11A

1(a) $\int x^6 \, dx$

$= \frac{x^7}{7} + c$

(c) $\int x^{-3} \, dx$

$= \frac{x^{-2}}{-2} + c$

$= -\frac{x^{-2}}{2} + c$

$= \frac{-1}{2x^2} + c$

(e) $\int 4x^3 \, dx$

$= \frac{4x^4}{4} + c$

$= x^4 + c$

(g) $\int \frac{3}{4} x^5 \, dx$

$= \frac{3}{4} \times \frac{x^6}{6} + c$

$= \frac{3x^6}{24} + c$

$= \frac{x^6}{8} + c$

(i) $\int x^{3/2} \, dx$

$= \frac{x^{5/2}}{5/2} + c$ $\quad \frac{1}{5/2} = 1 \times \frac{2}{5} = \frac{2}{5}$

$= \frac{2}{5} x^{5/2} + c$

(k) $\int x^{-1/2} \, dx$

$= \frac{x^{1/2}}{1/2} + c$ $\quad \frac{1}{1/2} = 1 \times \frac{2}{1} = 2$

$= 2x^{1/2} + c$

(m) $\int 6x^{1/2} \, dx$

$= \frac{6x^{3/2}}{3/2} + c$

$= \frac{2}{3} \times 6x^{3/2} + c$

$= 4x^{3/2} + c$

2(a) $\int 3x^2 + x - 1 \, dx$

$= \frac{3x^3}{3} + \frac{x^2}{2} - x + c$

$= x^3 + \frac{x^2}{2} - x + c$

11A

2(c) $\int \frac{3}{2}x^5 - \frac{1}{4}x - 4 \, dx$

$= \frac{3}{2}\frac{x^6}{6} - \frac{1}{4}\frac{x^2}{2} - 4x + c$

$= \frac{3x^6}{12} - \frac{x^2}{8} - 4x + c$

$= \frac{x^6}{4} - \frac{x^2}{8} - 4x + c$

3(a) $\int x^{\frac{1}{2}} - x^{-\frac{1}{2}} \, dx$

$= \frac{x^{3/2}}{3/2} - \frac{x^{+1/2}}{1/2} + c$

$= \frac{2x^{3/2}}{3} - 2x^{1/2} + c$

This would be fine for final answer.

$= \frac{2}{3}x^{1/2}(x-3) + c$

$= \frac{2}{3}\sqrt{x}(x-3) + c$

3(c) $\int \frac{2}{3}x^{2/3} - 4x^{-1/5} - 8x \, dx$

$= \frac{2}{3}\frac{x^{5/3}}{5/3} - \frac{4x^{4/5}}{4/5} - \frac{8x^2}{2} + c$

$= \frac{2}{\cancel{3}} \times \frac{\cancel{3}}{5} x^{5/3} - 4 \times \frac{5}{4} x^{4/5} - 4x^2 + c$

