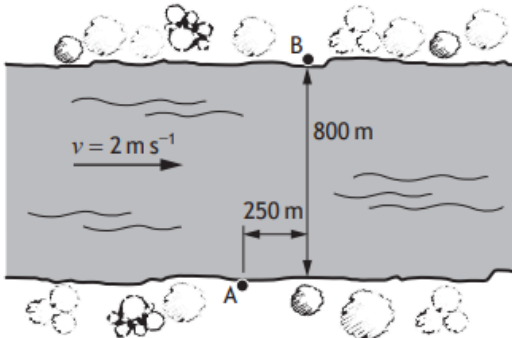


Y	Q	Vector Motion	
19	3	<p>A radio-controlled model boat moves from an origin with velocity <math>\mathbf{v}(t) = (4\mathbf{i} + (t+1)\mathbf{j}) \text{ ms}^{-1}</math>, where <math>t</math> is measured in seconds. The radio signal has a range of 80 metres.</p> <p>Determine whether the boat is still within range of the radio signal after 10 seconds.</p>	4
19	16	<p>A rower is crossing a river that is 800 metres wide. They set off from point A and need to reach point B as quickly as possible. B is 250 metres downstream.</p> <p>They row at a speed of <math>4 \text{ ms}^{-1}</math> in still water, and the river current flows at <math>2 \text{ ms}^{-1}</math>.</p>  <p>(a) Find at what angle to the bank the rower needs to steer.</p> <p>After rowing for one minute the rower gets tired and immediately reduces speed to <math>3 \text{ ms}^{-1}</math>, adjusting steering to maintain the same course.</p> <p>(b) (i) Find how far they are from B at this time.</p> <p>(ii) Calculate the total time it takes the rower to reach point B.</p>	4 3 3
18	11	<p>At 08:00 a port official records Boat A at point <math>(1.2, 1.6)</math> and Boat B at <math>(34.8, 1)</math>, where the distances are in kilometres relative to the port as an origin.</p> <p>At 08:06 the official records their points as <math>(6, 3)</math> and <math>(34, 2.5)</math> respectively.</p> <p>(a) Show that their average velocities over this period can be expressed, in <math>\text{km h}^{-1}</math>, as</p> $\mathbf{v}_A = 48\mathbf{i} + 14\mathbf{j} \text{ and } \mathbf{v}_B = -8\mathbf{i} + 15\mathbf{j}.$ <p>(b) (i) Assuming that each boat maintains a constant velocity, show that they are on a collision course.</p> <p>(ii) Find the location of the collision.</p>	2 3 1
17	3	<p>The velocity of a particle after <math>t</math> seconds of travel can be expressed as <math>\mathbf{v} = (3\sin 2t)\mathbf{i} + (\cos 2t - 3)\mathbf{j} \text{ ms}^{-1}</math> where <math>\mathbf{i}</math> and <math>\mathbf{j}</math> are unit vectors in horizontal and vertical directions respectively.</p> <p>Find the magnitude of the acceleration of the particle when <math>t = \frac{\pi}{6}</math> seconds.</p>	4

17	14	<p>A fishing boat, A, leaves a harbour with a constant speed of <math>10 \text{ km h}^{-1}</math> on a bearing of <math>060^\circ</math>.</p> <p>At the same time another fishing boat, B, is <math>12 \text{ km}</math> due east of A, moving with a constant speed of <math>10\sqrt{3} \text{ km h}^{-1}</math> on a bearing of <math>330^\circ</math>.</p> <p>(a) (i) Describe how the vectors <math>\mathbf{i}</math> and <math>\mathbf{j}</math> should be defined in this situation. <span style="float: right;">2</span></p> <p>(ii) Show that the position of boat A relative to boat B, <math>t</math> hours after A has left the harbour, can be written as <math>{}_A\mathbf{r}_B = (10\sqrt{3}t - 12)\mathbf{i} - 10t\mathbf{j}</math> kilometres. <span style="float: right;">3</span></p> <p>(b) Find for how long the two boats will be within <math>7 \text{ km}</math> of each other. Give your answer to the nearest minute. <span style="float: right;">5</span></p>																
16	12	<p>An aircraft flies <math>1080 \text{ km}</math> due east from Glasgow to Copenhagen in a time of <math>2\frac{1}{4}</math> hours.</p> <p>The aircraft sets a course on a bearing of <math>100^\circ</math> and the speed of the aircraft in still air is <math>450 \text{ km h}^{-1}</math>.</p> <p>(a) Calculate the magnitude and direction of the wind. <span style="float: right;">3</span></p> <p>(b) (i) Given that the velocity of the wind remains constant, explain why the return journey will take longer. <span style="float: right;">1</span></p> <p>(ii) Calculate how much longer the return journey will take, giving your answer to the nearest minute. <span style="float: right;">4</span></p>																
16 (Sp)	14	<p>Three vessels A, B and C are being tracked by coastguards at half-hour intervals. With distances measured in kilometres and speeds in kilometres per hour, they have the following displacement and velocity vectors:</p> <table border="1" data-bbox="368 1171 1249 1406" style="width: 100%; border-collapse: collapse; text-align: center;"> <thead> <tr> <th>Vessel</th> <th>A</th> <th>B</th> <th>C</th> </tr> </thead> <tbody> <tr> <td>Time</td> <td>10:00</td> <td>10:30</td> <td>11:00</td> </tr> <tr> <td>Position</td> <td><math>2\mathbf{i} + 7\mathbf{j}</math></td> <td><math>6\mathbf{i} + 9\mathbf{j}</math></td> <td><math>12\mathbf{i} + 9\mathbf{j}</math></td> </tr> <tr> <td>Velocity</td> <td><math>4\mathbf{i} + 5\mathbf{j}</math></td> <td><math>3\mathbf{i} + 4\mathbf{j}</math></td> <td><math>2\mathbf{i} + 6\mathbf{j}</math></td> </tr> </tbody> </table> <p>(a) Show that if A and C continue without changing course they will collide. Find the time and position of the collision. <span style="float: right;">5</span></p> <p>At the instant of the collision, vessel B changes course and then proceeds directly to the scene of the collision at its original speed.</p> <p>(b) Find the time, to the nearest minute, at which vessel B will arrive at the scene of the collision and state the bearing of its course to this point. <span style="float: right;">5</span></p>	Vessel	A	B	C	Time	10:00	10:30	11:00	Position	$2\mathbf{i} + 7\mathbf{j}$	$6\mathbf{i} + 9\mathbf{j}$	$12\mathbf{i} + 9\mathbf{j}$	Velocity	$4\mathbf{i} + 5\mathbf{j}$	$3\mathbf{i} + 4\mathbf{j}$	$2\mathbf{i} + 6\mathbf{j}$
Vessel	A	B	C															
Time	10:00	10:30	11:00															
Position	$2\mathbf{i} + 7\mathbf{j}$	$6\mathbf{i} + 9\mathbf{j}$	$12\mathbf{i} + 9\mathbf{j}$															
Velocity	$4\mathbf{i} + 5\mathbf{j}$	$3\mathbf{i} + 4\mathbf{j}$	$2\mathbf{i} + 6\mathbf{j}$															