdot 8 + 7 \times 0 \cdot 302 = 84 \cdot 9\text{m}$ | 3 |
| **Notes:** | | | | | |
| **Commonly Observed Responses:** | | | | | |
| **Alternative solution 2** | | | | | |
| | | (ii) | •[8] find new horizontal component<br><br>•[9] calculate time of flight<br><br>•[10] calculate range | •[8] $\ddot{x} = 0 \Rightarrow \dot{x} = v\cos\theta + 7$<br>$\Rightarrow x = vt\cos\theta + 7t = 21 \cdot 13$<br><br>•[9] $t = \frac{2v\sin\theta}{g} = \frac{2 \times 25 \cdot 8 \times \sin 35}{9 \cdot 8} = 0 \cdot 302$<br><br>•[10] $R = 21 \cdot 13 \times 3 \cdot 02 + 7 \times 3 \cdot 02 = 85 \cdot 0\text{m}$ | 3 |
| **Notes:** | | | | | |
| **Commonly Observed Responses:** | | | | | |

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|---|---|---|---|---|---|
| **Alternative solution 3** | | | | | |
| **9.** | (b) | (ii) | |  | **3** |
| | | | •[8] calculate resultant velocity | $a^2 = b^2 + c^2 - 2bc\cos A$<br>•[8] $= 25\cdot8^2 + 7^2 - 2(25\cdot8)(7)\cos145°$<br>$= 1010\cdot5$<br>$a = 31\cdot8\text{ms}^{-1}$ | |
| | | | •[9] calculate angle | •[9] $\dfrac{\sin145°}{31\cdot8} = \dfrac{\sin C}{7} \Rightarrow C = \sin^{-1}(0\cdot126) = 7\cdot25°$<br>$\theta = 35° - 7\cdot25° = 27\cdot7°$ | |
| | | | •[10] calculate range using formula | •[10] $R = \dfrac{v^2\sin2\theta}{g} = \dfrac{31\cdot8^2\sin(2\times27\cdot7)}{9\cdot8} = 85\cdot0\text{m}$ | |
| **Notes:** | | | | | |
| **Commonly Observed Responses:** | | | | | |

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|---|---|---|---|---|---|
| **10.** | (a) | | **Method1: Relative to A** | | **6** |
| | | | •1 derive expressions for the mass and centres of mass of the original lamina and the circular hole | •1 Original Lamina: $16\pi m$ $(4,0)$<br>Circular hole: $\pi m$ $(2,1)$ | |
| | | | •2 derive expressions for the mass and centres of mass of the semi-circular hole | •2 Semi-circular hole:<br>$2\pi m$ $\left(6, \frac{8}{3\pi}\right)$ $[6, 0\cdot849]$ | |
| | | | •3 take moments horizontally by equating with centre of mass of remaining shape | •3<br>$13\pi m\bar{x} = 16\pi m \times 4 - \pi m \times 2 - 2\pi m \times 6$ | |
| | | | •4 solve this equation to find horizontal value of centre of mass | •4 $\bar{x} = \frac{50}{13}$ $[3\cdot846]$ | |
| | | | •5 take moments vertically | •5 $13\pi m\bar{y} = 16\pi m(0) - \pi m \times 1 - 2\pi m \times \frac{8}{3\pi}$ | |
| | | | •6 solve this equation to find vertical value of centre of mass | •6 $\bar{y} = -0\cdot208$ | |

**Notes:**

1. •6 Position does not have to be specified as coordinates as moments were taken from *A*
2. Do not penalise omission of mass

**Commonly Observed Responses:**

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|---|---|---|---|---|---|
| **10.** | (a) | | **Method 2: Relative to C** | | **6** |
| | | | •[1] derive expressions for the mass and centres of mass of the original lamina and the circular hole | •[1] Original Lamina: $16\pi m$ $(4,0)$<br>Circular hole: $\pi m$ $(2,1)$ | |
| | | | •[2] derive expressions for the mass and centres of mass of the semi-circular hole | •[2] Semi-circular hole:<br>$2\pi m$ $\left(6,\dfrac{8}{3\pi}\right)$ $[6, 0\cdot849]$ | |
| | | | •[3] take moments horizontally by equating with centre of mass of remaining shape | •[3]<br>$13\pi m\bar{x} = 16\pi m\times 0 - \pi m\times -2 - 2\pi m\times 2$ | |
| | | | •[4] solve this equation to find horizontal value of centre of mass | •[4] $\bar{x} = \dfrac{-2}{13}$ $[-0\cdot154]$ | |
| | | | •[5] take moments vertically | •[5] $13\pi m\bar{y} = 16\pi m(0) - \pi m\times 1 - 2\pi m\times\dfrac{8}{3\pi}$ | |
| | | | •[6] solve this equation to find vertical value of centre of mass<br>And state coordinates relative to *A* | •[6] $\bar{y} = -0\cdot208$<br>$(3\cdot846, -0\cdot208)$ | |

