Marking instructions for each question

Question			Generic scheme	Illustrative scheme	Max mark
1.			•[1] use impulse = change in momentum •[2] calculate final velocity •[3] calculate magnitude of velocity •[4] calculate direction of velocity	•[1] $4\mathbf{v}-4(3\mathbf{i}+2\mathbf{j})=(6\mathbf{i}+\mathbf{j})$ •[2] $\mathbf{v}=\frac{18\mathbf{i}+9\mathbf{j}}{4}=\frac{9}{2}\mathbf{i}+\frac{9}{4}\mathbf{j}$ •[3] $\lvert\mathbf{v}\rvert=\sqrt{\left(\frac{9}{2}\right)^2+\left(\frac{9}{4}\right)^2}=5\cdot03$ •[4] $\tan^{-1}\left(\frac{9}{4}\div\frac{9}{2}\right)=26\cdot6°$	4

Notes:

1. Accept 153·4°

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
2.	(a)		•¹ start to use the product rule with one term correct	•¹ $1\times e^{-3x}\ldots$ or $\ldots-3xe^{-3x}$	**3**
			•² complete differentiation	•² $e^{-3x}-3xe^{-3x}$	
			•³ substitute $x=-1$	•³ $4e^{3}$	

Notes:

Commonly Observed Responses:

	(b)		•⁴ start differentiation with evidence of use of quotient rule with denominator and one term of numerator correct	•⁴ $\dfrac{3(2t+1)^2\ldots}{\left((2t+1)^2\right)^2}$ or $\dfrac{\ldots-3t\left(2(2t+1)\times 2\right)}{\left((2t+1)^2\right)^2}$	**3**
			•⁵ complete differentiation	•⁵ $\dfrac{3(2t+1)^2-3t\left(2(2t+1)\times 2\right)}{\left((2t+1)^2\right)^2}$	
			•⁶ simplify answer	•⁶ $\dfrac{3(1-2t)}{(2t+1)^3}$	

Notes:

1. •⁶ accept $\dfrac{3-6t}{(2t+1)^3}$
2. •⁶ is not available for a candidate who produces further incorrect simplification.

Commonly Observed Responses:

Alternative solution for (b) – Product rule

			•⁴ start differentiation with evidence of use of product rule with one term correct	•⁴ $3(2t+1)^{-2}\ldots$ or $\ldots-3t\left(2(2t+1)^{-3}\times 2\right)$	
			•⁵ complete differentiation	•⁵ $3(2t+1)^{-2}-3t\left(2(2t+1)^{-3}\times 2\right)$	
			•⁶ simplify answer	•⁶ $\dfrac{3(1-2t)}{(2t+1)^3}$	

Question			Generic scheme	Illustrative scheme	Max mark
3.			•1 integrate both components •2 evaluate constant(s) of integration •3 calculate displacement after 10 seconds •4 find distance and state if within range.	•1 $4t+c_1$ and $\frac{t^2}{2}+t+c_2$ •2 $c_1=c_2=0$ as boat starts at origin •3 $40\mathbf{i}+60\mathbf{j}$ •4 72·1 Yes, it is within range	4
Notes: If constants of integration are omitted at •1, award •1 but •2 is unavailable					
Commonly Observed Responses:					
4.			•1 use maximum speed and acceleration in appropriate formulae •2 state values of a and ω •3 derive or state equation for velocity at an instant •4 substitute to give value of velocity •5 interpret velocity	•1 $15=a\omega$ and $60=a\omega^2$ •2 $\omega=4$ $a=\frac{15}{4}$ •3 $a\omega\cos\omega t$ •4 $-2\cdot18$ •5 particle is moving in opposite direction to original movement	5
Notes: 1. •5 is unavailable for a positive answer at •4					
Commonly Observed Responses: Award •3 for $x=\frac{15}{4}\sin(4\times2)=3\cdot71$ **and** $v^2=4^2\left(\left(\frac{15}{4}\right)^2-3\cdot71^2\right)$ Subsequently, •4 can only be awarded for selecting the negative value with appropriate justification					