$= \frac{2x^{5/3}}{5} - 5x^{4/5} - 4x^2 + c$

4(a) $\int k^4 + 3k - 5 \, dk$

$= \frac{k^5}{5} + \frac{3k^2}{2} - 5k + c$

(c) $\int 9t^{1/2} + 4t^{-1/2} \, dt$

$= \frac{9t^{3/2}}{3/2} + \frac{4t^{1/2}}{1/2} + c$

$= \frac{2}{3}9t^{3/2} + 8t^{1/2} + c$

$= 6t^{3/2} + 8t^{1/2} + c$

$= 2t^{1/2}(t+4) + c$

$= 2\sqrt{t}(t+4) + c$

11B

1(a)(i) $(2x-1)(x-3)$

$= 2x^2 - 6x - x + 3$

$= 2x^2 - 7x + 3$

(ii) $\int 2x^2 - 7x + 3 \, dx$

$= \frac{2x^3}{3} - \frac{7x^2}{2} + 3x + c$

1(c)(i) $(x+2)(x^2+3x-4)$

$= x^3 + 3x^2 - 4x + 2x^2 + 6x - 8$

$= x^3 + 5x^2 + 2x - 8$

(ii) $\int x^3 + 5x^2 + 2x - 8 \, dx$

$= \frac{x^4}{4} + \frac{5x^3}{3} + x^2 - 8x + c$

1(e)(i) $(x-2)(x+3)^2$

$= (x-2)(x^2+6x+9)$

$= x^3 + 6x^2 + 9x - 2x^2 - 12x - 18$

$= x^3 + 4x^2 - 3x - 18$

(ii) $\int x^3 + 4x^2 - 3x - 18 \, dx$

$= \frac{x^4}{4} + \frac{4x^3}{3} - \frac{3x^2}{2} - 18x + c$

3(a) $\frac{6}{x^3}$

$= 6x^{-3}$

$\int 6x^{-3} \, dx$

$= \frac{6x^{-2}}{-2} + c$

$= -3x^{-2} + c$

$= \frac{-3}{x^2} + c$

(c) $y = \frac{7}{3x^8}$

$y = \frac{7x^{-8}}{3}$

$\int \frac{7x^{-8}}{3} \, dx$

$= \frac{7x^{-7}}{3 \times (-7)} + c$

$= \frac{-x^{-7}}{3} + c$

$= \frac{-1}{3x^7} + c$

11B

4(c) $6\sqrt[5]{x}$

$= 6x^{1/5}$

$\int 6x^{1/5}\,dx$

$= \dfrac{6x^{6/5}}{6/5} + c$

$= \dfrac{5}{\cancel{6}_1} \times \cancel{6}^1 x^{6/5} + c$

$= 5x^{6/5} + c$

4(e) $\dfrac{1}{x\sqrt{x}}$

$= \dfrac{1}{x\,x^{1/2}}$

$= \dfrac{1}{x^{3/2}}$

$= x^{-3/2}$

$\int x^{-3/2}\,dx$

$= \dfrac{x^{-1/2}}{-1/2} + c$

$= -2x^{-1/2} + c$

$= \dfrac{-2}{\sqrt{x}} + c$

4(h) $\dfrac{1}{2\sqrt[4]{x^3}}$

$= \dfrac{1}{2x^{3/4}}$

$= \dfrac{x^{-3/4}}{2}$

$\int \dfrac{x^{-3/4}}{2}\,dx$

$= \dfrac{x^{1/4}}{2 \times \frac{1}{4}} + c$

$= \dfrac{x^{1/4}}{\frac{1}{2}} + c$

$= 2x^{1/4} + c$

$= 2\sqrt[4]{x} + c$

11B

5(e) $\dfrac{x^4 - x - 3}{x^4}$

$= \dfrac{x^4}{x^4} - \dfrac{x}{x^4} - \dfrac{3}{x^4}$

$= 1 - \dfrac{1}{x^3} - \dfrac{3}{x^4}$

$= 1 - x^{-3} - 3x^{-4}$

$\int 1 - x^{-3} - 3x^{-4} \, dx$

$= x - \dfrac{x^{-2}}{-2} - \dfrac{3x^{-3}}{-3} + c$

$= x + \dfrac{x^{-2}}{2} + \dfrac{\cancel{3}^{1} x^{-3}}{\cancel{3}_{1}} + c$

$= x + \dfrac{1}{2x^2} + \dfrac{1}{x^3} + c$

5(e) $\dfrac{(x-2)(x+3)}{x^4}$

$= \dfrac{x^2 + x - 6}{x^4}$

$= \dfrac{x^2}{x^4} + \dfrac{x}{x^4} - \dfrac{6}{x^4}$

$= x^{-2} + x^{-3} - 6x^{-4}$

$\int x^{-2} + x^{-3} - 6x^{-4} \, dx$

5(e) continued