**Notes:**

**Commonly Observed Responses:**

•[1] •[2] Alternative presentation of data

| | Original $\pi m(4^2) = 16\pi m$ | Small Circle $\pi m(4^2) = 16\pi m$ | Semicircle $\frac{1}{2}\pi m(2^2) = 2\pi$ | Remaining $13\pi m$ |
|---|---|---|---|---|
| Moments from A: $\bar{x}$ | $\begin{pmatrix}4\\0\end{pmatrix}$ | $\begin{pmatrix}2\\1\end{pmatrix}$ | $\begin{pmatrix}6\\\frac{8}{3\pi}\end{pmatrix}$ | $\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$ |
| Moments from C: $\bar{y}$ | $\begin{pmatrix}0\\0\end{pmatrix}$ | $\begin{pmatrix}-2\\1\end{pmatrix}$ | $\begin{pmatrix}2\\\frac{8}{3\pi}\end{pmatrix}$ | $\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$ |

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|---|---|---|---|---|---|
| | (b) | | •[7] interpret rotation | •[7] $\tan\theta = \dfrac{0\cdot208}{3\cdot846}$ $\theta = 3\cdot1°$ | **1** |

**Notes:**

**Commonly Observed Responses:**

<table>
<tr><th colspan="3">Question</th><th>Generic scheme</th><th>Illustrative scheme</th><th>Max mark</th></tr>
<tr><td>11.</td><td>(a)</td><td></td><td>•1 calculate the displacement of $A$ and $B$ in 6 minutes<br><br>•2 calculate velocity of $A$ and $B$</td><td>•1 $\mathbf{r}_A = 4\cdot8\mathbf{i} + 1\cdot4\mathbf{j}$<br>$\mathbf{r}_B = -0\cdot8\mathbf{i} + 1\cdot5\mathbf{j}$<br><br>•2 $\mathbf{v}_A = \frac{4\cdot8}{0\cdot1}\mathbf{i} + \frac{1\cdot4}{0\cdot1}\mathbf{j} = 48\mathbf{i} + 14\mathbf{j}$<br>$\mathbf{v}_B = \frac{-0\cdot8}{0\cdot1}\mathbf{i} + \frac{1\cdot5}{0\cdot1}\mathbf{j} = -8\mathbf{i} + 15\mathbf{j}$</td><td>2</td></tr>
<tr><td></td><td>(b)</td><td>(i)</td><td>•3 express displacement of $A$ and $B$ as functions of time<br><br>•4 equate $\mathbf{i}$-components<br><br>•5 equate $\mathbf{j}$-components and form conclusion</td><td>•3 $\mathbf{r}_A = (1\cdot2 + 48t)\mathbf{i} + (1\cdot6 + 14t)\mathbf{j}$<br>$\mathbf{r}_B = (34\cdot8 - 8t)\mathbf{i} + (1 + 15t)\mathbf{j}$<br><br>•4 $1\cdot2 + 48t = 34\cdot8 - 8t$<br>$\mathbf{i}$ components equal when<br>$t = 0\cdot6$ hours<br><br>•5 $1\cdot6 + 14t = 1 + 15t$<br>$t = 0\cdot6$ hours<br>$\mathbf{i}$ and $\mathbf{j}$ components are equal at<br>$t = 0\cdot6$ so boats collide</td><td>3</td></tr>
<tr><td colspan="6">Notes:<br>1. Horizontal marking can apply at •4 and •5.</td></tr>
<tr><td colspan="6">Commonly Observed Responses:</td></tr>
<tr><td></td><td></td><td>(ii)</td><td>•6 find the position of collision</td><td>•6 $\begin{pmatrix}30\\10\end{pmatrix}$ or $(30,10)$</td><td>1</td></tr>
<tr><td colspan="6">Notes:</td></tr>
<tr><td colspan="6">Commonly Observed Responses:</td></tr>
</table>

