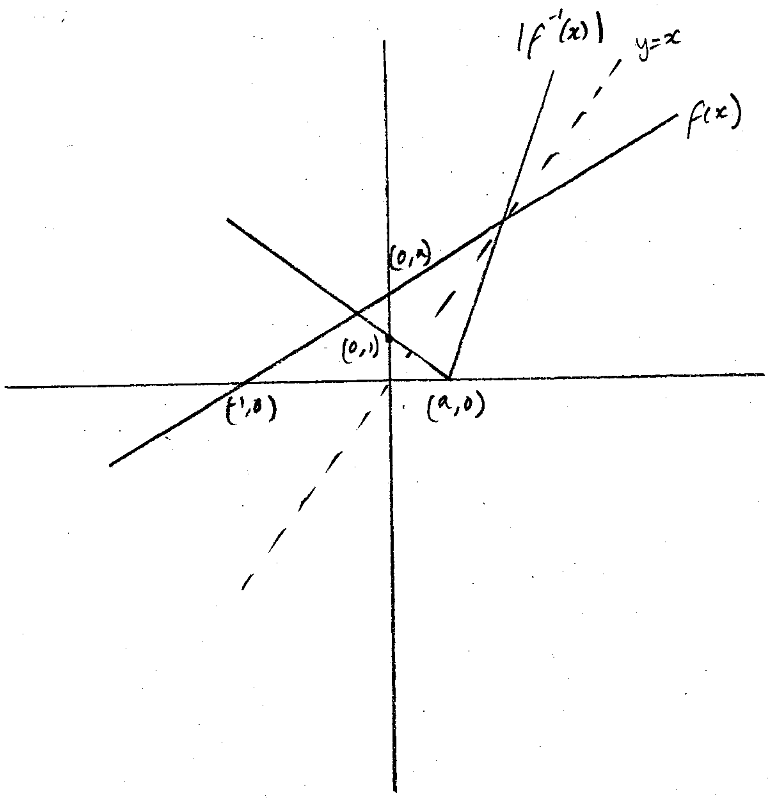
Properties of Functions.

1.



- Shape
- $(0,1)$
- $(a,0)$

2. $f(-x) = (-x)^2 \cdot \sin(-x)$

$= x^2 \cdot -\sin x$

$= -x^2 \sin x$

$= -f(x)$ The function is odd.

- substitute
- algebraic manipulation
- conclusion

3. $x \neq \pm\sqrt{3}$ vertical asymptotes at $x = -\sqrt{3}$ and $x = \sqrt{3}$

- vertical asymptotes

$$\begin{array}{r|l} & x \\ \hline x^2-3 & x^3 \\ & x^3 - 3x \\ \hline & 3x \end{array}$$

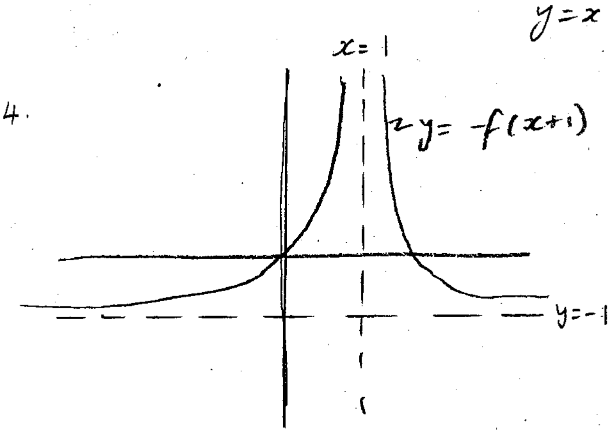
$y = x + \dfrac{3x}{x^2-3}$

$x \to \infty \quad \dfrac{3x}{x^2-3} \to \dfrac{3}{x} \to 0$ so $y \to x$

$y=x$ is a non-vertical asymptote.

- Rewrite y
- Non-vertical asymptote.

4.



- Shift right
- Reflect
- Modulus
- label asymptotes.