$= \dfrac{x^{-1}}{-1} + \dfrac{x^{-2}}{-2} - \dfrac{6x^{-3}}{-3} + c$

$= -x^{-1} - \dfrac{x^{-2}}{2} + 2x^{-3} + c$

$= -\dfrac{1}{x} - \dfrac{1}{2x^2} + \dfrac{2}{x^3} + c$

11B

6(c) $(1-x^2)\left(2+\frac{1}{\sqrt{x}}\right)$

$= 2 + \frac{1}{\sqrt{x}} - 2x^2 - \frac{x^2}{\sqrt{x}}$

$= 2 + \frac{1}{x^{1/2}} - 2x^2 - x^{3/2}$

$= 2 + x^{-1/2} - 2x^2 - x^{3/2}$

$\int 2 + x^{-1/2} - 2x^2 - x^{3/2}\,dx$

$= 2x + \frac{x^{1/2}}{\frac{1}{2}} - \frac{2x^3}{3} - \frac{x^{5/2}}{5/2} + c$

$= 2x + 2x^{1/2} - \frac{2x^3}{3} - \frac{2}{5}x^{5/2} + c$

$= 2x + 2\sqrt{x} - \frac{2x^3}{3} - 2\sqrt{x^5} + c$

6 (e) $\frac{1-x^3}{\sqrt{x}}$

$= \frac{1}{\sqrt{x}} - \frac{x^3}{\sqrt{x}}$

$= \frac{1}{x^{1/2}} - \frac{x^3}{x^{1/2}}$

$= x^{-1/2} - x^{5/2}$

6(e) continued

$\int x^{-1/2} - x^{5/2}\,dx$

$= \frac{x^{1/2}}{1/2} - \frac{x^{7/2}}{7/2} + c$

$= 2x^{1/2} - \frac{2}{7}x^{7/2} + c$

$= 2\sqrt{x} - \frac{2\sqrt{x^7}}{7} + c$

11B

7(f) $\dfrac{3-x^4}{x^3}$

$= \dfrac{3}{x^3} - \dfrac{x^4}{x^3}$

$= 3x^{-3} - x$

$\int 3x^{-3} - x$

$= \dfrac{3x^{-2}}{-2} - \dfrac{x^2}{2} + c$

$= \dfrac{-3x^{-2}}{2} - \dfrac{x^2}{2} + c$

$= \dfrac{-3}{2x^2} - \dfrac{x^2}{2} + c$

7(h) $\dfrac{1}{u^2} - 3\sqrt{u} + 2$

$= u^{-2} - 3u^{1/2} + 2$

$\int u^{-2} - 3u^{1/2} + 2\, du$

$= \dfrac{u^{-1}}{-1} - \dfrac{3u^{3/2}}{3/2} + 2u + c$

$= -u^{-1} - \frac{2}{3} \times 3u^{3/2} + 2u + c$

$= \dfrac{-1}{u} - 2u^{3/2} + 2u + c$

7(i) $\dfrac{3}{4\sqrt[5]{x}}$

$= \dfrac{3}{4x^{1/5}}$

$= \dfrac{3x^{-1/5}}{4}$

$\int \dfrac{3x^{-1/5}}{4}\, dx$

$= \dfrac{3x^{4/5}}{4 \times \frac{4}{5}} + c$

$= \dfrac{3x^{4/5}}{\frac{16}{5}} + c$

$= \dfrac{5}{16} \times 3x^{4/5} + c$

$= \dfrac{15}{16} x^{4/5} + c$

11C

1(f) $\int (5x-2)^7\,dx$

$$= \frac{(5x-2)^8}{5\times 8} + c$$

$$= \frac{(5x-2)^8}{40} + c$$

2(f) $\int (2x+1)^{-2}\,dx$

$$= \frac{(2x+1)^{-1}}{2\times(-1)} + c$$

$$= \frac{(2x+1)^{-1}}{-2} + c$$

$$= -\frac{(2x+1)^{-1}}{2} + c$$

$$= \frac{-1}{2(2x+1)} + c$$

3(c) $\int 12(x+6)^{\frac{1}{5}}\,dx$

$$= \frac{12(x+6)^{6/5}}{1\times\frac{6}{5}} + c$$

$$= \frac{5}{6}\,12(x+6)^{6/5} + c$$

$$= 10(x+6)^{6/5} + c$$

3(k) $\int \frac{5}{8}(6x+1)^{\frac{1}{4}}\,dx$

$$= \frac{5}{8}\,\frac{(6x+1)^{5/4}}{6\times 5/4} + c$$

$$= \frac{\cancel{5}^{1}}{\cancel{8}_{2}}\times\frac{\cancel{4}^{1}}{\cancel{5}_{1}}\,\frac{(6x+1)^{5/4}}{6} + c$$

