

10A

1(d) $y = x^3 + 3x^2 - 2x + 4$ (2, 20)

$$\frac{dy}{dx} = 3x^2 + 6x - 2$$

$$\frac{dy}{dx}(2) = 3(2)^2 + 6(2) - 2$$

$$\frac{dy}{dx}(2) = 22$$

$$\underline{m = 22} \quad \text{at } (2, 20)$$

$$y - b = m(x - a)$$

$$y - 20 = 22(x - 2)$$

$$y - 20 = 22x - 44$$

$$\underline{y = 22x - 24}$$

2(c) $y = 4 \sin x$ $(\frac{\pi}{4}, 2\sqrt{2})$

$$\frac{dy}{dx} = 4 \cos x$$

$$\frac{dy}{dx}\left(\frac{\pi}{4}\right) = 4 \cos\left(\frac{\pi}{4}\right)$$

$$= 4 \cdot \frac{1}{\sqrt{2}}$$

$$= \frac{4}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{4\sqrt{2}}{2} = 2\sqrt{2}$$

$$m = 2\sqrt{2} \quad \text{at } \left(\frac{\pi}{4}, 2\sqrt{2}\right)$$

$$y - b = m(x - a)$$

$$y - 2\sqrt{2} = 2\sqrt{2}\left(x - \frac{\pi}{4}\right)$$

10A

2(c) continued

common factor of
 $2\sqrt{2}$.

$$y = 2\sqrt{2}x + 2\sqrt{2} \cdot \frac{\pi}{4} + 2\sqrt{2}$$

$$y = 2\sqrt{2}x + 2\sqrt{2} \left(1 + \frac{\pi}{4}\right)$$

3(e) $y = 8 - x^3$

$$\frac{dy}{dx} = -3x^2$$

$$\begin{aligned} \frac{dy}{dx}(-3) &= -3(-3)^2 \\ &= \underline{\underline{-27}} \end{aligned}$$

$$\begin{aligned} y(-3) &= 8 - (-3)^3 \\ &= \underline{\underline{35}} \end{aligned}$$

$$m = -27 \text{ at } (-3, 35)$$

$$y - b = m(x - a)$$

$$y - 35 = -27(x - -3)$$

$$y - 35 = -27x - 81$$

$$\underline{\underline{y = -27x - 46}}$$

10A

$$4(d) \quad y = 6 \sin\left(x - \frac{\pi}{4}\right) \quad \text{at } x = \frac{\pi}{4}$$

$$\begin{aligned} y\left(\frac{\pi}{4}\right) &= 6 \sin\left(\frac{\pi}{4} - \frac{\pi}{4}\right) \\ &= 6 \sin 0 \\ &= \underline{0} \quad \left(\frac{\pi}{4}, 0\right) \end{aligned}$$

$$\frac{dy}{dx} = 6 \cos\left(x - \frac{\pi}{4}\right) \times \frac{d}{dx}\left(x - \frac{\pi}{4}\right)$$

$$\frac{dy}{dx} = 6 \cos\left(x - \frac{\pi}{4}\right)$$

$$\begin{aligned} \frac{dy}{dx}\left(\frac{\pi}{4}\right) &= 6 \cos\left(\frac{\pi}{4} - \frac{\pi}{4}\right) \\ &= 6 \cos(0) \\ &= \underline{6} \quad m = 6 \quad \text{at } \left(\frac{\pi}{4}, 0\right) \end{aligned}$$

$$y - 0 = 6\left(x - \frac{\pi}{4}\right)$$

$$y = 6x - \frac{6\pi}{4}$$

$$y = 6x - \frac{3\pi}{2}$$



10A

⑦ $y = f(x)$

$$y = 8 - 2(x-4)^2$$

at A and B $y = 0$

$$0 = 8 - 2(x-4)^2$$

$$2(x-4)^2 = 8$$

$$(x-4)^2 = \frac{8}{2} = 4$$

$$x-4 = \pm\sqrt{4}$$

$$x-4 = \pm 2$$

$$x-4 = 2$$

$$x-4 = -2$$

$$\underline{x_B = 6}$$

$$\underline{x_A = 2}$$

at B (6, 0)

$$\frac{dy}{dx} = -4(x-4)^1 \times \frac{d}{dx}(x-4)^1$$

$$= -4(x-4)$$

$$= -4x + 16$$

$$\frac{dy}{dx}(6) = -4(6) + 16$$

$$= -8$$

$$\underline{m = -8}$$

$$y - 0 = -8(x - 6)$$

$$\underline{y = -8x + 48}$$

10A

(11)

$$f(x) = 2 \cos\left(2x + \frac{\pi}{2}\right)$$

$$f'(x) = -2 \sin\left(2x + \frac{\pi}{2}\right) \times \frac{d}{dx}\left(2x + \frac{\pi}{2}\right)$$

