17A

1(f) $\int_{\pi/6}^{\pi/4} 2\cos x \, dx$

$= \left[2\sin x\right]_{\pi/6}^{\pi/4}$

$= 2\sin\frac{\pi}{4} - 2\sin\frac{\pi}{6}$

$= 2\cdot\frac{1}{\sqrt{2}} - 2\cdot\frac{1}{2}$

$= \frac{2}{\sqrt{2}} - 1$

$= \underline{\underline{\sqrt{2} - 1}}$ square units

2(d) $\int_{-1}^{1} \frac{1}{(2x+3)^5} - 1 \, dx$

$= \int_{-1}^{1} (2x+3)^{-5} - 1 \, dx$

$= \left[\frac{(2x+3)^{-4}}{(2)(-4)} - x\right]_{-1}^{1}$

$= \left[\frac{-1}{8(2x+3)^4} - x\right]_{-1}^{1}$

$= \left(\frac{-1}{8(2(1)+3)^4} - 1\right) - \left(\frac{-1}{8(2(-1)+3)^4} - (-1)\right)$

17A

2(d) continued

$$= \left(\frac{-1}{8\times625} - 1\right) - \left(\frac{-1}{8} + 1\right)$$

$$= \left(\frac{-1}{5000} - \frac{5000}{5000}\right) - \left(\frac{-625}{5000} + \frac{5000}{5000}\right)$$

$$= \frac{-5001}{5000} - \frac{4375}{5000}$$

$$= \frac{-9376}{5000}$$

$$= -\frac{1172}{625}$$ (-ve sign indicates area below x-axis)

$$= \frac{1172}{625}$$ square units

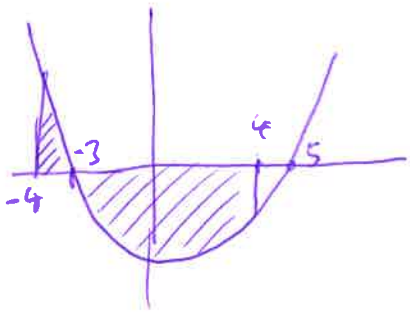
17A

③(a) Find roots

$$x^2-2x-15=0$$

$$(x-5)(x+3)=0$$

$$\underline{x=5,\ -3}$$



must split into 2 different integrals if areas are both above and below the x-axis.

$$A_1=\int_{-4}^{-3} x^2-2x-15\,dx$$

$$=\left[\frac{x^3}{3}-\frac{\cancel{2}x^2}{\cancel{2}}-15x\right]_{-4}^{-3}$$

$$=\left(\frac{(-3)^3}{3}-(-3)^2-15(-3)\right)-\left(\frac{(-4)^3}{3}-(-4)^2-15(-4)\right)$$