$$= \frac{(6x+1)^{5/4}}{2\times 6} + c$$

$$= \frac{(6x+1)^{5/4}}{12} + c$$

4(f) $\int \frac{9}{(5x-3)^2}\,dx$

$$= \int 9(5x-3)^{-2}\,dx$$

$$= \frac{9(5x-3)^{-1}}{5\times(-1)} + c$$

$$= \frac{9(5x-3)^{-1}}{-5} + c$$

$$= \frac{-9}{5(5x-3)} + c$$

$$= \frac{-9}{25x-15} + c$$

$$= \frac{9}{15-25x} + c$$

11C

5(h) $\int \frac{1}{\sqrt{(x-2)^3}}\,dx$

$= \int \frac{1}{(x-2)^{3/2}}\,dx$

$= \int (x-2)^{-3/2}\,dx$

$= \frac{(x-2)^{-1/2}}{-\frac{1}{2}} + c$

$= -2(x-2)^{-1/2} + c$

$= \frac{-2}{(x-2)^{1/2}} + c$

$= \frac{-2}{\sqrt{x-2}} + c$

5(l) $\int \frac{4}{5\sqrt[6]{(x-1)^5}}\,dx$

$= \int \frac{4}{5(x-1)^{5/6}}\,dx$

$= \int \frac{4}{5}(x-1)^{-5/6}\,dx$

$= \frac{4}{5}\,\frac{(x-1)^{1/6}}{1 \times \frac{1}{6}} + c$

$= \frac{4}{5} \times 6(x-1)^{1/6} +$ $\quad = \frac{24(x-1)^{1/6}}{5} + c$

11C

7(g) $\int \frac{1}{(6x-5)^4}\,dx$

$= \int (6x-5)^{-4}\,dx$

$= \frac{(6x-5)^{-3}}{6\times(-3)} + c$

$= \frac{(6x-5)^{-3}}{-18} + c$

$= -\frac{(6x-5)^{-3}}{18} + c$

$= \frac{-1}{18(6x-5)^3} + c$ ✓ $\left(= \frac{1}{18(5-6x)^3} + c \right)$

11E

1(a) $\int 8\cos x \, dx$

$= \underline{\underline{8\sin x + c}}$

1(c) $\int -4\sin x \, dx$

$= -4\times(-\cos x) + c$

$= \underline{\underline{4\cos x + c}}$

(g) $\int 4\cos\left(x - \frac{\pi}{3}\right) dx$

$= \underline{\underline{4\sin\left(x - \frac{\pi}{3}\right)}} + c$

(j) $\int \sin 4x \, dx$

$= \frac{1}{4} \times -\cos 4x + c$

$= -\frac{1}{4}\cos 4x + c$

(l) $\int \frac{1}{2}\cos(3x - \pi) \, dx$

$= \frac{1}{2} \times \frac{1}{3}\sin(3x - \pi) + c$

$= \frac{1}{6}\sin(3x - \pi) + c$ ✓

$= -\frac{1}{6}\sin 3x +$

$\sin(A-B) = \sin A\cos B - \cos A\sin B$

note: $\sin(3x - \pi) = \sin 3x \cancelto{-1}{\cos\pi} - \cos 3x \cancelto{0}{\sin\pi}$

$= -\sin 3x$

11E

2(e) $\int (4x+1)^5 - \cos 3x \, dx$

$= \dfrac{(4x+1)^6}{4 \times 6} - \dfrac{1}{3} \sin 3x + c$

$= \dfrac{(4x+1)^6}{24} - \dfrac{1}{3} \sin 3x + c$

(f) $\int \dfrac{x-1}{x^3} + 4 \sin(x-1) \, dx$

$= \int \dfrac{x}{x^3} - \dfrac{1}{x^3} + 4 \sin(x-1) \, dx$

$= \int x^{-2} - x^{-3} + 4 \sin(x-1) \, dx$

$= \dfrac{x^{-1}}{-1} - \dfrac{x^{-2}}{-2} + 4 \times (-\cos(x-1)) + c$

$= -\dfrac{1}{x} + \dfrac{1}{2x^2} - 4 \cos(x-1) + c$ ✓ (good enough)