$$f'(x) = -4 \sin\left(2x + \frac{\pi}{2}\right)$$

$$M=2 \Rightarrow -4 \sin\left(2x + \frac{\pi}{2}\right) = 2$$

$$\sin\left(2x + \frac{\pi}{2}\right) = -\frac{1}{2} \quad \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$$

$$2x + \frac{\pi}{2} = \pi + \frac{\pi}{6}, 2\pi - \frac{\pi}{6} \quad \begin{array}{c} S/A \\ T/C \\ \checkmark \end{array}$$

$$2x + \frac{\pi}{2} = \frac{7\pi}{6}, \frac{11\pi}{6}$$

$$2x = \frac{4\pi}{6}, \frac{8\pi}{6} \quad \left(\frac{\pi}{2} = \frac{3\pi}{6}\right)$$

$$x = \frac{4\pi}{12}, \frac{8\pi}{12}$$

$$x = \frac{\pi}{3}, \frac{2\pi}{3}$$

Assume point P has $x = \frac{\pi}{3}$

$x = \frac{\pi}{3}$ find y coordinate

$$y\left(\frac{\pi}{3}\right) = 2 \cos\left(2\frac{\pi}{3} + \frac{\pi}{2}\right)$$

$$= 2 \cos\left(\frac{7\pi}{6}\right)$$

$$= -2 \cos\left(\frac{\pi}{6}\right)$$

$$= -2 \cdot \frac{\sqrt{3}}{2}$$

$$= -\sqrt{3}$$

$$\left(\frac{\pi}{3}, -\sqrt{3}\right)$$



10A

⑪ continued

$$m=2 \text{ at } \left(\frac{\pi}{3}, -\sqrt{3}\right)$$

$$y - -\sqrt{3} = 2\left(x - \frac{\pi}{3}\right)$$

$$y + \sqrt{3} = 2x - \frac{2\pi}{3}$$

$$\underline{\underline{y = 2x - \frac{2\pi}{3} - \sqrt{3}}}$$

10A

(15)

$$y = (3x+2)^5$$

$$\frac{dy}{dx} = 5(3x+2)^4 \times \frac{d}{dx}(3x+2)$$

$$= 15(3x+2)^4$$

$$\frac{dy}{dx}(-1) = 15(3(-1)+2)^4$$

$$= 15(-1)^4$$

$$= \underline{15}$$

$$m = 15$$

at $x = -1$ y coordinate

$$y(-1) = (3(-1)+2)^5$$

$$= (-1)^5$$

$$= \underline{-1}$$

$$m = 15 \text{ at } (-1, -1)$$

$$(y - -1) = 15(x - -1)$$

$$y + 1 = 15(x + 1)$$

$$y + 1 = 15x + 15$$

$$y = 15x + 14$$



10A

(19) $y = (\sin x)^3$

$$\begin{aligned} y\left(\frac{7\pi}{6}\right) &= \left(\sin \frac{7\pi}{6}\right)^3 \\ &= \left(-\sin \frac{\pi}{6}\right)^3 \\ &= \left(-\frac{1}{2}\right)^3 \\ &= -\frac{1}{8} \end{aligned}$$



point $\left(\frac{7\pi}{6}, -\frac{1}{8}\right)$

$$\begin{aligned} \frac{dy}{dx} &= 3(\sin x)^2 \times \frac{d}{dx} \sin x \\ &= 3 \sin^2 x \cos x \end{aligned}$$

$$\begin{aligned} \frac{dy}{dx}\left(\frac{7\pi}{6}\right) &= 3 \sin^2\left(\frac{7\pi}{6}\right) \cos\left(\frac{7\pi}{6}\right) \\ &= 3 \left(-\sin^2\left(\frac{\pi}{6}\right)\right) \left(\cos\left(-\frac{\pi}{6}\right)\right) \\ &= 3 \cdot \frac{1}{4} \cdot -\frac{\sqrt{3}}{2} \\ &= -\frac{3\sqrt{3}}{8} \end{aligned}$$



continued over.

10A

(19)

continued

$$y - b = m(x - a)$$

$$y - -\frac{1}{8} = -\frac{3\sqrt{3}}{8} \left(x - \frac{7\pi}{6} \right)$$

$$y + \frac{1}{8} = -\frac{3\sqrt{3}}{8} \left(x - \frac{7\pi}{6} \right)$$

$$y + \frac{1}{8} = -\frac{3\sqrt{3}}{8}x + \frac{21\sqrt{3}\pi}{48} \quad \leftarrow \frac{1}{8}$$

