3A

See Leckie & Leckie answers for topic 3A.

Scroll down for 3B onwards.

3B Page 66

1(a) x^2+4x

$= \underline{\underline{(x+2)^2-4}}$

$\left[(x+2)^2 = (x+2)(x+2) = x^2+4x+4\right]$

(b) $x^2-6x+10$

$= (x-3)^2-9+10$

$= \underline{\underline{(x-3)^2+1}}$

(c) x^2+8x-1

$= (x+4)^2-16-1$

$= \underline{\underline{(x+4)^2-17}}$

(d) $5+2x+x^2$

$= x^2+2x+5$

$= (x+1)^2-1+5$

$= \underline{\underline{(x+1)^2+4}}$

(e) $10+4x-x^2$

$= -(x^2-4x)+10$

$= -\left[(x-2)^2-4\right]+10$

$= \underline{\underline{-(x-2)^2+14}}$

(f) $7-6x-x^2$

$= -x^2-6x-7 = -(x^2+6x)-7$

$= -\left[(x+3)^2-9\right]-7$

$= \underline{\underline{-(x+3)^2+2}}$

(g) $6x-3-x^2$

$= -x^2+6x-3$

$= -(x^2-6x)-3$

$= -\left[(x-3)^2-9\right]-3$

$= \underline{\underline{-(x-3)^2+6}}$

(h) $10-3x-x^2$

$= -x^2-3x+10$

$= -(x^2+3x)+10$

$= -\left[\left(x+\frac{3}{2}\right)^2-\left(\frac{3}{2}\right)^2\right]+10$

$= \underline{\underline{-\left(x+\frac{3}{2}\right)^2+12\frac{1}{4}}}$

3B

② (a) $2x^2+8x+3$

$= 2(x^2+4x) + 3$

$= 2[(x+2)^2-4] + 3$

$= \underline{\underline{2(x+2)^2 - 5}}$

(b) $2x^2-4x+15$

$= \cancel{2[(x-1)^2}$

$= 2[x^2-2x] + 3$

$= 2[(x-1)^2 - 1] + 3$

$= \underline{\underline{2(x-1)^2 + 1}}$

(c) $3x^2+12x-5$

$= 3(x^2+4x) - 5$

$= 3[(x+2)^2-4] - 5$

$= \underline{\underline{3(x+2)^2 - 17}}$

(d) $5x^2-30x+36$

$= 5(x^2-6x) + 36$

$= 5[(x-3)^2-9] + 36$

$= \underline{\underline{5(x-3)^2 - 9}}$

(e) $4x^2-12x+1$

$= 4(x^2-3x) + 1$

$= 4\left[\left(x-\frac{3}{2}\right)^2-\left(\frac{3}{2}\right)^2\right]+1$

$= 4\left(x-\frac{3}{2}\right)^2 - 9 + 1$

$= \underline{\underline{4\left(x-\frac{3}{2}\right)^2 - 8}}$

(f) $2x^2+6x-9$

$= 2(x^2+3x) - 9$

$= 2\left[\left(x+\frac{3}{2}\right)^2-\left(\frac{3}{2}\right)^2\right] - 9$

$= 2\left(x+\frac{3}{2}\right)^2 - \frac{9}{2} - 9$

$= \underline{\underline{2\left(x+\frac{3}{2}\right)^2 - 13\frac{1}{2}}}$

(g) $4+7x-7x^2$

$= -7x^2+7x+4$

$= -7(x^2-x) + 4$

$= -7\left[\left(x-\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2\right]+4$