$$=(-9-9+45)-\left(-\frac{64}{3}-16+60\right)$$

$$=27-22\tfrac{2}{3}$$

$$=\underline{\underline{4\tfrac{1}{3}}}$$

17A (3a) continued

$$A_2 = \int_{-3}^{4} x^2 - 2x - 15 \, dx$$

$$= \left[\frac{x^3}{3} - x^2 - 15x\right]_{-3}^{4}$$

evaluated in A_1

$$= \left(\frac{(4)^3}{3} - (4)^2 - 15(4)\right) - (27)$$

$$= \left(\frac{64}{3} - 16 - 60\right) - (27)$$

$$= \left(21\tfrac{1}{3} - 76\right) - 27$$

$$= -54\tfrac{2}{3} - 27$$

$$= -81\tfrac{2}{3}$$

negative area as below x-axis

$A_2 = 81\tfrac{2}{3}$ units^2

$$A = A_1 + A_2$$

$$= 4\tfrac{1}{3} + 81\tfrac{2}{3}$$

$$= \underline{\underline{86 \text{ units}^2}}$$

17A/3(f) $y = -(x-1)(x-4)^2$

on x-axis $y=0 \Rightarrow 0 = -(x-1)(x-4)^2$

roots $x-1=0 \quad x-4=0$

$\underline{x=1} \quad \underline{x=4}$

$$A_1 = \int_{-1}^{1} -(x-1)(x-4)^2 \, dx$$

$$= \int_{-1}^{1} -(x-1)(x^2-8x+16) \, dx$$

$$= \int_{-1}^{1} -(x^3 - 8x^2 + 16x - x^2 + 8x - 16) \, dx$$

$$= \int_{-1}^{1} -x^3 + 9x^2 - 24x + 16 \, dx$$

$$= \left[-\frac{x^4}{4} + 3x^3 - 12x^2 + 16x \right]_{-1}^{1}$$

$$= \left(-\frac{(1)^4}{4} + 3(1)^3 - 12(1)^2 + 16(1) \right) - \left(-\frac{(-1)^4}{4} + 3(-1)^3 - 12(-1)^2 + 16(-1) \right)$$

$$= \left(6\tfrac{3}{4}\right) - \left(-31\tfrac{1}{4}\right)$$

$$= \underline{\underline{38}}$$

$$A_2 = \int_{1}^{4} -(x-1)(x-4)^2 \, dx$$

$$= \left[-\frac{x^4}{4} + 3x^3 - 12x^2 + 16x \right]_{1}^{4}$$ * same as A_1

$$= \left(-\frac{(4)^4}{4} + 3(4)^3 - 12(4)^2 + 16(4)\right) - \left(6\tfrac{3}{4}\right)$$

calculated in A_1

$$= (0) - 6\tfrac{3}{4}$$

$$= -6\tfrac{3}{4}$$ (-ve sign signifies area below x-axis)

$$A_2 = 6\tfrac{3}{4}$$

$$A_3 = \int_4^5 -(x-1)(x-4)^2 \, dx$$

$$= \left[-\frac{x^4}{4} + 3x^3 - 12x^2 + 16x\right]_4^5$$

$$= \left(-\frac{5^4}{4} + 3(5)^3 - 12(5)^2 + 16(5)\right) - (0)$$

calculated in A_2

$$= -\frac{625}{4} + 375 - 300 + 80$$

$$= -156\tfrac{1}{4} + 375 - 300 + 80$$

$$= -1\tfrac{1}{4}$$ (-ve signifies area below x-axis)

$$A_3 = 1\tfrac{1}{4}$$

$$A = A_1 + A_2 + A_3$$

$$= 38 + 6\tfrac{3}{4} + 1\tfrac{1}{4}$$

$$= \underline{\underline{46 \text{ square units}}}$$

17A

6 (a) (i)

$$\begin{array}{r|rrrr} -2 & 1 & -3 & -6 & 8 \\ & & -2 & 10 & -8 \\ \hline & 1 & -5 & +4 & \boxed{0} \end{array}$$

$x = -2$ is a root and $(x+2)$ is a factor of $f(x)$.

(ii) $(x+2)(x^2 - 5x + 4)$

$= \underline{(x+2)(x-1)(x-4)}$

(b) (i) $(x+2)(x-1)(x-4) = 0$

$\underline{x = -2, \ x = 1, \ x = 4}$

$\underline{K(-2, 0), \ L(1, 0), \ M(4, 0)}$

(ii) $A_1 = \int_{-2}^{1} x^3 - 3x^2 - 6x + 8 \, dx$

$= \left[\frac{x^4}{4} - x^3 - 3x^2 + 8x\right]_{-2}^{1}$

$= \left(\frac{(1)^4}{4} - (1)^3 - 3(1)^2 + 8(1)\right) - \left(\frac{(-2)^4}{4} - (-2)^3 - 3(-2)^2 + 8(-2)\right)$

$= \left(4\tfrac{1}{4}\right) - (-16)$

$= \underline{20\tfrac{1}{4}}$

$A_2 = \int_{1}^{4} x^3 - 3x^2 - 6x + 8 \, dx$

$= \left[\frac{x^4}{4} - x^3 - 3x^2 + 8x\right]_{1}^{4}$

$= \left(\frac{(4)^4}{4} - (4)^3 - 3(4)^2 + 8(4)\right) - \left(4\tfrac{1}{4}\right)$ calculated as part of A_1