$$y = -\frac{3\sqrt{3}}{8}x + \frac{7\sqrt{3}\pi}{16} - \frac{1}{8}$$

108

1 (c) $h(x) = x^4 - 2x^3 - 2x^2 - 3$

$$h'(x) = 4x^3 - 6x^2 - 4x$$

$$h'(-1) = 4(-1)^3 - 6(-1)^2 - 4(-1)$$

$$= \underline{\underline{-6}}$$

$h'(-1) < 0 \quad \therefore$ $h(x)$ decreasing at $x = -1$

1 (d) $p(x) = 6x - x^4$

$$p'(x) = 6 - 4x^3$$

$$p'(-2) = 6 - 4(-2)^3$$

$$p'(-2) = 38$$

$p'(-2) > 0 \quad \therefore$ $p(x)$ increasing at $x = -2$

2 (a) $f(x) = \frac{1}{3}x^3 - x^2 - 3x + 2$

$$f'(x) = x^2 - 2x - 3$$

when increasing $f'(x) > 0$

$$x^2 - 2x - 3 > 0$$

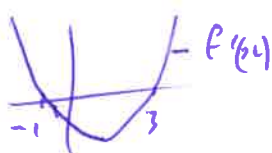
solve $x^2 - 2x - 3 = 0$

$$(x-3)(x+1) = 0$$

$$x-3=0 \quad x+1=0$$

$$x=3 \quad x=-1$$

sketch



$f'(x) > 0$ $x < -1$ and $x > 3$
(i.e. $f(x)$ increasing)

10B

$$2 \text{ (d)} \quad K(x) = x^4 + 32x$$

$$K'(x) = 4x^3 + 32$$

$$\text{If decreasing } 4x^3 + 32 < 0$$

$$4x^3 < -32$$

$$x^3 < -8$$

$$\underline{\underline{x < -2}}$$

$$5 \text{ (a)} \quad h(x) = (3x-1)^4$$

$$h'(x) = 4(3x-1)^3 \times \frac{d}{dx}(3x-1)$$

$$h'(\frac{1}{2}) = 4(3(\frac{1}{2})-1)^3 \times 3$$

$$= 12(\frac{1}{2})^3$$

$$= 12 \cdot \frac{1}{8}$$

$$= \frac{3}{2}$$

$$h'(\frac{1}{2}) > 0 \quad \therefore h(x) \text{ increasing at } x = \frac{1}{2}$$

$$\text{(b) when decreasing } h'(x) < 0$$

$$12(3x-1)^3 < 0$$

($\div 12$)

$$(3x-1)^3 < 0$$

($\div 12$)

($\sqrt[3]{}$)

$$3x-1 < 0$$

($\sqrt[3]{}$)

$$3x < 1$$

$$\underline{\underline{x < \frac{1}{3}}}$$

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$$\textcircled{8} \quad h(x) = 4 \sin x + 3 \cos \left(x + \frac{\pi}{6}\right)$$

$$h'(x) = 4 \cos x - 3 \sin \left(x + \frac{\pi}{6}\right) \times \frac{d}{dx} \left(x + \frac{\pi}{6}\right)$$

$$h'(x) = 4 \cos x - 3 \sin \left(x + \frac{\pi}{6}\right)$$

$$h'\left(\frac{\pi}{6}\right) = 4 \cos\left(\frac{\pi}{6}\right) - 3 \sin\left(\frac{\pi}{6} + \frac{\pi}{6}\right)$$

$$= 4 \cos \frac{\pi}{6} - 3 \sin\left(\frac{\pi}{3}\right)$$

$$= 4 \cdot \frac{\sqrt{3}}{2} - 3 \cdot \frac{\sqrt{3}}{2}$$

$$= \frac{\sqrt{3}}{2}$$

$h'\left(\frac{\pi}{6}\right) > 0 \quad \therefore h(x)$ increasing at $x = \frac{\pi}{6}$

$$\textcircled{11} \quad f(x) = \frac{1}{3}x^3 - 3x^2 + 10x - 5$$

$$f'(x) = x^2 - 6x + 10$$

$$f'(x) = x^2 - 6x + 9 + 1$$

$$f'(x) = (x-3)^2 + 1$$

$$(x-3)^2 + 1 > 0 \quad \text{for all } x$$

$\therefore f(x)$ is always increasing

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$$(12) \quad h(x) = \frac{2}{3}x^3 + 2x^2 + 2x - 3$$

$$\begin{aligned} h'(x) &= 2x^2 + 4x + 2 \\ &= 2(x^2 + 2x + 1) \\ &= 2(x+1)^2 \end{aligned}$$

$2(x+1)^2 \geq 0$ for all x \therefore $h(x)$ is never decreasing.

$$(15) \quad y = 2 \sin^3 x$$

$$y = 2 (\sin x)^3$$

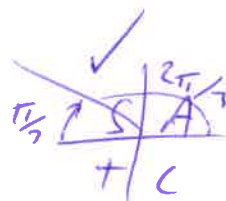
$$\begin{aligned} \frac{dy}{dx} &= 2 \times 3 (\sin x)^2 \times \frac{d}{dx} \sin x \\ &= 6 \sin^2 x \cos x \end{aligned}$$

$$\begin{aligned} \frac{dy}{dx} &= 6 \sin^2 \frac{2\pi}{3} \cos \frac{2\pi}{3} \\ &= 6 \left(\sin^2 \frac{\pi}{3} \right) \left(-\cos \frac{\pi}{3} \right) \end{aligned}$$

$$= 6 \left(\frac{\sqrt{3}}{2} \right)^2 \left(-\frac{1}{2} \right)$$

$$= 6 \times \frac{3}{4} \times \left(-\frac{1}{2} \right)$$

$$= -\frac{18}{8} = -\frac{9}{4}$$



$$\sin \frac{2\pi}{3} = \sin \frac{\pi}{3}$$

$$\cos \frac{2\pi}{3} = -\cos \frac{\pi}{3}$$

$\frac{dy}{dx} < 0$ at $x = \frac{2\pi}{3}$ \therefore y is decreasing.