$= -7\left(x-\frac{1}{2}\right)^2 + \frac{7}{4} + 4$

$= \underline{\underline{-7\left(x-\frac{1}{2}\right)^2 + 5\frac{3}{4}}}$

(h) $-2x^2-6x-3$

$= -2(x^2+3x) - 3$

$= -2\left[\left(x+\frac{3}{2}\right)^2-\left(\frac{3}{2}\right)^2\right] - 3$

$= -2\left(x+\frac{3}{2}\right)^2 + \frac{9}{2} - 3$

$= \underline{\underline{-2\left(x+\frac{3}{2}\right)^2 + \frac{3}{2}}}$

3B

Q3 (a) $15-4x-2x^2$

$= -2x^2-4x+15$

$= -2(x^2+2x)+15$

$= -2[(x+1)^2-1^2]+15$

$= -2(x+1)^2+2+15$

$= \underline{17-2(x+1)^2}$

(b) $12x-x^2$

$= -x^2+12x$

$= -(x^2-12x)$

$= -[(x-6)^2-36]$

$= \underline{36-(x-6)^2}$

(c) $3+8x-4x^2$

$= -4x^2+8x+3$

$= -4(x^2-2x)+3$

$= -4[(x-1)^2-1]+3$

$= -4(x-1)^2+4+3$

$= \underline{7-4(x-1)^2}$

(d) $-3x^2+12x-8$

$= -3(x^2-4x)-8$

$= -3[(x-2)^2-4]-8$

$= -3(x-2)^2+12-8$

$= \underline{4-3(x-2)^2}$

(e) $1+6x-2x^2$

$= -2x^2+6x+1$

$= -2(x^2-3x)+1$

$= -2[(x-\frac{3}{2})^2-(\frac{3}{2})^2]+1$

$= -2(x-\frac{3}{2})^2+\frac{9}{2}+1$

$= -2(x-\frac{3}{2})^2+\frac{11}{2}$

$= \underline{\frac{11}{2}-2(x-\frac{3}{2})^2}$

(f) $19-20x-4x^2$

$= -4x^2-20x+19$

$= -4(x^2+5x)+19$

$= -4[(x+\frac{5}{2})^2-(\frac{5}{2})^2]+19$

$= -4[(x+\frac{5}{2})^2-\frac{25}{4}]+19$

$= -4(x+\frac{5}{2})^2+25+19$

$= \underline{44-4(x+\frac{5}{2})^2}$

(g) $4+7x-7x^2$

$= -7x^2+7x+4$

$= -7(x^2-x)+4$

$= -7((x-\frac{1}{2})^2-(\frac{1}{2})^2)+4$

$= -7(x-\frac{1}{2})^2+\frac{7}{4}+4$

$= \underline{5\frac{3}{4}-7(x-\frac{1}{2})^2}$

(h) $-2x^2-6x-3$

$= -2(x^2+3x)-3$

$= -2[(x+\frac{3}{2})^2-(\frac{3}{2})^2]-3$

$= -2(x+\frac{3}{2})^2+\frac{9}{2}-3$

$= \underline{\frac{3}{2}-2(x+\frac{3}{2})^2}$

3B

Q4 (a) $(2-x)(3+2x)$