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|---|---|---|---|---|---|
| **Alternative Solution (relative position vector)** | | | | | |
| | (b) | (i) | •³ express displacement of $A$ and $B$ as functions of time<br>•⁴ find relative position vector and set vector or either component to zero<br>•⁵ find time of collision and form conclusion | •³ $\mathbf{r}_A = \begin{pmatrix} 48t+1\cdot2 \\ 14t+1\cdot6 \end{pmatrix}, \mathbf{r}_B = \begin{pmatrix} -8t+34\cdot8 \\ 15t+1 \end{pmatrix}$<br>•⁴ ${}_A\mathbf{r}_B = \begin{pmatrix} 56t-33\cdot6 \\ -t+0\cdot6 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$<br>or $56t-33\cdot6=0$ or $-t+0\cdot6=0$<br>•⁵ $-t+0\cdot6=0 \Rightarrow t=0\cdot6$<br>$56t-33\cdot6=0 \Rightarrow t=0\cdot6$<br>$\mathbf{i}$ and $\mathbf{j}$ components are equal at $t=0\cdot6$ so boats collide | **3** |
| | | (ii) | •⁶ find the position of collision | •⁶ $\begin{pmatrix} 30 \\ 10 \end{pmatrix}$ or $(30,10)$ | **1** |
| **Notes:** | | | | | |
| **Commonly Observed Responses:** | | | | | |
| **Alternative solution (parallel vectors)** | | | | | |
| | (b) | (i) | •³ expression to indicate method of bringing *B* to rest with substitution<br>•⁴ expression for $A_1B_1$<br>•⁵ *A* and *B* will collide if $v_{A-B}$ is parallel to $A_1B_1$ | $v_{A-B} = v_A - v_B$<br>•³ $= \begin{pmatrix} 48 \\ 14 \end{pmatrix} - \begin{pmatrix} -8 \\ 15 \end{pmatrix} = \begin{pmatrix} 56 \\ -1 \end{pmatrix}$<br>•⁴ $A_1B_1 = \begin{pmatrix} 34\cdot8 \\ 1 \end{pmatrix} - \begin{pmatrix} 1\cdot2 \\ 1\cdot6 \end{pmatrix} = \begin{pmatrix} 33\cdot6 \\ -0\cdot6 \end{pmatrix}$<br>•⁵ $\frac{3}{5}\begin{pmatrix} 56 \\ -1 \end{pmatrix} = \begin{pmatrix} 33\cdot6 \\ -0\cdot6 \end{pmatrix}$ or $A_1B_1 = 0\cdot6v_{A-B}$<br>So boats collide | **3** |
| | | (ii) | •⁶ use $t=0\cdot6$ to find the position of collision and state as coordinate | •⁶ $\begin{pmatrix} 30 \\ 10 \end{pmatrix}$ or $(30,10)$ | **1** |
| **Notes:** | | | | | |
| **Commonly Observed Responses:** | | | | | |

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|---|---|---|---|---|---|
| 12. | (a) | | •1 use Newton's second law parallel to wire<br><br>•2 resolve perpendicular to the cable and combine equations and simplify expression for acceleration<br><br>•3 use appropriate equation of motion with some substitution<br><br>•4 substitute all values and calculate speed | •1 $mg\sin\theta - \mu R = ma$<br><br>•2 $R = mg\cos\theta$<br>$a = g\left(\sin\theta - \mu\cos\theta\right)$ $[0\cdot589]$<br><br>•3 $v^2 = u^2 + 2\left(g\left(\sin\theta - \mu\cos\theta\right)\right)s$<br><br>•4<br>$v^2 = 2^2 + 2\left(g\left(\sin 20° - 0\cdot3\cos 20°\right)\right)\times 20$<br>$v = 5\cdot25\,\text{ms}^{-1}$ | 4 |
| **Notes:** | | | | | |
| **Commonly Observed Responses:** | | | | | |
| **Alternative solution (work/energy principle)** | | | | | |
| | (a) | | •1 calculate height and find expression for energy at top<br><br>•2 find expression for energy at bottom and calculate change in energy<br><br>•3 calculate work done against friction and use work/energy principle<br><br>•4 substitute and solve to find speed | $h = 20\sin 20°\left(\approx 6\cdot84\right)$<br>•1<br>and $mg \times 20\sin 20° + \frac{1}{2}m \times 2^2$<br><br>•2 $20mg\sin 20° + 2m - \frac{1}{2}mv^2$<br><br>$W = 0\cdot3mg\cos 20° \times 20$<br>•3<br>$W = 20mg\sin 20° + 2m - \frac{1}{2}mv^2$<br><br>$6mg\cos 20°$<br>•4 $= 20mg\sin 20° + 2m - \frac{1}{2}mv^2$<br>$v = 5\cdot25\,\text{ms}^{-1}$ | 4 |
| **Notes:** | | | | | |
| **Commonly Observed Responses:** | | | | | |