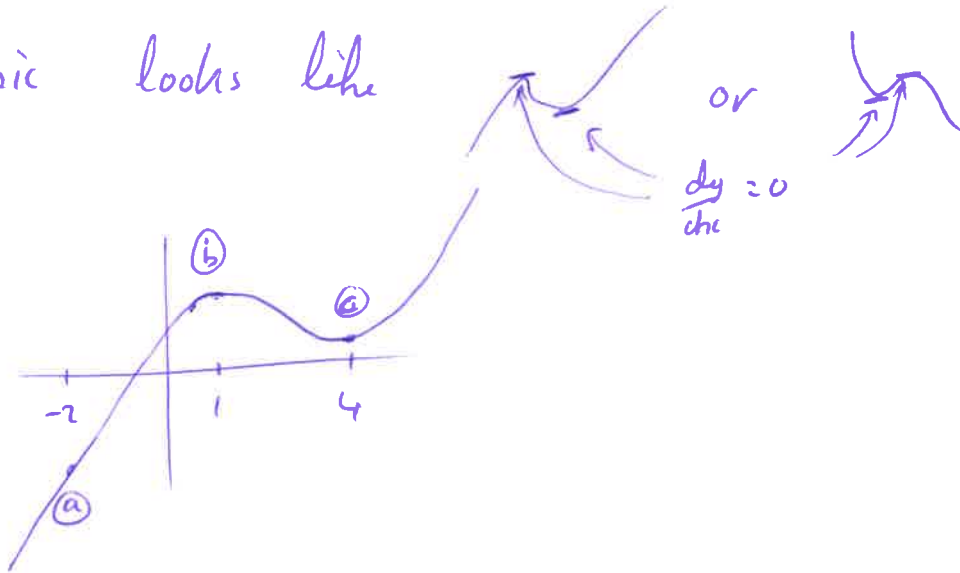
108

(18) (a) $f'(-2) > 0$ at $x = -2$ the curve is increasing

(b) at $x = 1$ the gradient is 0

(c) at $x = 4$ the gradient is 0

cubic looks like



A correct

B incorrect

C incorrect

D incorrect (curve's gradient gets steeper & steeper after $x = 4$)

E correct.

IOC

positive cubic

$$1(d) \quad y = x^3 - 2x^2 - 4x + 1$$

$$\frac{dy}{dx} = 3x^2 - 4x - 4$$

at stationary points $\frac{dy}{dx} = 0$

$$3x^2 - 4x - 4 = 0$$

$$(3x + 2)(x - 2) = 0$$

$$3x + 2 = 0 \quad x - 2 = 0$$

$$3x = -2 \quad \underline{x = 2}$$

$$\underline{x = -\frac{2}{3}}$$

$$y\left(-\frac{2}{3}\right) = \left(-\frac{2}{3}\right)^3 - 2\left(-\frac{2}{3}\right)^2 - 4\left(-\frac{2}{3}\right) + 1$$
$$= \frac{67}{27}$$

$$y(2) = (2)^3 - 2(2)^2 - 4(2) + 1$$
$$= -7$$

stationary points $\left(-\frac{2}{3}, \frac{67}{27}\right), (2, -7)$

Nature table

x	$\rightarrow -\frac{2}{3}$	$\rightarrow 2$	\rightarrow		
$\frac{dy}{dx} = (3x+2)(x-2)$	+	0	-	0	+
slope	/	-	\	-	/
	$\left(-\frac{2}{3}, \frac{67}{27}\right)$	$(2, -7)$			
	maximum	minimum			

IOC

negative cubic

$$(2)(d) \quad y = x^2(2-x)$$

$$y = 2x^2 - x^3$$

$$\frac{dy}{dx} = 4x - 3x^2$$

$\frac{dy}{dx} = 0$ for stationary point

$$4x - 3x^2 = 0$$

$$x(4-3x) = 0$$

$$\underline{x=0}, \quad 4-3x=0$$

$$4 = 3x$$

$$\frac{4}{3} = x$$

$$\underline{x = \frac{4}{3}}$$

$$y(0) = (0)^2(2-0) = 0$$

$$y\left(\frac{4}{3}\right) = \left(\frac{4}{3}\right)^2\left(2 - \frac{4}{3}\right) = \frac{32}{27}$$

stationary points at $(0,0)$ and $\left(\frac{4}{3}, \frac{32}{27}\right)$

Nature table

	x	$\rightarrow 0$	$\rightarrow \frac{4}{3}$	\rightarrow
$\frac{dy}{dx} = x(4-3x)$		-	0	+ \rightarrow -
slope		\	-	/ - \
		$(0,0)$		$\left(\frac{4}{3}, \frac{32}{27}\right)$
		minimum		maximum