$= -\dfrac{1}{x} \times \dfrac{2x}{2x} + \dfrac{1}{2x^2} - 4 \cos(x-1) + c$

$= \dfrac{-2x}{2x^2} + \dfrac{1}{2x^2} - 4 \cos(x-1) + c$

$= \dfrac{1-2x}{2x^2} - 4 \cos(x-1) + c$

11E

2(j) $\int \sqrt{1-4x} + 2\sin\left(3x+\frac{\pi}{4}\right)\, dx$

$= \int (1-4x)^{\frac{1}{2}} + 2\sin\left(3x+\frac{\pi}{4}\right)\, dx$

$= \dfrac{(1-4x)^{3/2}}{(-4)\times\frac{3}{2}} + 2 \times \left(-\frac{1}{3}\cos\left(3x+\frac{\pi}{4}\right)\right) + c$

$= \dfrac{(1-4x)^{3/2}}{-6} - \dfrac{2}{3}\cos\left(3x+\frac{\pi}{4}\right) + c$

$= -\frac{1}{6}(1-4x)^{3/2} - \frac{2}{3}\cos\left(3x+\frac{\pi}{4}\right) + c$

3(a) $\cos 2x = 2\cos^2 x - 1$

$\cos 2x + 1 = 2\cos^2 x$

$\frac{1}{2}(\cos 2x + 1) = \cos^2 x$

$\cos^2 x = \frac{1}{2}\cos 2x + \frac{1}{2}$

$\cos^2 x = \frac{1}{2} + \frac{1}{2}\cos 2x$

$\int \cos^2 x\, dx$

$= \int \frac{1}{2} + \frac{1}{2}\cos 2x\, dx$

$= \frac{1}{2}x + \frac{1}{2}\times\frac{1}{2}\sin 2x + c$

$= \frac{1}{2}x + \frac{1}{4}\sin 2x + c$

11C

1(f) $\int (5x-2)^7 \, dx$

$$= \frac{(5x-2)^8}{8 \times 5} + c$$

$$= \frac{(5x-2)^8}{40} + c$$

2(f) $\int (2x+1)^{-2} \, dx$

$$= \frac{(2x+1)^{-1}}{-1 \times 2} + c$$

$=$

$$\sin(3x-\pi) = \sin 3x \overset{-1}{\cancel{\cos\pi}} - \cos 3x \overset{0}{\cancel{\sin\pi}}$$

$$= -\sin 3x$$

11 E

5(a) $\int 4\sin^2 x$

$= \int 4 \times \left(\frac{1}{2} - \frac{1}{2}\cos 2x\right) dx$

$1 - 2\sin^2 x = \cos 2x$

$1 = \cos 2x + 2\sin^2 x$

$1 - \cos 2x = 2\sin^2 x$

$\frac{1}{2} - \frac{1}{2}\cos 2x = \sin^2 x$

$= \int 2 - 2\cos 2x \, dx$

$= 2x - 2 \times \frac{1}{2}\sin 2x + c$

$= 2x - \sin 2x + c$

5 (d) $\int \cos^2 x - \frac{2}{3}x^2 \, dx$

$= \int \frac{1}{2} + \frac{1}{2}\cos 2x - \frac{2}{3}x^2 \, dx$

* See 3(a) for $\cos^2 x = \frac{1}{2} + \frac{1}{2}\cos 2x$

$= \frac{1}{2}x + \frac{1}{2} \times \frac{1}{2}\sin 2x - \frac{2}{3}\frac{x^3}{3} + c$