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|---|---|---|---|---|---|
| **12.** | (b) | | •5 find total initial energy<br><br>•6 find total final energy<br><br>•7 use conservation of energy to form equation<br><br>•8 substitute values and calculate angle | •5 setting zero PE level at seat $E_K + E_P = \frac{1}{2}mu^2 + 0 = 13 \cdot 8m$<br><br>•6 $E_K + E_P = 0 + mg(r - r\cos\theta)$<br><br>•7 $13 \cdot 8m = mgr(1 - \cos\theta)$<br><br>•8 $\cos\theta = 1 - \frac{5 \cdot 25^2}{2 \times 9 \cdot 8 \times 1 \cdot 8}$<br>$\theta = 77{\cdot}4°$ | **4** |
| **Notes:** | | | | | |
| **Commonly Observed Responses:** | | | | | |
| **Alternative solution (work/energy principle)** | | | | | |
| | (b) | | •1 use conservation of energy<br><br>•2 substitute to find height<br><br>•3 find vertical distance below centre of rotation<br><br>•4 calculate angle | •1 $\frac{1}{2}mv^2 = mgh$<br><br>•2 $h = \frac{5 \cdot 25^2}{2 \times 9 \cdot 8} = 1 \cdot 406$<br><br>•3 $1 \cdot 8 - 1 \cdot 406 = 0 \cdot 394$<br><br>•4 $\cos^{-1}\left(\frac{0 \cdot 394}{1 \cdot 8}\right) = 77 \cdot 4°$ | **4** |
| **Notes:** | | | | | |
| **Commonly Observed Responses:** | | | | | |

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|---|---|---|---|---|---|
| 13. | | | •1 differentiate $u$ with respect to $x$ | •1 $\frac{du}{dx}=2x$ | 6 |
| | | | •2 evaluate new limits | •2 $x=0 \Rightarrow u=4,\ x=\sqrt{5} \Rightarrow u=9$ | |
| | | | •3 find new integral | •3 $\int_4^9 \frac{u-4}{u^{\frac{1}{2}}}du$ | |
| | | | •4 express in integrable form | •4 $\int_4^9 \left(u^{\frac{1}{2}}-4u^{-\frac{1}{2}}\right)du$ | |
| | | | •5 integrate | •5 $\left[\frac{2}{3}u^{\frac{3}{2}}-8u^{\frac{1}{2}}\right]_4^9$ | |
| | | | •6 evaluate | •6 $\frac{14}{3}$ | |

**Notes:**

1. •5 only $\frac{14}{3}$ or $4\frac{2}{3}$ are acceptable since the exact value is requested.
2. •2 can be awarded for resubstituting for $x$ instead of evaluating new limits.

**Commonly Observed Responses:**

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|---|---|---|---|---|---|
| **14.** | | | •[1] model EPE in stretched rope<br><br>•[2] equate potential and elastic potential energy at lowest point<br><br>•[3] set up quadratic equation in $d$<br><br>•[4] solve for $d$<br><br>•[5] select appropriate solution and find height above water | •[1] $\text{EPE} = \frac{1}{2}\frac{\lambda x^2}{l} = 50d^2$<br><br>•[2] $E_p = mg(10+d)$<br>$= 70 \times 9.8 \times (10+d) = 50d^2$<br><br>•[3] $6860 + 686d = \frac{1}{2} \times \frac{1000}{10} d^2$<br>$50d^2 - 686d - 6860 = 0$<br><br>•[4] $d = \frac{686 \pm \sqrt{686^2 + 4 \times 50 \times 6860}}{2 \times 50}$<br>$= 20{\cdot}43\ldots$ or $-6{\cdot}71\ldots$<br><br>•[5] total length = $10 + 20{\cdot}43 = 30{\cdot}43$<br>height above water<br>$40 - 30{\cdot}43 = 9{\cdot}57\,\text{m}$ | **5** |
| **Alternative for •[1]** | | | | | |
| | | | •[1] calculate work done to stretch $d$ | •[1] $W = \int_0^d F\,dx = \int_0^d \left(\frac{\lambda}{10}x\right)dx = \frac{1}{2}\frac{\lambda}{10}d^2$ | |
| **Notes:** | | | | | |
| **Commonly Observed Responses:** | | | | | |