100

$$3d \quad y = \frac{1}{2} x^2 (2 - x^2)$$

$$= x^2 - \frac{1}{2} x^4$$

$$\frac{dy}{dx} = 2x - 2x^3$$

$\frac{dy}{dx} = 0$ for stationary point

$$2x - 2x^3 = 0$$

$$2x(1 - x^2) = 0$$

$$2x(1-x)(1+x) = 0$$

$$2x = 0 \quad 1-x = 0 \quad 1+x = 0$$

$$x = 0 \quad x = 1 \quad x = -1$$

$$y(-1) = \frac{1}{2} (1)^2 (2 - 1^2)$$

$$= \frac{1}{2}$$

$$(-1, \frac{1}{2})$$

$$y(0) = \frac{1}{2} (0)^2 (2 - (0)^2)$$

$$= 0$$

$$(0, \frac{1}{2})$$

$$y(1) = \frac{1}{2} (1)^2 (2 - (1)^2)$$

$$= \frac{1}{2}$$

$$(1, \frac{1}{2})$$

Nature table

x	\rightarrow	-1	\rightarrow	0	\rightarrow	1	\rightarrow
$\frac{dy}{dx} = 2x(1-x)(1+x)$	+	0	-	0	+	0	-
slope	/	-	\	-	/	-	\
		<u>$(-1, \frac{1}{2})$ Maximum</u>		<u>$(0, 0)$ Minimum</u>		<u>$(1, \frac{1}{2})$ Maximum</u>	

10C

$$\textcircled{4} \text{ (a) } f(x) = (4x-1)^5$$
$$f'(x) = 5(4x-1)^4 \times \frac{d}{dx}(4x-1)$$
$$= \underline{20(4x-1)^4}$$

$f'(x) = 0$ for stationary point

$$20(4x-1)^4 = 0$$

$$(4x-1)^4 = 0$$

$$4x-1 = 0$$

$$4x = 1$$

$$x = \underline{\underline{\frac{1}{4}}}$$

$$f\left(\frac{1}{4}\right) = \left(4\left(\frac{1}{4}\right) - 1\right)^5$$

$$= (0)^5$$

$$= 0$$

stationary point $\left(\frac{1}{4}, 0\right)$

(b)	x	$\rightarrow \frac{1}{4} \rightarrow$
$\frac{dy}{dx} = 20(4x-1)^4$		+ 0 +
slope		/ — /

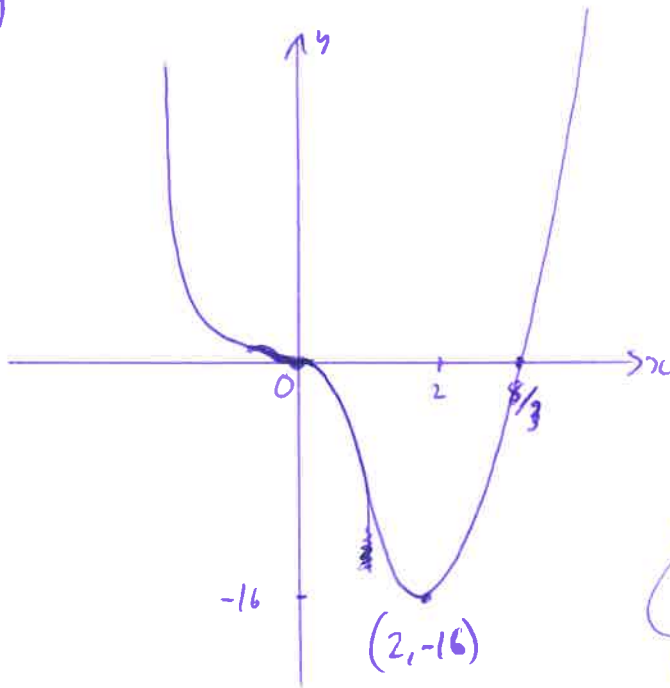
$\left(\frac{1}{4}, 0\right)$ point of inflexion

LOD

1g(ii) continued

x	$\leftarrow 0 \rightarrow 2 \rightarrow$
$\frac{dy}{dx} = 12x^2(x-2)$	$- \quad 0 \quad - \quad 0 \quad +$
slope	$\diagdown \quad \text{---} \quad \diagup \quad \text{---} \quad \diagdown$
	$(0,0)$ $(2,-16)$
	falling point of inflexion minimum

1g(iii)



Note: Annotate points or you will drop marks.