5. (a) $\frac{dy}{dx} = 5x^4 - 15x^2$

$\frac{dy}{dx} = 0 \quad 5x^2(x^2-3)=0$

$x=0$ or $x = \pm\sqrt{3}$

$\frac{d^2y}{dx^2} = 20x^3 - 30x$

$= 10x(2x^2-3)$

$x=\sqrt{3} \quad \frac{d^2y}{dx^2} > 0$ min T.P. $\quad y = x^3(x^2-5) = -6\sqrt{3}$

$x = -\sqrt{3} \quad \frac{d^2y}{dx^2} < 0$ max T.P. $\quad y = 6\sqrt{3}$

$\frac{d^2y}{dx^2} = 0 \quad x = 0$ or $x = \pm\sqrt{\frac{3}{2}}$ $\quad y = \pm\frac{3}{2}\sqrt{\frac{3}{2}}\left(\frac{3}{2} - \frac{10}{2}\right)$

$= \pm\frac{21}{4}\sqrt{\frac{3}{2}}$

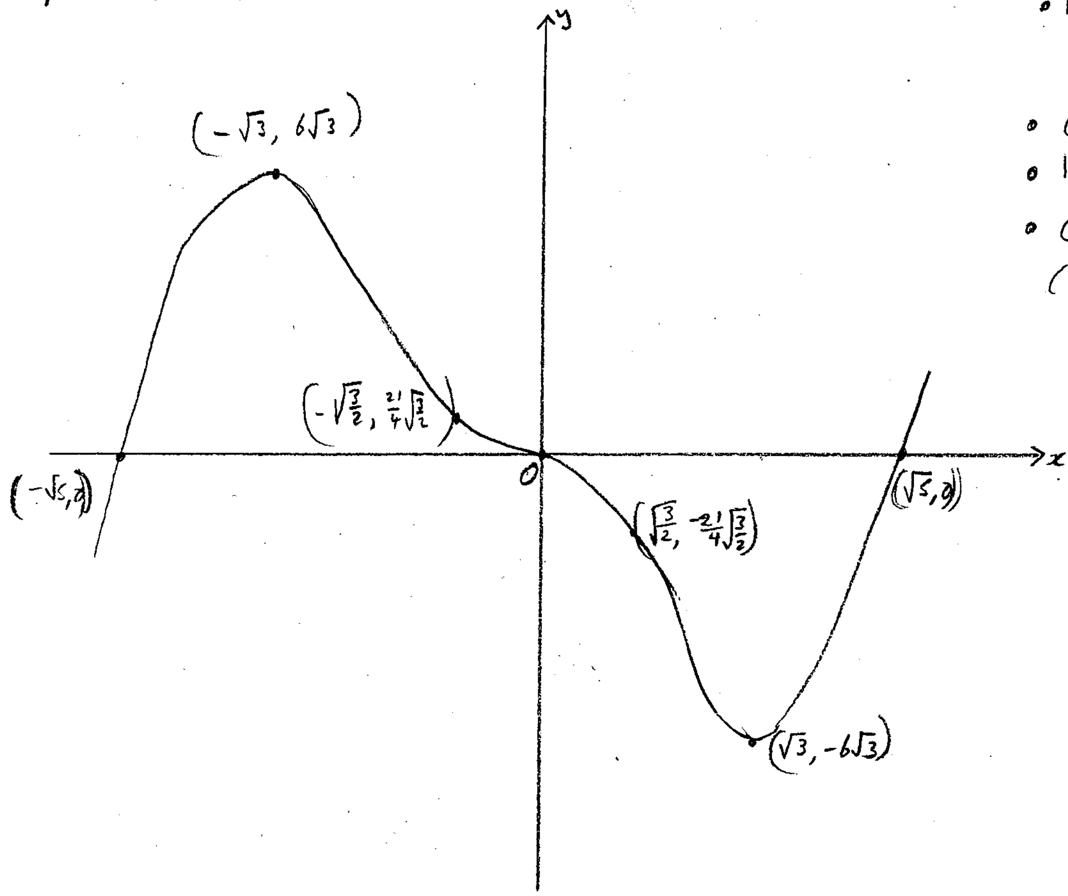
x	$\to$	$-\sqrt{\frac{3}{2}}$	$\to$	0	$\to$	$\sqrt{\frac{3}{2}}$	$\to$
$\frac{d^2y}{dx^2}$	$-$		$+$	0	$-$		$+$
Concavity			Up		Down		

$y=0$

$x^3(x^2-5)=0$

$x=0$ or $x = \pm\sqrt{5}$

Three points of inflexion. At $x=0$ $\frac{dy}{dx}=0$ hence $(0,0)$ is the horizontal PoI



- horizonal point of inflexion.
- Correct turning points
- Intersections with axes.
- Overall shape (odd function).

$f(-x) = (-x)^5 - 5(-x)^3$

$= -x^5 + 5x^3$

$= -f(x)$

Function is odd.