Complex Numbers - Solutions

1. (a) (i) $z = 5 - 5\sqrt{3}i$

$$|z| = \sqrt{5^2 + (5\sqrt{3})^2} = \sqrt{100} = 10$$

$$\arg z = \tan^{-1}\left(-\frac{5\sqrt{3}}{5}\right) = -\frac{\pi}{3}$$

(ii) $z = (3-i)(2+3i)$

$$= 6 + 9i - 2i - 3i^2$$

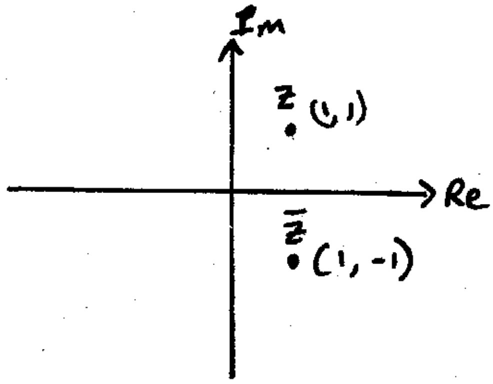
$$= 9 + 7i$$

$$|z| = \sqrt{9^2 + 7^2} = \sqrt{\cancel{67}130}$$

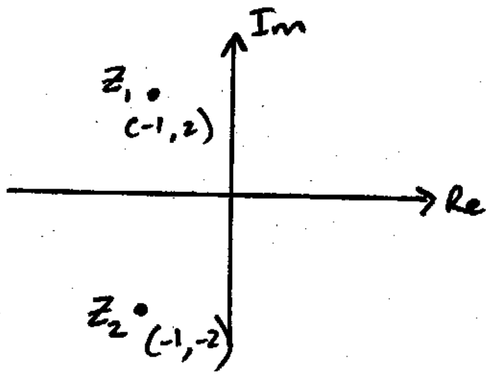
$$\arg z = \tan^{-1}\left(\frac{7}{9}\right) = 0.661$$

(b) $z = \frac{4+2i}{3-i} \cdot \frac{3+i}{3+i} = \frac{12 + 4i + 6i + 2i^2}{9 - i^2} = \frac{10 + 10i}{10} = 1 + i$

$\bar{z} = 1 - i$



2. $z = \frac{-2 \pm \sqrt{2^2 - 4 \times 5}}{2} = \frac{-2 \pm \sqrt{-16}}{2} = \frac{-2 \pm 4i}{2} = -1 \pm 2i$



3. $|z| = \sqrt{1^2 + (\sqrt{3})^2} = \sqrt{4} = 2$

$$\arg z = \tan^{-1} -\sqrt{3} = -\frac{\pi}{3}$$

4.

$$\begin{array}{r|rrrrr} 1+i & 1 & 0 & 3 & -6 & 10 \\ & & 1+i & 2i & 1+5i & -10 \\ \hline & 1 & 1+i & 3+2i & -5+5i & \boxed{0} \end{array}$$

If $z = 1+i$ is a root, then $z = 1-i$ is a root

$$\begin{array}{r|rrrr} 1-i & 1 & 1+i & 3+2i & -5+5i \\ & & 1-i & 2-2i & 5-5i \\ \hline & 1 & 2 & 5 & \boxed{0} \end{array}$$

$z^2 + 2z + 5 = 0$

$z = -1+2i$ or $z = -1-2i$ (solved in Q2).

The roots are $z_1 = 1+i$, $z_2 = 1-i$, $z_3 = -1+2i$, $z_4 = -1-2i$

5.(a) $(1+ic)^6 = 1 + 6ic + 15i^2c^2 + 20i^3c^3 + 15i^4c^4 + 6i^5c^5 + i^6c^6$

$= 1 + 6ic - 15c^2 - 20ic^3 + 15c^4 + 6ic^5 - c^6$

(b) If z is real then $6ic - 20ic^3 + 6ic^5 = 0$

$2c(3 - 10c^2 + 3c^4) = 0$

$2c(3c^2-1)(c^2-3) = 0$

$c = 0$ or $c^2 = \frac{1}{3}$ or $c^2 = 3$

$c = \pm\frac{1}{\sqrt{3}}$ $\quad c = \pm\sqrt{3}$