$= 6+4x-3x-2x^2$

$= -2x^2+x+6$

$= -2\left(x^2-\frac{x}{2}\right)+6$

$= -2\left[\left(x-\frac{1}{4}\right)^2-\left(\frac{1}{4}\right)^2\right]+6$

$= -2\left(x-\frac{1}{4}\right)^2+\frac{1}{8}+6$

$= -2\left(x-\frac{1}{4}\right)^2+6\frac{1}{8}$

(b) $(3x+2)(2x+1)$

$= 6x^2+3x+4x+2$

$= 6x^2+7x+2$

$= 6\left(x^2+\frac{7}{6}x\right)+2$

~~$= 6\left(x+\frac{7}{12}\right)^2\left(\frac{7}{12}\right)^2+2$~~

$= 6\left[\left(x+\frac{7}{12}\right)^2-\left(\frac{7}{12}\right)^2\right]+2$

$= 6\left[\left(x+\frac{7}{12}\right)^2-\frac{49}{144}\right]+2$

$= 6\left(x+\frac{7}{12}\right)^2-\frac{49}{24}+2$

$= 6\left(x+\frac{7}{12}\right)^2+\frac{1}{24}$

(c) $3x(x+2)-5$

$= 3x^2+6x-5$

$= 3(x^2+2x)-5$

$= 3\left[(x+1)^2-1\right]-5$

$= 3(x+1)^2-3-5$

$= 3(x+1)^2-8$

(d) $(x+3)^2 ~~4~~ -2x+5$

$= x^2+6x+9-2x+5$

$= x^2+4x+14$

$= (x+2)^2-4+14$

$= (x+2)^2+10$

3C

1(a) $y = (x-3)^2 + 2$

$\rightarrow 3 \quad \uparrow 2$

positive $(\ )^2$

so min t.p. at $(3,2)$

(b) $y = \underset{+ve}{2}(x+3)^2 - 7$

$\leftarrow 3 \quad \downarrow 7$

min t.p. at $(-3,-7)$

(c) $y = 12 - 2(x+4)^2$

$\uparrow 12 \quad -ve \quad \leftarrow 4$

max t.p. at ~~(4,12)~~ $(-4, 12)$

(d) $y = 3 + \underset{+ve}{2}(x-5)^2$

$\uparrow 3 \quad 5 \rightarrow$

min t.p. at $(5,3)$

(e) $y = 5 - 3(x-5)^2$

$\uparrow 5 \quad -ve \quad 5 \rightarrow$

max t.p. at $(5,5)$

(f) $y = \underset{-ve}{-4}(x+5)^2 + 7$

$\leftarrow 5 \quad 7 \uparrow$

max t.p. at $(-5,7)$

(g) $y = 9 + (x-2)^2$

$9 \uparrow \quad 2 \rightarrow$

min t.p. at $(2,9)$

(h) $y = 13 - (x+7)^2$

$-ve$

$13 \uparrow \quad \leftarrow 7$

max t.p. at $(-7,13)$

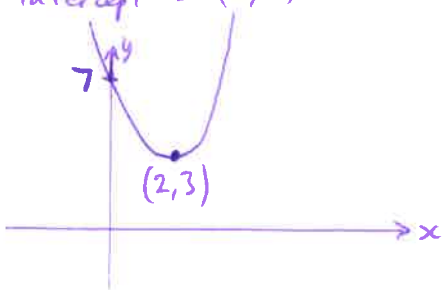