$= -16 - 4\tfrac{1}{4}$

$= -20\tfrac{1}{4}$ (-ve sign show area is under x-axis

$A_2 = \underline{20\tfrac{1}{4}}$

$A = A_1 + A_2 = \underline{\underline{40\tfrac{1}{2}}}$ square units

17A

10(a) T.P. at $(3,-2)$ so $b=-3$ $c=-2$

$$y = a(x-3)^2 - 2$$

y-int: $x=0$ $y=\frac{5}{2}$

$$\frac{5}{2} = a(0-3)^2 - 2$$

$$\frac{5}{2} = 9a - 2$$

$$\frac{9}{2} = 9a$$

$$a = \frac{1}{2}$$

(b)(i) $y = \frac{1}{2}(x-3)^2 - 2$

x-int when $y=0$

$$0 = \frac{1}{2}(x-3)^2 - 2$$

$$0 = (x-3)^2 - 4$$ (* or expand, simplify & factorise)

$$4 = (x-3)^2$$

$$x-3 = \pm\sqrt{4}$$

$x-3 = -2$ $\quad$ $x-3=2$

$x = 1$ $\quad$ $x = 5$

$R(1,0)$ $\quad$ $S(5,0)$

(b)(ii) $A_1 = \int_0^1 \frac{1}{2}(x-3)^2 - 2$

$$= \int_0^1 \frac{1}{2}(x^2-6x+9) - 2$$

$$= \int_0^1 \frac{1}{2}x^2 - 3x + \frac{5}{2} \, dx$$

17A

10 b(ii) continued

$$= \left[\frac{1}{6}x^3 - \frac{3x^2}{2} + \frac{5x}{2}\right]_0^1$$

$$= \left(\frac{1}{6}(1)^3 - \frac{3(1)^2}{2} + \frac{5(1)}{2}\right) - (0)$$

$$= \frac{7}{6}$$

$$A_2 = \int_1^5 \frac{1}{2}(x-3)^2 - 2 \, dx$$

$$= \left[\frac{1}{6}x^3 - \frac{3x^2}{2} + \frac{5x}{2}\right]_1^5$$

$$= \left(\frac{1}{6}(5)^3 - \frac{3(5)^2}{2} + \frac{5(5)}{2}\right) - \left(\frac{7}{6}\right)$$ calculater in A_1

$$= \left(\frac{125}{6} - \frac{75}{2} + \frac{25}{2}\right) - \left(\frac{7}{6}\right)$$

$$= \left(20\tfrac{5}{6} - 25\right) - \left(\frac{7}{6}\right)$$

$$= -5\tfrac{2}{6}$$ (-ve area as below x-axis)

$$= 5\tfrac{2}{6}$$

$$A = A_1^* + A_2$$

$$= \frac{7}{6} + 5\tfrac{2}{6}$$

$$= 6\tfrac{3}{6}$$

$= 6\frac{1}{2}$ square units

17B

1(d) $\int_{-2}^{4} (\text{top curve} - \text{bottom curve})\, dx$

$= \int_{-2}^{4} (-x^2+2x+9) - (2x^2-4x-15)\, dx$

$= \int_{-2}^{4} -3x^2+6x+24\, dx$

$= \left[-x^3+3x^2+24x\right]_{-2}^{4}$

-ve sign

$= \left(-(4)^3+3(4)^2+24(4)\right) - \left(-(-2)^3+3(-2)^2+24(-2)\right)$

$=$ ~~108~~ square units

108 square units

$=$ ~~124 square units~~

2(b)(i) $y=y$

$x^2-3x+1 = -2x+3$

$x^2-x-2=0$

$(x-2)(x+1)=0$

$x=2,\ x=-1$

(ii) $\int_{-1}^{2} (\text{top curve} - \text{bottom curve})\, dx$

$= \int_{-1}^{2} (-2x+3) - (x^2-3x+1)\, dx$

$= \int_{-1}^{2} -x^2+x+2\, dx$

17B

2(b) continued

$$= \left[-\frac{x^3}{3} + \frac{x^2}{2} + 2x \right]_{-1}^{2}$$

$$= \left(-\frac{(2)^3}{3} + \frac{(2)^2}{2} + 2(2) \right) - \left(-\frac{(-1)^3}{3} + \frac{(-1)^2}{2} + 2(-1) \right)$$