100

$$(9) (i) y = 3x^4 - 8x^3$$

$$y\text{-int: } y(0) = 3(0)^4 - 8(0)^3$$

$$y(0) = 0$$

$$\underline{(0, 0)}$$

$$x\text{-int: } 0 = 3x^4 - 8x^3$$

$$(y=0) \quad 0 = x^3(3x-8)$$

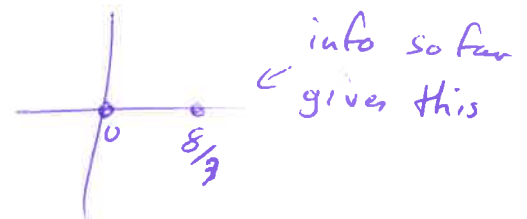
$$x^3 = 0 \quad 3x - 8 = 0$$

$$x = 0 \quad 3x = 8$$

$$x = \frac{8}{3}$$

$$\underline{(0, 0)}$$

$$\underline{\left(\frac{8}{3}, 0\right)}$$



$$(ii) \quad \frac{dy}{dx} = 12x^3 - 24x^2$$

$$\frac{dy}{dx} = 0 \quad \text{for stationary point}$$

$$12x^3 - 24x^2 = 0$$

$$12x^2(x-2) = 0$$

$$12x^2 = 0 \quad x - 2 = 0$$

$$x^2 = 0 \quad \underline{x = 2}$$

$$\underline{x = 0}$$

$$y(0) = 3(0)^4 - 8(0)^3$$

$$= 0$$

$$\underline{(0, 0)}$$

$$y(2) = 3(2)^4 - 8(2)^3$$

$$= -16$$

$$\underline{(2, -16)}$$

continued over

101

(2) (a) (i)
$$\begin{array}{ccc|ccc} 1 & 3 & 0 & -4 & & \\ & 1 & 4 & 4 & & \\ \hline 1 & 4 & 4 & 0 & r=0 & \end{array} \therefore x=1 \text{ is a root and } (x-1) \text{ is a factor of } f(x).$$

(ii) $f(x) = (x-1)(x^2+4x+4)$

$f(x) = (x-1)(x+2)(x+2)$

(b) $f'(x) = 3x^2 + 6x$

$f'(x) = 0$ for stationary point

$3x^2 + 6x = 0$

$3x(x+2) = 0$

$3x = 0 \quad (x+2) = 0$

$x = 0 \quad x = -2$

$f(0) = (0)^3 + 3(0)^2 - 4$
 $= -4$

$(0, -4)$

$f(-2) = (-2)^3 + 3(-2)^2 - 4$
 $= 0$

$(-2, 0)$

Nature

x	$\rightarrow -2$	$\rightarrow 0$	\rightarrow
$\frac{dy}{dx} = 3x(x+2)$	$\begin{array}{c} + \\ 0 \end{array}$	$-$	$\begin{array}{c} 0 \\ + \end{array}$
Slope	\diagup	\diagdown	\diagup

maximum at $(-2, 0)$

minimum at $(0, -4)$

100

2(c) y-int : $f(0) = (0)^3 + 3(0)^2 - 4$
 $= -4$

$(0, -4)$

x-int $f(x) = 0$

$(x-1)(x+2)(x+2) = 0$

From part (a)

$x-1 = 0$

$x+2 = 0$

$x+2 = 0$

$x = 1$

$x = -2$

$x = -2$

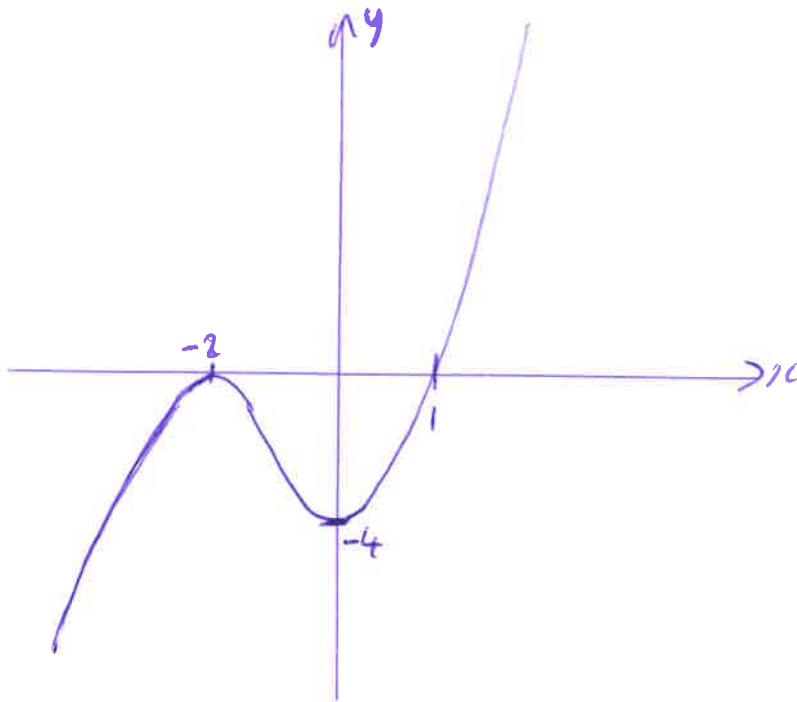
Note: $x = -2$ is a repeated root so curve will touch but not cross.