2(a) $y = (x-2)^2 + 3$

$2 \rightarrow \quad 3 \uparrow$

min t.p. $(2,3)$

$y(0) = (0-2)^2 + 3$
$= 7$

y-intercept $= (0,7)$



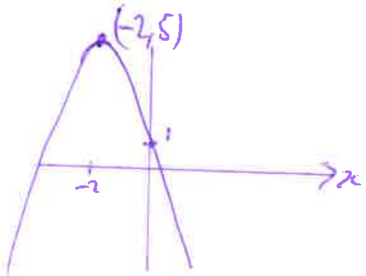
3C

2(b) $y = 5 - (x+2)^2$

$\uparrow \quad \leftarrow$

max t.p. $(-2, 5)$

$y(0) = 5 - (0+2)^2$
$= 1$

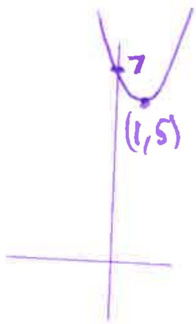


2(c) $y = 2(x-1)^2 + 5$

$\rightarrow 1 \quad \uparrow 5$

min t.p. $(1, 5)$

$y(0) = 2(0-1)^2 + 5$
$= 7$

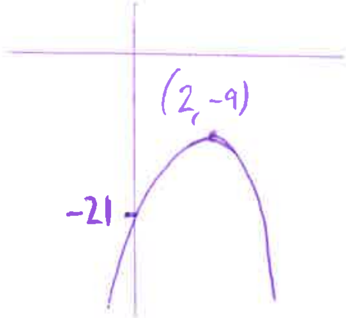


2(d) $y = -3(x-2)^2 - 9$

-ve $\quad 2 \rightarrow \quad 9 \downarrow$

max t.p $(2, -9)$

$y(0) = -3(0-2)^2 - 9$
$= -21$

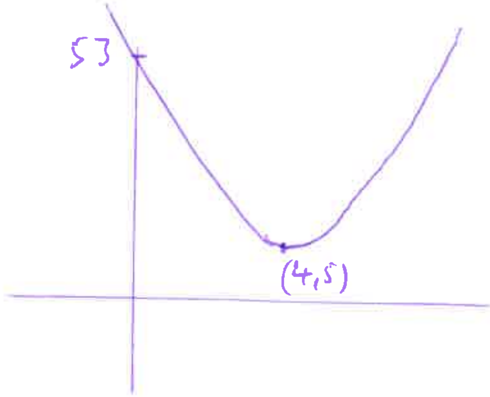


2(e) $y = 5 + 3(x-4)^2$

$5 \uparrow \quad 4 \rightarrow$

min t.p. at $(4, 5)$

$y(0) = 5 + 3(0-4)^2$
$= 53$

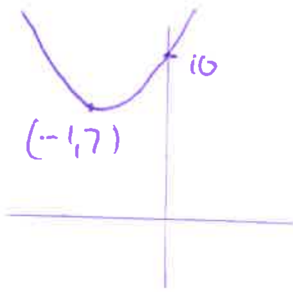


2(f) $y = 3(x+1)^2 + 7$

$1 \leftarrow \quad 7 \uparrow$

min t.p $(-1, 7)$

$y(0) = 3(0+1)^2 + 7$
$= 10$



3C

2(g) $y = 4 - 2(x+3)^2$

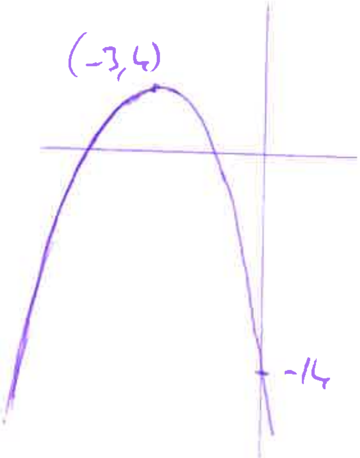
$4\uparrow \quad 3\leftarrow$

max t.p. at $(-3,4)$

$y(0) = 4 - 2(0+3)^2$

$= 4 - 18$

$= -14$



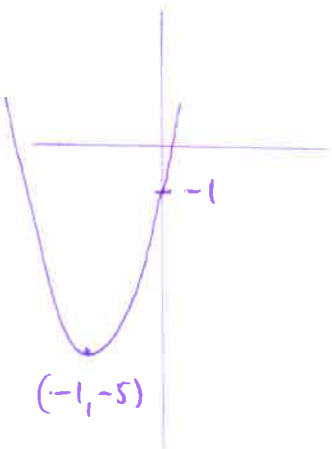
2(h) $y = 4(x+1)^2 - 5$

$1\leftarrow \quad 5\downarrow$

min t.p. $(-1,-5)$

$y(0) = 4(0+1)^2 - 5$

$= -1$



(3) $3x^2 + 6x + 10$

$= 3(x^2 + 2x) + 10$

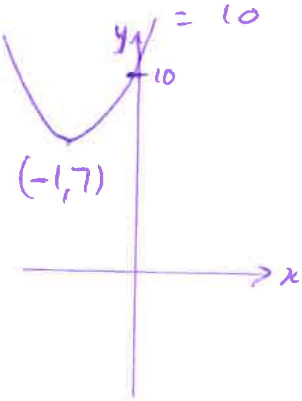
$= 3[(x+1)^2 - 1] + 10$

$= 3(x+1)^2 + 7$

(b) $1\leftarrow \quad 7\uparrow$

$y(0) = 3(0+1)^2 + 7$

$= 10$



(4)(a) $15 - 4x - 2x^2$

$= -2x^2 - 4x + 15$

$= -2(x^2 - 2x) + 15$

$= -2[(x-1)^2 - 1] + 15$

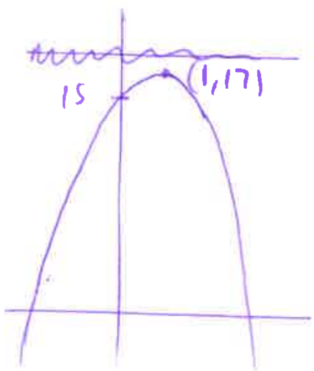
$= -2(x-1)^2 + 2 + 15$

$= 17 - 2(x-1)^2$

(b) $\uparrow 17 \quad \rightarrow 1$