Question			Generic scheme	Illustrative scheme	Max mark
5.			•1 state auxiliary equation	•1 $m^2-3m+2=0$	5
			•2 solve auxiliary equation and state general solution	•2 $y=Ae^x+Be^{2x}$	
			•3 differentiate general solution	•3 $\frac{dy}{dx}=Ae^x+2Be^{2x}$	
			•4 substitute values into general solution and derivative to obtain 2 equations in A and B	•4 $1=A+B$ $3=A+2B$	
			•5 solve for A and B and state solution	•5 $y=-e^x+2e^{2x}$	

Notes:
1. "$\ldots=0$ " must appear for •1 to be awarded
2. " $y=$ …" need not appear at •2, but must appear in the final answer for •5 to be awarded

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
6.	(a)		•1 take moments about support	•1 $10g\times4+5g\times1-12g\times2$	3
			•2 find magnitude of turning effect	•2 $45g-24g=21g$	
			•3 interpret answer	•3 anticlockwise	
	(b)		•4 take moments about any point	•4 $10g(4-x)+5g(1-x)$ or $30gx+12g(x+2)$	3
			•5 equate to moments in opposite direction	•5 $10g(4-x)+5g(1-x)=30gx+12g(x+2)$	
			•6 calculate required distance	•6 $\frac{21}{57}$ or $0\cdot368$	

Notes:
•2 Accept 206

Alternative solution for (b)

			•4 calculate total mass and start to take moments about A	•4 $57x=\ldots$	
			•5 complete moments about A	•5 $57x=5\times3+12\times6+30\times4$	
			•6 calculate required distance	•6 $x=3\cdot632\Rightarrow0\cdot368$	

Question			Generic scheme	Illustrative scheme	Max mark
7.			•1 begin to differentiate log function •2 differentiate either trig term •3 complete differentiation •4 simplify	•1 $\frac{1}{(\sec 2t + \tan 2t)}\ldots$ •2 $2\sec 2t \tan 2t$ or $2\sec^2 2t$ •3 $\frac{2\sec 2t \tan 2t + 2\sec^2 2t}{(\sec 2t + \tan 2t)}$ •4 $2\sec 2t$	4

Notes:

•4 accept $\frac{2}{\cos 2t}$

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
8.			•1 set up integral •2 begin integration by parts •3 complete integration and include constant of integration •4 determine value of c from initial conditions •5 determine value of velocity	•1 $\int 2t(2t+1)^{\frac{1}{2}}\,dt$ •2 $2t \times \frac{(2t+1)^{\frac{3}{2}}}{\frac{3}{2}\times 2} - \ldots$ •3 $2t \times \frac{(2t+1)^{\frac{3}{2}}}{\frac{3}{2}\times 2} - \frac{2}{3} \times \frac{(2t+1)^{\frac{5}{2}}}{\frac{5}{2}\times 2} + c$ •4 $c = \frac{2}{15}$ •5 $v = 39\cdot 7$	5

Notes:

1. Alternative method for •3 •4 •5 could involve using limits of integration. In this case •4 is awarded for correct limits.
2. $\ldots dt$ must appear somewhere in the working for •1 to be awarded

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
9.			•1 resolve forces parallel to the plane	•1 $F\cos\theta + 25 = mg\sin 40$	5
			•2 resolve forces perpendicular to the plane	•2 $F\sin\theta + 30 = mg\cos 40$	
			•3 use equations from •1 and •2 to eliminate F	•3 $\frac{\sin\theta°}{\cos\theta°} = \frac{5g\cos 40 - 30}{5g\sin 40 - 25}$	
			•4 solve to find θ	•4 $49\cdot2°$	
			•5 substitute value for θ into either equation for F and solve	•5 $9\cdot95$	

Notes:

1. For •5 accept $9\cdot94$ or $9\cdot96$

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
10.			•1 start to differentiate using product rule	•1 $\ldots 2xe^{2y}\ldots$ or $\ldots 2x^2e^{2y}\frac{dy}{dx}\ldots$	4
			•2 complete differentiation	•2 $3\frac{dy}{dx} + 2xe^{2y} + 2x^2e^{2y}\frac{dy}{dx} = 0$	
			•3 determine value of x when $y = 0$	•3 $x = 3$	
			•4 evaluate gradient	•4 $-\frac{2}{7}$ or $-0\cdot286$	

Notes:

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
11.			•[1] use Newton's second law with substitution to set up equation •[2] separate variables and set up integration •[3] integrate with constant of integration (or use of limits) •[4] find constant of integration •[5] substitute and rearrange equation for v	•[1] $-0\cdot2v^2 = 2v\frac{dv}{dx}$ •[2] $\int -0\cdot1 dx = \int \frac{1}{v} dv$ •[3] $-0\cdot1x + c = \ln\lvert v\rvert$ •[4] $c = \ln 5$ •[5] $-0\cdot1x + \ln 5 = \ln\lvert v\rvert$ $v = 5e^{-0\cdot1x}$	5.