$f(x) = (x)^3 + 3(x)^2 - 4$

$(1, 0)$

$(-2, 0)$

2(d)



101)

$$5(a) \quad -2 \left| \begin{array}{cccc} 1 & -1 & -1 & 10 \\ & -2 & 6 & -10 \\ \hline 1 & -3 & 5 & 0 \end{array} \right.$$

$r=0 \therefore x = -2$ is a root and $(x+2)$ is a factor of $f(x)$.

$$f(x) = \underline{\underline{(x+2)(x^2-3x+5)}}$$

$\leftarrow b^2 - 4ac$

$$= (-3)^2 - 4(1)(5)$$

$$= -11 < 0 \therefore \text{no real roots}$$

i.e. cannot be factored further.

(b) $y = x^3 - x^2 - x + 10$

$$\frac{dy}{dx} = 3x^2 - 2x - 1$$

$\frac{dy}{dx} = 0$ for stationary points

$$3x^2 - 2x - 1 = 0$$

$$(3x+1)(x-1) = 0$$

$$3x+1=0 \quad x-1=0$$

$$3x = -1 \quad x = 1$$

$$x = -\frac{1}{3}$$

$$y\left(-\frac{1}{3}\right) = \left(-\frac{1}{3}\right)^3 - \left(-\frac{1}{3}\right)^2 - \left(-\frac{1}{3}\right) + 10$$

$$= 10\frac{5}{27}$$

$\left(-\frac{1}{3}, 10\frac{5}{27}\right)$ maximum

$$y(1) = (1)^3 - (1)^2 - (1) + 10$$

$$= 9$$

$(1, 9)$ minimum

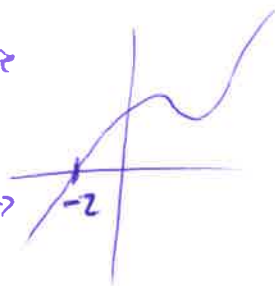
Nature	x	$\rightarrow -\frac{1}{3}$	$\rightarrow 1$	\rightarrow
$\frac{dy}{dx} = (3x+1)(x-1)$		+	0	-
slope		/	\	/

10D

10(E)

5(c) $f(x)$ is a positive cubic and there is only one root ($x+2=0$) at $x=-2$, stationary points occur after $x=-2$ so y coordinates must be positive

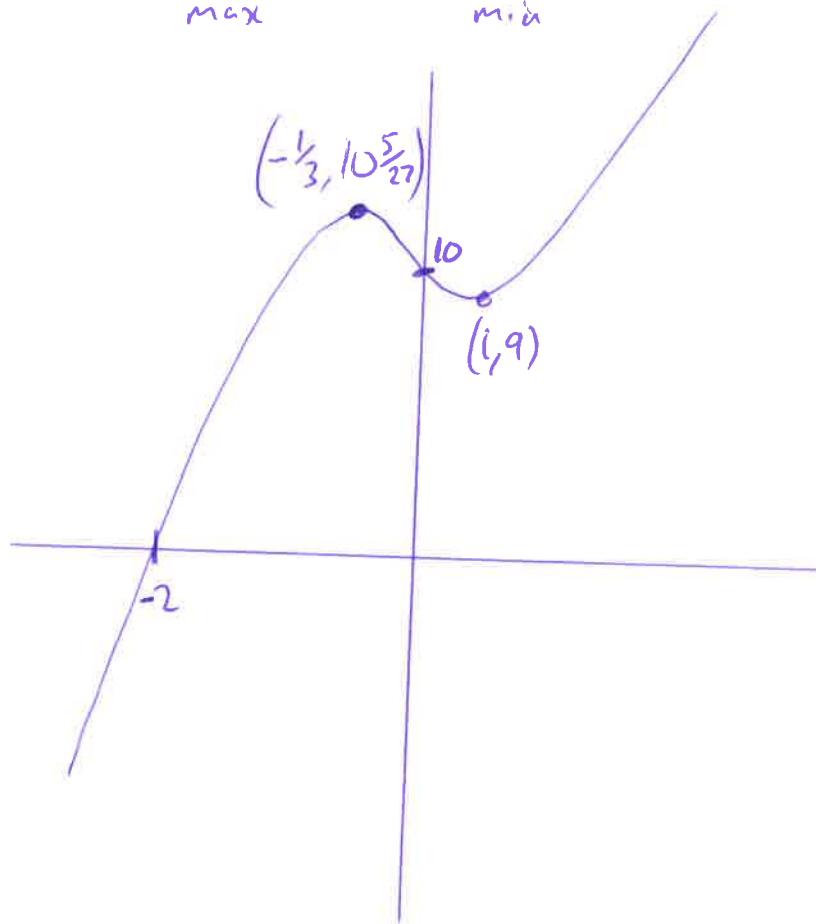
only root \rightarrow
so graph does not drop below x -axis again.