$$= \left(\frac{10}{3} \right) - \left(-\frac{7}{6} \right)$$

$$= \frac{20}{6} - \left(-\frac{7}{6} \right)$$

$$= \frac{27}{6}$$

$$= \frac{9}{2} \text{ square units}$$

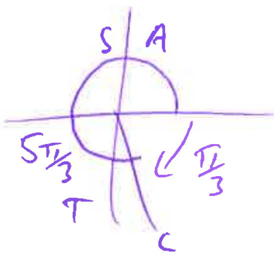
17B

2(e) $\int_{\frac{\pi}{2}}^{\frac{5\pi}{6}} 3\sin 2x \, dx$

$= \left[-\frac{3}{2}\cos 2x\right]_{\frac{\pi}{2}}^{\frac{5\pi}{6}}$

$= \left(-\frac{3}{2}\cos\left(2\left(\frac{5\pi}{6}\right)\right)\right) - \left(-\frac{3}{2}\cos\left(2\left(\frac{\pi}{2}\right)\right)\right)$

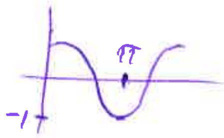
$= \left(-\frac{3}{2}\cos\left(\frac{5\pi}{3}\right)\right) - \left(-\frac{3}{2}\cos(\pi)\right)$



$= \left(-\frac{3}{2}\cos\left(\frac{\pi}{3}\right)\right) - \left(-\frac{3}{2}\cos(\pi)\right)$

$\cos\frac{5\pi}{3} = \cos\frac{\pi}{3}$

$= \left(-\frac{3}{2}\times\frac{1}{2}\right) - \left(-\frac{3}{2}\times -1\right)$



$= \frac{-3}{4} - \frac{3}{2}$

$= -\frac{9}{4}$

Note: $-\frac{9}{4}$ shows the area is $\frac{9}{4}$ but it is below the x-axis

Area $= \frac{9}{4}$ units^2

17B

(3) $A_1 = \int_{-2}^{1}$ top curve − bottom curve dx

$$= \int_{-2}^{1} (x^3 - 2x^2 - 6x + 8) - (2x^2 + x - 2)\,dx$$

$$= \int_{-2}^{1} x^3 - 4x^2 - 7x + 10\,dx$$

$$= \left[\frac{x^4}{4} - \frac{4x^3}{3} - \frac{7x^2}{2} + 10x\right]_{-2}^{1}$$

$$= \left(\frac{(1)^4}{4} - \frac{4(1)^3}{3} - \frac{7(1)^2}{2} + 10(1)\right) - \left(\frac{(-2)^4}{4} - \frac{4(-2)^3}{3} - \frac{7(-2)^2}{2} + 10(-2)\right)$$

$$= 24\tfrac{3}{4}$$

$A_2 = \int_{1}^{5}$ (top curve − bottom curve) dx

$$= \int_{1}^{5} (2x^2 + x - 2) - (x^3 - 2x^2 - 6x + 8)\,dx$$

$$= \int_{1}^{5} -x^3 + 4x^2 + 7x - 10\,dx$$

$$= \left[-\frac{x^4}{4} + \frac{4x^3}{3} + \frac{7x^2}{2} - 10x\right]_{1}^{5}$$

$$= \left(-\frac{(5)^4}{4} + \frac{4(5)^3}{3} + \frac{7(5)^2}{2} - 10(5)\right) - \left(-\frac{(1)^4}{4} + \frac{4(1)^3}{3} + \frac{7(1)^2}{2} - 10(1)\right)$$

$$= 53\tfrac{1}{3}$$

$$A = A_1 + A_2$$

$= 78\tfrac{1}{12}$ square units

17B

5 (a)

$$\begin{array}{c|cccc} -3 & 1 & -1 & -8 & 12 \\ & & -3 & 12 & -12 \\ \hline & 1 & -4 & 4 & \boxed{0} \end{array}$$

$x = -3$ is a root and $(x+3)$ is a factor of $f(x)$.