$y(0) = 17 - 2(0-1)^2$

$= 15$



3C

5) (a) $\rightarrow 2 \uparrow 1$

$y = a(x-2)^2+1$

$x=0 \quad y=5$

$5 = a(0-2)^2+1$

$5 = 4a+1$

$a=1$

$y = (x-2)^2+1$

(b) max t.p. $\rightarrow 3 \uparrow 27$

$y = -a(x-3)^2+27$

$x=0 \quad y=9$

$9 = -a(0-3)^2+27$

$9 = -9a+27$

$-18 = -9a$

$a=2$

$y = -2(x-3)^2+27$

(c) $\leftarrow 5 \uparrow 1$

$y = a(x+5)^2+1$

$76 = a(0+5)^2+1$

$76 = 25a+1$

$a=3$

$y = 3(x+5)^2+1$

(d) $y = a(x-2)^2+5$

$17 = a(0-2)^2+5$

$17 = 4a+5$

$a=3$

$y = 3(x-2)^2+5$

(e) $y = -a(x-3)^2-1$

$x=4, y=-3$

$-3 = -a(4-3)^2-1$

$-3 = -a-1$

$a = -1+3$

$a=2$

$y = -2(x-3)^2-1$

(f) $y = -a(x-2)^2+9$

$1 = -a(0-2)^2+9$

$1 = -4a+9$

$a = +2$

$y = -2(x-2)^2+9$

3C

6) $f(x) = 3x^2 + 6x - 2$

$= 3(x^2 + 2x) - 2$

$= 3[(x+1)^2 - 1] - 2$

$= 3(x+1)^2 - 3 - 2$

$= \underline{\underline{3(x+1)^2 - 5}}$

min ~~value~~ t.p. $= (-1, -5)$

hence min value (lowest value of y) $= \underline{\underline{-5}}$

(7) $f(x) = 5 + 3x - x^2$

$= -(x^2 - 3x) + 5$

$= -\left[\left(x - \frac{3}{2}\right)^2 + \left(\frac{3}{2}\right)^2\right] + 5$

$= -\left(x - \frac{3}{2}\right)^2 + \frac{9}{4} + 5$

$= \underline{\underline{7\frac{1}{4} - \left(x - \frac{3}{2}\right)^2}}$

max t.p. at $\left(7\frac{1}{4}, \frac{3}{2}\right)$

Hence max value $= \underline{\underline{7\frac{1}{4}}}$

(8) $3x^2 + 12x + 54$

$= 3[x^2 + 4x] + 54$

$= 3[(x+2)^2 - 4] + 54$

$= 3(x+2)^2 - 12 + 54$

$= \underline{\underline{3(x+2)^2 + 42}}$

$\dfrac{1}{3x^2 + 12x + 54}$

$= \dfrac{1}{3(x+2)^2 + 42}$

max value occurs when denominator at lowest value $\therefore$ max value $= \underline{\underline{\frac{1}{42}}}$ when $x = -2$.