5(d) x int: $x=-2$

y int ($x=0$) $y=10$

S.P. $(-\frac{1}{3}, 10\frac{5}{27})$ and $(1, 9)$
max min



(Not to scale)

10D

7(a)

$$\begin{array}{r|rrrr} -2 & 1 & k & k+1 & 18 \\ & & -2 & -2k+4 & +2k-10 \\ \hline & 1 & k-2 & -k+5 & \end{array} \quad \begin{array}{l} 8+2k=0 \\ \text{as } x=-2 \text{ is a root.} \end{array}$$

$$\begin{array}{l} 8+2k=0 \\ 8 = -2k \\ k = \underline{\underline{-4}} \end{array} \quad \begin{array}{l} \rightarrow 2k = -8 \\ k = \underline{\underline{-4}} \end{array}$$

$$\begin{aligned} f(x) &= (x+2)(x^2 + (k-2)x + (-k+5)) \\ &= (x+2)(x^2 - 6x + 9) \quad (\text{as } k=-4) \\ &= \underline{\underline{(x+2)(x-3)^2}} \end{aligned}$$

(b) Repeated root at $x=3 \therefore x=3$ is a stationary point ($m=0$) and the x -axis will be a tangent to the curve.

(c) $f(x) = x^3 - 4x^2 + 3x + 18$ (as $k=-4$)

$$f'(x) = 3x^2 - 8x + 3$$

$f'(x) = 0$ for stationary points

$$3x^2 - 8x + 3 = 0$$

$$(3x+1)(x-3) = 0$$

$$3x+1 = 0 \quad x-3 = 0$$

$$3x = -1 \quad \underline{\underline{x = -\frac{1}{3}}}$$

$$\underline{\underline{x = -\frac{1}{3}}}$$

10)

7(c) continued

$$f\left(-\frac{1}{3}\right) = \left(-\frac{1}{3}\right)^3 - 4\left(-\frac{1}{3}\right)^2 - 3\left(-\frac{1}{3}\right) + 18$$
$$= 18\frac{14}{27}$$

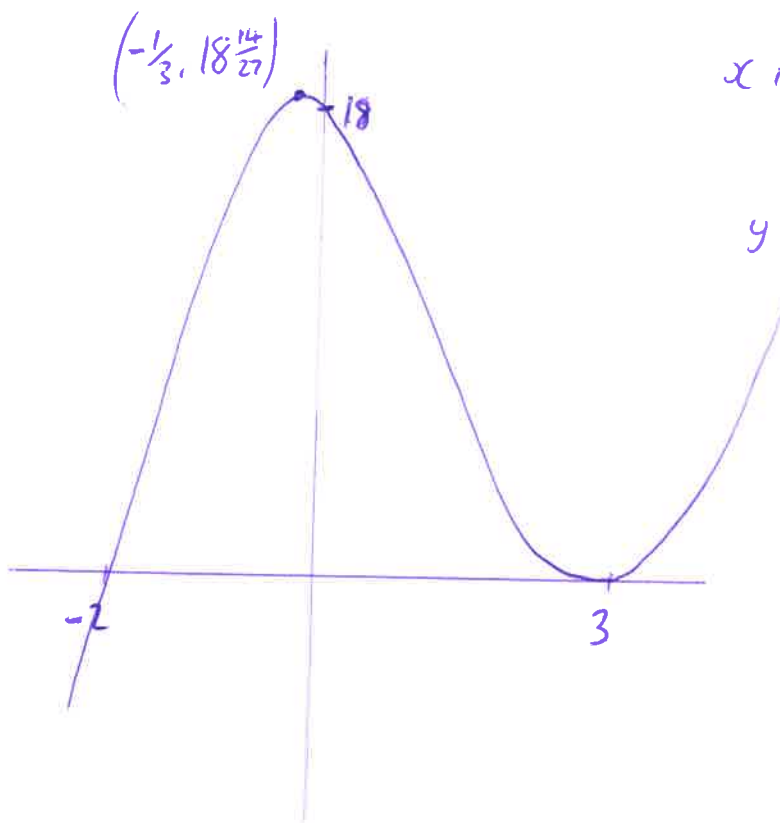
$$f(3) = (3)^3 - 4(3)^2 - 3(3) + 18$$
$$= 0$$

$$\left(-\frac{1}{3}, 18\frac{14}{27}\right) \quad (3, 0)$$

Nature:

	x	\rightarrow	$-\frac{1}{3}$	\rightarrow	3	\rightarrow	
$\frac{dy}{dx} = (3x+1)(x-3)$			+	0	-	0	+
slope			/	-	\	-	/
			$\left(-\frac{1}{3}, 18\frac{14}{27}\right)$		$(3, 0)$		
			maximum		minimum		

(d)



$$x \text{ int: } (x+2)(x-3)^2 = 0$$

$$x = -2, x = 3$$

$$y \text{ int: } y(0) = 18$$