Notes:

1. If c is omitted at •[3], then •[3], •[4] and •[5] are not available.
2. Do not withhold •[3] or •[5] for the omission of the modulus sign
3. Alternative method for •[3] •[4] •[5] could involve using limits of integration
4. for •[1] accept $-0\cdot2v^2 = 2\frac{dv}{dt}$. All marks are still available for appropriate working.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
12.	(a)		•1 resolve forces vertically •2 apply Newton's 2nd law for horizontal forces •3 substitute and eliminate R •4 substitute in expression for v and use trig identity for $\tan\theta°$ •5 rearrange to required answer	•1 $R\cos\theta° + \mu R\sin\theta° = mg$ •2 $R\sin\theta° - \mu R\cos\theta° = \dfrac{mv^2}{r}$ •3 $\dfrac{\sin\theta° - \mu\cos\theta°}{\cos\theta° + \mu\sin\theta°} = \dfrac{v^2}{gr}$ •4 $\dfrac{\tan\theta° - \mu}{1+\mu\tan\theta°} = \dfrac{1}{100}$ •5 $100\tan\theta° - 100\mu = 1 + \mu\tan\theta°$ $\mu\tan\theta° + 100\mu = 100\tan\theta° - 1$ $\mu = \dfrac{100\tan\theta° - 1}{\tan\theta° + 100}$.	5

Notes:
1. •5 is unavailable for candidates who write down the correct expression without justification

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		•6 resolve forces for friction acting down the slope •7 substitute and eliminate R •8 find maximum speed •9 find minimum speed and state conclusion	•6 $R\cos\theta° - \mu R\sin\theta° = mg$ $R\sin\theta° + \mu R\cos\theta° = \dfrac{mv^2}{r}$ •7 $\dfrac{\sin\theta° + \mu\cos\theta°}{\cos\theta° - \mu\sin\theta°} = \dfrac{v^2}{gr}$ •8 $v = 30\cdot3$ •9 $2\cdot8$ and motorcyclist will not slip.	4

Notes:
1. Accept •7 stated immediately from (a) as understanding of slipping up the slope

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
	(c)		•10 state reason with justification	•10 eg worn tyres - alter value of coefficient of friction	1

Notes:
1. •10 cannot be awarded for any reference to mass

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
13.	(a)		•¹ resolve perpendicular to the slope	•¹ $R = mg\cos\theta$	**4**
			•² apply Newton's second law parallel to the slope	•² $-\mu R - mg\sin\theta = ma$	
			•³ find expression for acceleration	•³ $a = -g(\mu\cos\theta + \sin\theta)$	
			•⁴ substitute into equation of motion and complete	•⁴ $0 = V^2 + 2(-g(\mu\cos\theta + \sin\theta))s$ $s = \dfrac{V^2}{2g(\mu\cos\theta + \sin\theta)}$	
Notes:					
Commonly Observed Responses:					
	(b)		•⁵ find work done against friction in terms of given variables	•⁵ $W = \mu mg\cos\theta \times \dfrac{V^2}{2g(\mu\cos\theta + \sin\theta)}$	**3**
			•⁶ substitute for W and start simplification	•⁶ $\dfrac{1}{8} = \dfrac{\mu\cos\theta}{2(\mu\cos\theta + \sin\theta)}$	
			•⁷ state expression for μ	•⁷ $\mu = \dfrac{1}{3}\tan\theta$	
Notes: •⁷ **accept** $\mu = \dfrac{\sin\theta}{3\cos\theta}$					
Commonly Observed Responses:					