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|---|---|---|---|---|---|
| **Alternative solution (SHM)** | | | | | |
| **14.** | | | •[1] calculate speed at point cord becomes tense | $v^2 = u^2 + 2as$<br>•[1] $\Rightarrow v^2 = 0^2 + 2\times 9\cdot 8 \times 10$<br>$v = 14$ | **5** |
| | | | •[2] calculate equilibrium extension | •[2] $\frac{\lambda x_e}{l} = mg \Rightarrow \frac{1000x_e}{10} = 70g$<br>$x_e = 0\cdot 7g = 6\cdot 86$ | |
| | | | •[3] use Newton's second law to set up equation and calculate $\omega$ | •[3] $70g - \frac{1000(x + 0\cdot 7g)}{10} = 70\ddot{x}$<br>$\ddot{x} = -\frac{10}{7}x \Rightarrow \omega = \sqrt{\frac{10}{7}}$ | |
| | | | •[4] calculate amplitude of motion | •[4] $14^2 = \left(\sqrt{\frac{10}{7}}\right)^2 \left(a^2 - (0\cdot 7g)^2\right)$<br>$a = 13\cdot 574$ | |
| | | | •[5] calculate height above water | •[5] $40 - (10 + 6\cdot 86 + 13\cdot 574)$<br>$= 9\cdot 57\,\text{m}$ | |
| **Notes:** | | | | | |
| **Commonly Observed Responses:** | | | | | |

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|---|---|---|---|---|---|
| **Alternative solution (Newton's Second Law and splitting the variables)** | | | | | |
| **14.** | | | •[1] apply Newton's Second Law and Hooke's Law | •[1] $mg - \frac{\lambda x}{l} = ma$ <br> $70g - \frac{1000x}{10} = 70v\frac{dv}{dx}$ | **5** |
| | | | •[2] separate variables and integrate | •[2] $\int v\,dv = \int\left(g - \frac{10}{7}x\right)dx$ <br> $\frac{v^2}{2} + c = gx - \frac{5}{7}x^2$ | |
| | | | •[3] calculate speed at point cord becomes tense and substitute to find constant of integration | •[3] $v^2 = 0^2 + 2\times 9\cdot 8\times 10 \Rightarrow v = 14$ <br> $x = 0, v = 14 \Rightarrow c = -98$ <br> $\therefore \frac{v^2}{2} - 98 = gx - \frac{5}{7}x^2$ | |
| | | | •[4] substitute $v = 0$ and solve quadratic | •[4] $5x^2 - 7gx - 686 = 0$ <br> $\ldots \Rightarrow x = 20\cdot 43,\ x = -6\cdot 71$ | |
| | | | •[5] select solution and calculate height above water | •[5] $40 - 10 - 20\cdot 43$ <br> $= 9\cdot 57$ m | |
| **Notes:** | | | | | |
| **Commonly Observed Responses:** | | | | | |

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|---|---|---|---|---|---|
| **15.** | (a) | | •[1] set up auxiliary equation | •[1] $m^2 + 0 \cdot 4m + 0 \cdot 04 = 0$ | **5** |
| | | | •[2] solve quadratic equation to give general solution | •[2] $(m + 0 \cdot 2)(m + 0 \cdot 2) = 0 \Rightarrow m = -0 \cdot 2$ repeated $x = Ae^{-0 \cdot 2t} + Bte^{-0 \cdot 2t}$ | |
| | | | •[3] initial condition $x = 1 \cdot 5$ when $t = 0$ | •[3] $A = 1 \cdot 5$ | |
| | | | •[4] differentiate to use initial condition | •[4] $\frac{dx}{dt} = -0 \cdot 2Ae^{-0 \cdot 2t} + Be^{-0.2t} - 0 \cdot 2Bte^{-0 \cdot 2t}$ | |
| | | | •[5] substitution to obtain $B$ and particular solution | •[5] $-0 \cdot 5 = -0 \cdot 3 + B$<br>$B = -0 \cdot 2$<br>Hence $x = 1 \cdot 5e^{-0 \cdot 2t} - 0 \cdot 2te^{-0 \cdot 2t}$ | |