(9) $(2x-5)(2x+3)$

$= 4x^2 - 4x - 15$

$= 4(x^2 - x) - 15$

$= 4\left[\left(x - \frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2\right] - 15$

$= 4\left(x - \frac{1}{2}\right)^2 + 1 - 15$

$= 4\left(x - \frac{1}{2}\right)^2 - 14$

$\dfrac{1}{(2x-5)(2x+3)}$

$= \dfrac{1}{4\left(x - \frac{1}{2}\right)^2 - 14}$

3C

(10) x^2+4x+9

$= (x+2)^2-4+9$

$= (x+2)^2+5 \geq 5$

for all values of x as $(x+2)^2 \geq 0$ for all values of x.

(11) $f(x) = 15-2x-x^2$

$= -(x^2+2x)+15$

$= -[(x+1)^2-1]+15$

$= -(x+1)^2+1+15$

$= 16-(x+1)^2$

$16-(x+1)^2 < 20$ for all values of x as $(x+1)^2 \geq 0$ for all values of x. i.e. subtracting 0 or a positive number from 16 will always give a number less than 20.

(12) $f(x) = 2x^2+6x+11$

$= 2(x^2+3x)+11$

$= 2[(x+\frac{3}{2})^2-(\frac{3}{2})^2]+11$

$= 2[(x+\frac{3}{2})^2-\frac{9}{4}]+11$

$= 2(x+\frac{3}{2})^2-\frac{9}{2}+11$

$= 2(x+\frac{3}{2})^2+\frac{13}{2} \geq 0$

for all values of x.

3D

Q1

Graphical solution available on Leckie &Leckie solution section of madrasmaths.com