(b) $(x+3)(x^2-4x+4)$

$= (x+3)(x-2)(x-2)$

(c) (i) $y = y$

$x^3 - x^2 - 6x + 9 = 2x - 3$

$x^3 - x^2 - 8x + 12 = 0$

$(x+3)(x-2)^2 = 0$

$\underline{x = -3}$, $\underline{x = 2}$, $\underline{x = 2}$

$x = 2$ is a repeated root so line is a tangent to curve at this point

(ii) $y(2) = 2(2) - 3$

$= 1$

$\underline{M(2, 1)}$

(d) at N $x = -3$

$y(-3) = 2(-3) - 3$

$= -9$

$\underline{N(-3, -9)}$

17B

5(e) $\int_{-3}^{2}$ top curve - bottom curve dx

$= \int_{-3}^{2} (x^3 - x^2 - 6x + 9) - (2x - 3)\, dx$

$= \int_{-3}^{2} x^3 - x^2 - 8x + 12\, dx$

$= \left[\frac{x^4}{4} - \frac{x^3}{3} - \frac{8x^2}{2} + 12x \right]_{-3}^{2}$

$= \left(\frac{(+2)^4}{4} - \frac{(2)^3}{3} - \frac{8(2)^2}{2} + 12(2) \right) - \left(\frac{(-3)^4}{4} - \frac{(-3)^2}{3} - 4(-3)^2 + 12(-3) \right)$

$= 52\frac{1}{12}$ square units

17B

(12) (a) $y = y$

$3\cos 2x = -7\cos x + 2$

$3\cos 2x + 7\cos x - 2 = 0$ $\qquad\qquad \cos 2x = 2\cos^2 x - 1$

$3(2\cos^2 x - 1) + 7\cos x - 2 = 0$

$6\cos^2 x - 3 + 7\cos x - 2 = 0$

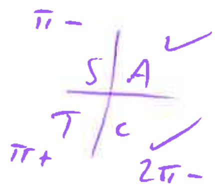
$6\cos^2 x + 7\cos x - 5 = 0$

$(3\cos x + 5)(2\cos x - 1) = 0$

$3\cos x + 5 = 0$ $\qquad\qquad 2\cos x - 1$

$3\cos x = -5$ $\qquad\qquad 2\cos x = 1$

$\cos x = -\frac{5}{3}$ $\qquad\qquad \cos x = \frac{1}{2}$



no solutions $\qquad\qquad x = \frac{\pi}{3}, \frac{5\pi}{3}$

$x = \frac{\pi}{3}$ $\quad y = 3\cos\left(2\left(\frac{\pi}{3}\right)\right)$ $\qquad\qquad x = \frac{5\pi}{3}$ $\quad y = 3\cos\left(2\left(\frac{5\pi}{3}\right)\right)$

$y = -\frac{3}{2}$ $\qquad\qquad = 3\cos\frac{10\pi}{3}$

$A\left(\frac{\pi}{3}, -\frac{3}{2}\right)$ $\qquad\qquad B\left(\frac{\pi}{3}, -\frac{3}{2}\right)$ $= -\frac{3}{2}$

(b) $\displaystyle\int_{\frac{\pi}{3}}^{\frac{5\pi}{3}} (\text{top curve} - \text{bottom curve})\, dx$

$= \displaystyle\int_{\frac{\pi}{3}}^{\frac{5\pi}{3}} (-7\cos x + 2) - (3\cos 2x)\, dx$

17B

12(b)

$$= \left[-7\sin x + 2x - \frac{3}{2}\sin 2x \right]_{\pi/3}^{5\pi/3}$$

$$= \left(-7\sin\left(\frac{5\pi}{3}\right) + 2\left(\frac{5\pi}{3}\right) - \frac{3}{2}\sin\left(2\left(\frac{5\pi}{3}\right)\right) \right)$$

$$- \left(-7\sin\left(\frac{\pi}{3}\right) + 2\left(\frac{\pi}{3}\right) - \frac{3}{2}\sin\left(2\left(\frac{\pi}{3}\right)\right) \right)$$

$$= \frac{8\pi}{3} + 7\sqrt{3} \text{ square units}$$