$= \frac{1}{2}x + \frac{1}{4}\sin 2x - \frac{2x^3}{9} + c$

11G

(1) $\frac{dy}{dx} = 3x^2 - 2x + 4$

$y = \int 3x^2 - 2x + 4 \, dx$

$y = x^3 - x^2 + 4x + c$

$x = -2 \quad y = 5$

$5 = (-2)^3 - (-2)^2 + 4(-2) + c$

$5 = -8 - 4 - \cancel{12}\,8 + c$

$5 = c - 20$

$\underline{\underline{c = 25}}$

$y = x^3 - x^2 + 4x + 25$

(3) $g'(x) = 4 \sin 2x$

$g(x) = \int 4 \sin 2x \, dx$

$g(x) = 4 \times -\frac{1}{2} \cos 2x + c$

$g(x) = -2 \cos 2x + c$

$x = \frac{\pi}{3} \quad y = g(x) = 4$

$4 = -2 \cos\left(2 \frac{\pi}{3}\right) + c$

$4 = -2 \times \left(-\cos\left(\frac{\pi}{3}\right)\right) + c$

$4 = -2 \times -\frac{1}{2} + c$

$4 = 1 + c$

$c = 3 \qquad \Rightarrow \underline{\underline{g(x) = -2\cos 2x + 3}}$

11G

5) $f'(x) = 4(x-1)$

$f'(x) = 4x-4$

$f(x) = \int 4x-4 \, dx$

$f(x) = \frac{4x^2}{2} - 4x + c$

$f(x) = 2x^2 - 4x + c$

$f(0)=7$ i.e. when $x=0$ $f(x)=7$

$7 = 2(0)^2 - 4(0) + c$

$c = 7$

$f(x) = 2x^2 - 4x + 7$

(b) $2(x^2-2x) + 7$

$= 2((x-1)^2-1) + 7$

$= 2(x-1)^2 - 2 + 7$

$= 2(x-1)^2 + 5$

min value $= 5$ at $x=1$ $\therefore$ $f(x)$ will not cross x-axis

or use b^2-4ac to show no real roots.

11G

(7) $f'(x) = 3x^2 + 4x - 4$

$$f(x) = \frac{3x^3}{3} + \frac{4x^2}{2} - 4x + c$$

$$= x^3 + 2x^2 - 4x + c$$

$x = -1 \quad y = f(x) = -3$

$$-3 = (-1)^3 + 2(-1)^2 - 4(-1) + c$$

$$-3 = -1 + 2 + 4 + c$$

$$-3 = 5 + c$$

$$c = -8$$

$$f(x) = x^3 + 2x^2 - 4x - 8$$

(b) (i)

$$\begin{array}{c|cccc} 2 & 1 & 2 & -4 & -8 \\ & & 2 & 8 & 8 \\ \hline & 1 & 4 & 4 & \boxed{0} \end{array}$$

$r = 0 \therefore$ $x = 2$ is a root and $(x-2)$ is a factor of $f(x)$.

(ii) $f(x) = (x-2)(x^2 + 4x + 4)$

$f(x) = (x-2)(x+2)^2$

(c) $(x+2)^2 \Rightarrow x = -2$ is a repeated root so there will be a ~~turning~~ stationary point at $x = -2$ (i.e. gradient $= 0$) $\therefore$ the x-axis will be a tangent to the ~~x-axis~~ curve at ~~x=2~~ $(-2, 0)$.

11G

(11) (a) $f'(x) = a\cos bx$

$a = 4$ (amplitude of 4)

$b = 2$ (graph of $y = \cos x$ has been compressed horizontally by factor of 2).

(b) $f'(x) = 4\cos 2x$

$f(x) = \int 4\cos 2x \, dx$

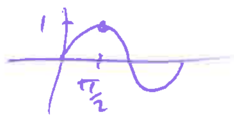
$f(x) = 4 \times \frac{1}{2}\sin 2x + c$

$f(x) = 2\sin 2x + c$

$x = \frac{\pi}{4}$ $y = f(x) = 3$

$3 = 2\sin\left(2\frac{\pi}{4}\right) + c$

$3 = 2 \cancel{\sin\frac{\pi}{2}}^{1} + c$



$3 = 2 + c$

$c = 1$

$\underline{\underline{f(x) = 2\sin 2x + 1}}$

(c) on x-axis $y = f(x) = 0$

$2\sin 2x + 1 = 0$

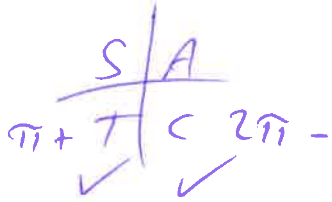
$2\sin 2x = -1$

11G

11(c) continued

$$\sin 2x = -\tfrac{1}{2}$$

$$\sin^{-1}\left(\tfrac{1}{2}\right) = \tfrac{\pi}{6}$$



$$2x = \pi + \tfrac{\pi}{6}\ ,\ 2\pi - \tfrac{\pi}{6}$$

$$2x = \frac{7\pi}{6}\ ,\ \frac{11\pi}{6}\ ,\ \frac{19\pi}{6}\ ,\ \ldots$$

$$x = \frac{7\pi}{12}\ ,\ \frac{11\pi}{12}\ ,\ \cancel{\frac{19\pi}{12}}$$ out of range.

coordinates $\left(\frac{7\pi}{12}, 0\right)$ $\left(\frac{11\pi}{12}, 0\right)$