**Notes:**

1. •[1] only available for correct quadratic expression equated to zero.
2. •[2] only available if the general solution is expressed in terms of $t$

**Commonly Observed Responses:**

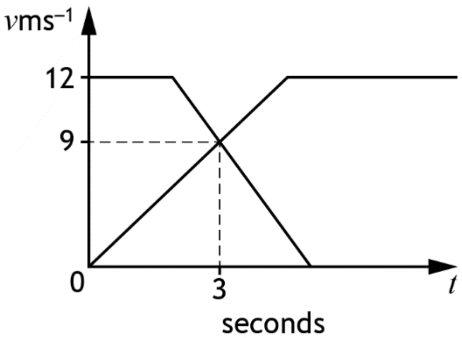
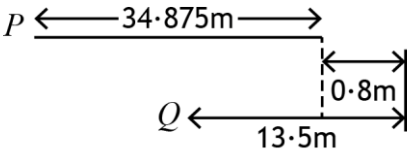
•[2] $x = Ae^{-0 \cdot 2t} + Be^{-0 \cdot 2t}$, leading to $A + B = 1 \cdot 5$ only •[1] and •[3] are available.

•[5] $\frac{dx}{dt} = +0 \cdot 5$ leading to $B = 0 \cdot 8$

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|---|---|---|---|---|---|
| | (b) | | •[6] substitute $t = 2$ into expression for $x$ and calculate distance moved. | •[6] $x = 1 \cdot 5e^{-0 \cdot 4} - 0 \cdot 4e^{-0 \cdot 4}$<br>$x = 0 \cdot 737$<br>distance moved $1 \cdot 5 - 0 \cdot 737 = 0 \cdot 763$ | **1** |

**Notes:**

**Commonly Observed Responses:**

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|---|---|---|---|---|---|
| **16.** | (a) | (i) | •1 sketch graph showing speed increase/decrease of both runners and annotation of meeting after 3 seconds<br><br>•2 sketch complete with relevant annotation | •1 •2<br><br> | **2** |
| | | (ii) | •3 use equations of motion under constant acceleration to find time for deceleration of P | $s=\ t=\ u=12\ \ v=9\ \ a=4$<br>•3 $v=u+at$<br>$9=12-4t$<br>$t=0\cdot75$ s | **1** |
| **Notes:**<br>1. Must show v/t graph beyond t=3 and a maximum speed for Q of $12\text{ms}^{-1}$<br>2. Graph Q .. allow variations after t=3s but maximum speed must not exceed $12\text{ms}^{-1}$ as constant acceleration is not specified. | | | | | |
| **Commonly Observed Responses:** | | | | | |
| **16.** | (b) | | •4 expression for area under the graph for *P*<br><br>•5 correct displacement<br><br>•6 find displacement for *Q* in three seconds<br><br>•7 explain displacements<br><br>•8 calculate distance | •4 $P:\ 27+\frac{1}{2}(2\cdot25+3)\times3$<br><br>•5 $34\cdot875$ metres<br><br>•6 $\frac{1}{2}\times3\times9=13\cdot5$ metres<br><br>•7<br><br><br><br>•8 $34\cdot875+0\cdot8-13\cdot5=22\cdot175$m | **5** |
| **Notes:** | | | | | |
| **Commonly Observed Responses:** | | | | | |

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|---|---|---|---|---|---|
| **17.** | | | •[1] use $F = ma$ with substitution of $\frac{dv}{dt}$ for acceleration<br><br>•[2] equate Impulse with change in Momentum<br><br>•[3] separate variables and start integration<br><br>•[4] use initial conditions with substitution<br><br>•[5] complete proof | •[1] $m\frac{dv}{dt} = -kv^2$<br><br>•[2] $I = mv$<br><br>•[3] $\int \frac{mdv}{v^2} = -kt + c$<br><br>•[4] $t = 0 \quad v = \frac{I}{m}$<br>$\frac{-m}{v} = -kt - \frac{m^2}{I}$<br><br>•[5] $\frac{m}{v} = \frac{ktI + m^2}{I}$<br>$v = \frac{mI}{ktI + m^2}$ | **5** |

**Notes:**

1. Use of $c = \frac{m^2}{I}$ may appear in •[4]

**Commonly Observed Responses:**

**[END OF MARKING INSTRUCTIONS]**