Question			Generic scheme	Illustrative scheme	Max mark
Alternative solutions for 13. (a)					
			•[1] state force acting down slope •[2] find work done against friction to travel *s* metres up slope •[3] resolve perpendicular to slope and substitute for *R* •[4] use work energy principle to find expression for *s*	•[1] $F = mg\sin\theta + \mu R$ •[2] $(mg\sin\theta + \mu R)s$ •[3] $R = mg\cos\theta$ $(mg\sin\theta + \mu mg\cos\theta)s$ •[4] $\frac{1}{2}mV^2 = mg(\sin\theta + \mu\cos\theta)s$ $s = \frac{V^2}{2g(\sin\theta + \mu\cos\theta)}$	
			•[1] find work done against gravity •[2] find work done against friction •[3] use work/energy principle •[4] find expression for *s*	•[1] $mg \times s\sin\theta$ •[2] $\mu mg \times s\cos\theta$ •[3] $\frac{1}{2}mV^2 = mgs\sin\theta + \mu mgs\cos\theta$ •[4] $s = \frac{V^2}{2g(\sin\theta + \mu\cos\theta)}$	

Question			Generic scheme	Illustrative scheme	Max mark
14.	(a)		•[1] consider energy at A	•[1] $E_k + E_p = \frac{1}{2}mu^2 + 0$	4
			•[2] consider energy at P, and substitute for h	•[2] $E_k + E_p = \frac{1}{2}mv^2 + mgh$ $= mgr(1-\cos\theta)$	
			•[3] use conservation of energy	•[3] $\frac{1}{2}mu^2 = mgr(1-\cos\theta)$	
			•[4] substitute and calculate angle	•[4] $6\cdot125 = 3\cdot92(1-\cos\theta)$ $\theta = 124\cdot2^\circ$	

Notes:

1. Accept $\theta = 124^\circ$
2. •[1] and •[2] may be implied by •[3]
3. If •[3] does not appear then evidence for •[1] and •[2] must include $E_k + E_p$ or "energy at A" or similar

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		•[5] state requirements for complete circle	•[5] $v > 0$ when angle = 180°	3
			•[6] set up inequality with initial kinetic energy greater than final potential energy	•[6] $\frac{1}{2}mu^2 > 2mgr$	
			•[7] solve for u	•[7] $u > \sqrt{\frac{8g}{5}}$	

Notes:

1. •[5] may be implied by •[6]
2. •[5] and •[6] can be awarded for equalities
3. •[7] accept $u > 3\cdot96$
4. •[7] do not accept $u \geq 3\cdot96$ or $u \geq \sqrt{\frac{8g}{5}}$

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
	(c)		•[8] state assumption	•[8] ball is of the same radius as tubing or does not spin or ball is smooth.	1

Notes:

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
15.	(a)		•¹ state condition for maximum height	•¹ $v=0$ stated or implied by •²	**3**
			•² find vertical component of initial velocity and substitute into vertical equation of motion	•² $0=u^2\sin^2\theta-2\times g\times s$	
			•³ introduce inequality and complete proof	•³ $\sin\theta<\frac{\sqrt{2\times g\times 3}}{u}$ $\sin\theta<\frac{\sqrt{6g}}{u}$	

Notes:

1. Only accept $\sin\theta=\frac{\sqrt{2gs}}{u}$ leading to inequality if further explanation is given

Alternative solution for (a)

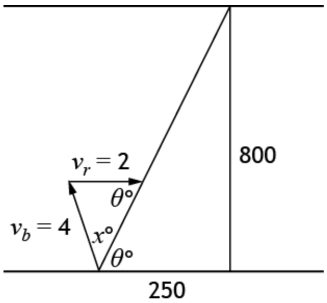
			•¹ state expression for height	•¹ $ut\sin\theta-\frac{1}{2}gt^2$	
			•² state expression for time and start substitution	•² $t=\frac{u\sin\theta}{g}$ $u\left(\frac{u\sin\theta}{g}\right)\sin\theta-\frac{1}{2}g\left(\frac{u\sin\theta}{g}\right)^2$	
			•³ introduce inequality and complete proof	•³ $\ldots<3$ and working leading to $\sin\theta<\sqrt{\frac{6g}{u}}$	

Question			Generic scheme	Illustrative scheme	Max mark
15.	(b)	(i)	•4 state time of flight	•4 $\frac{2u\sin\theta}{g}$	5
			•5 substitute into expression for range	•5 $\frac{2u^2\sin\theta\cos\theta}{g}$	
			•6 obtain expression for $\cos\theta$	•6 $\cos\theta = \frac{\sqrt{u^2-6g}}{u}$	
			•7 substitute expressions for $\sin\theta$ and $\cos\theta$ into expression for range	•7 $\frac{2u^2}{g}\times\frac{\sqrt{6g}}{u}\times\frac{\sqrt{u^2-6g}}{u}$	
			•8 simplify and complete	•8 valid working leading to $R = 12\sqrt{\frac{u^2-6g}{6g}}$	
Alternative solution for (b) (i)					
			•4 substitute into 2 equations of motion	•4 $3 = \frac{(u+v)t}{2}$ $6 = u\sin\theta\times t$	
			•5 combine equations to eliminate $\sin\theta$	•5 $\frac{6}{ut} = \frac{\sqrt{6g}}{u}$	
			•6 find expression for total time of flight	•6 Total time of flight $= \frac{12}{\sqrt{6g}}$	
			•7 find expression for horizontal component of velocity	•7 $u\cos\theta = \sqrt{u^2(1-\sin^2\theta)}$ $u\cos\theta = \sqrt{u^2-6g}$ $u\cos\theta = \sqrt{u^2-6g}$	
			•8 use expression for range and simplify as required	•8 Range $= \frac{12}{\sqrt{6g}}u\cos\theta$ Range $= \frac{12\sqrt{u^2-6g}}{\sqrt{6g}} = 12\sqrt{\frac{u^2-6g}{6g}}$	
		(ii)	•9 state constraint	•9 $u > \sqrt{6g}$	1

Notes:

Accept $u \geq \sqrt{6g}$, $u^2 \geq 6g$ or $u^2 > 6g$

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
16.	(a)		•[1] calculate the angle for direct route	•[1] $\tan\theta° = \dfrac{800}{250}$ $\theta° = 72{\cdot}6°$  	**4**
			•[2] use sine rule	•[2] $\dfrac{\sin x°}{2} = \dfrac{\sin 72{\cdot}6°}{4}$	
			•[3] determine angle inside velocity components triangle	•[3] $x = 28{\cdot}5$	
			•[4] interpret solution	•[4] angle to bank is $101{\cdot}1°$ or $78{\cdot}9°$	
Notes: •[4] accept $101{\cdot}2°$ or $78{\cdot}8°$					
Commonly Observed Responses:					
	(b)	(i)	•[5] calculate resultant speed before slowing	•[5] $v_{resultant} = 4{\cdot}11$	**3**
			•[6] calculate distance from A of rower after 60 seconds	•[6] 247	
			•[7] calculate remaining distance after slowing	•[7] 591	
Alternative solution for (b) (i)					
			•[5] set up distance triangle and use sine/cosine rule	•[5] $\dfrac{x}{\sin 78{\cdot}8} = \dfrac{120}{\sin 28{\cdot}5} = \dfrac{240}{\sin 72{\cdot}6}$ or $x^2 = 120^2 + 240^2 - 2\times120\times240\times\cos 78{\cdot}8$	
			•[6] calculate full or partial distance	•[6] 247 after 1 minute or 838 to B	
			•[7] calculate remaining distance	•[7] 591	
Notes: Accept 592 for •[7]					
Commonly Observed Responses:					

Question			Generic scheme	Illustrative scheme	Max mark
16.	(b)	(ii)	•[8] calculate the new angle with the river bank or angle marked x •[9] calculate resultant velocity after slowing •[10] calculate remaining and total times	•[8] $67\cdot8^\circ$ or $112\cdot2^\circ$ or 39·5 •[9] $v = 2\cdot91\,\text{ms}^{-1}$ •[10] $t = 203$ seconds, Total time = 263 seconds	**3**
Notes:					
Commonly Observed Responses:					
17.	(a)		•[1] recognise form of integral and integrate correctly	•[1] $\tan\left(e^t\right) + c$	**1**
Notes: constant of integration not required					
Commonly Observed Responses:					
	(b)		•[2] recognise expression for velocity •[3] explain why original function cannot ever equal zero	•[2] $v = e^t \sec^2\left(e^t\right)$ •[3] neither $\sec\left(e^t\right)$ nor e^t can ever equal zero, so product can never be zero and hence particle never at rest	**2**
Notes:					
Commonly Observed Responses:					

[END OF MARKING INSTRUCTIONS]