Scroll down for Q2 onwards

3D

2 (a) $y = a^x$

$x = 1 \quad y = 6$

$6 = a^1$

$a = 6$

$y = 6^x$

(b) $y = a^x$

$9 = a^2$

$a = 3$

$y = 3^x$

(c) $y = a^x$

$64 = a^3$

$a = \sqrt[3]{64}$

$a = 4$

(d) $y = a^x$

$32 = a^5$

$a = \sqrt[5]{32}$

$a = 2$

$y = 2^x$

(e) $y = a^x$

$\frac{1}{4} = a^2$

$a = \sqrt{\frac{1}{4}}$

$a = \frac{1}{2}$

$y = \left(\frac{1}{2}\right)^x$

(f) $y = a^x$

$\frac{1}{27} = a^3$

$a = \sqrt{\frac{1}{27}}$

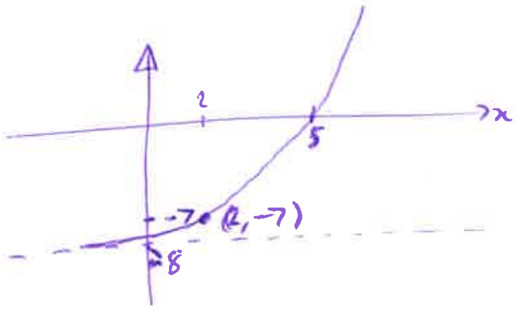
$a = \frac{1}{3}$

$y = \left(\frac{1}{3}\right)^x$

30

(4) $y = 2^{(x-2)} - 8$

Moves $y = 2^{x}$ $2 \Rightarrow$ $8 \downarrow$



x-intercept when $y = 0$

$$0 = 2^{(x-2)} - 8$$

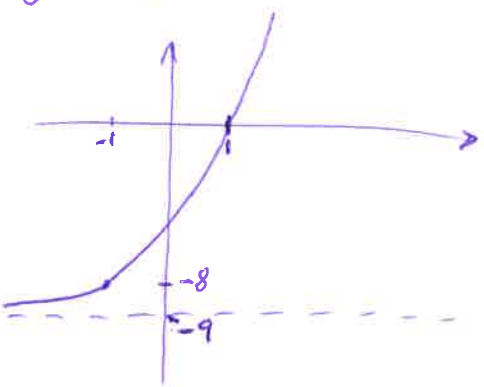
we know $2^3 = 8$

so $x - 2 = 3$

$\underline{x = 5}$ $\underline{5,0}$

(5) $y = 3^{(x+1)} - 9$

$y = 3^{x}$ $1 \leftarrow$ $\downarrow 9$



x-intercept when $y = 0$

$$3^{(x+1)} - 9 = 0$$

we know $3^2 = 9$

hence $x = 1$

$\underline{(1,0)}$

(6) $y = a^{x} + b$

$x = 0$ $y = 3$

$$3 = a^{0} + b$$

$$3 = 1 + b$$

$$b = 2$$

$x = 3$ $y = 29$

$$29 = a^{3} + 2$$

$$a^{3} = 27$$

$$a = \sqrt[3]{27}$$

$\underline{a = 3}$

$\underline{y = 3^{x} + 2}$

3E

Q1 see Leckie & Leckie solns.

Q2 (a) $y = \log_6 x$

(b) $y = \log_{10} x$

$y = \log_{3/4} x$

$y = \log_{1/5} x$

Q3 see Leckie & Leckie solns for graph but here is help with transformation

(a) $y = \log_4 x \quad \rightarrow 2$

(b) $y = \log_7 x \quad \leftarrow 4$

(c) $y = \log_4 x \quad \updownarrow 2$ (stretch)

(d) $y = \log_2 x \quad \uparrow 3$

(e) $y = \log_7 x \quad \updownarrow 3 \quad \rightarrow 1$

(f) $y = -2\log_5 x + 3$

reflect in x-axis

stretch by 2

move up 3

(g) $y = \log_7 x \quad \updownarrow 2, \leftarrow 2, \downarrow 1$

(h) $y = -\log_2 x + 3$

$y = \log_2 x$ reflect x-axis, $\uparrow 3$

Q4

(a) $y = \log_2 x^3$

$= \underline{3\log_2 x} \quad \updownarrow 3$

(b) $y = \log_4 x^2$

$= 2\log_4 x \quad \updownarrow 2$

(c) $y = \log_4\left(\frac{1}{x}\right)$

$= \log_4 x^{-1}$

$= -\log_4 x$ reflect in x-axis

(d) $y = \log_7 x^2$

$y = 2\log_7 x \quad \updownarrow 2$

(e) $y = \log_2(8x)$

$= \log_2 x + \log_2 8$

$= \log_2 x + \log_2 2^3$

$= \log_2 x + 3\cancel{\log_2 2}$

$= \log_2 x + 3 \quad \uparrow 3$

(f) $y = \log_2(32x)$

$= \log_2 x + \log_2 32$

$= \log_2 x + \log_2 2^5$

$= \log_2 x + 5\log_2 2$

$= \log_2 x + 5 \quad \uparrow 5$

see Leckie & Leckie solution for actual graphs

3E

4(g) $y = \log_5 \frac{5}{x}$

$= \log_5 5 - \log_5 x$

$= 1 - \log_5 x$

$= -\log_5 x + 1$

reflect in x-axis and move up 1

4(h) $y = \log_3 \left(\frac{27}{x}\right)$

$y = \log_3 27 - \log_3 x$

$= -\log_3 x + \log_3 3^3$

$= -\log_3 x + 3 \cancel{\log_3 3}$

∴ reflect in x-axis & move up 3.

5 $y = \log_a (x - b)$

$b = 4$ as curve should cross at $(1, 0)$

as $b = 4$ point $8, 1$ moves to $(4, 1)$ hence $a = 4$

$y = \log_4 (x - 4)$

3F

1(a) $y = 3\sin(x-30) + 1$

$\updownarrow 3 \quad \rightarrow 30 \quad \uparrow 1$

so $(x, y) \rightarrow (x+30, 3\times y + 1)$

Key points of $y = \sin x$

$(0,0) \rightarrow (30°, 1)$

$(90,1) \rightarrow (120°, 4)$

$(180,0) \rightarrow (210°, 1)$

$(270,-1) \rightarrow (300°, -2)$

$(360,0) \rightarrow (390°, 1)$

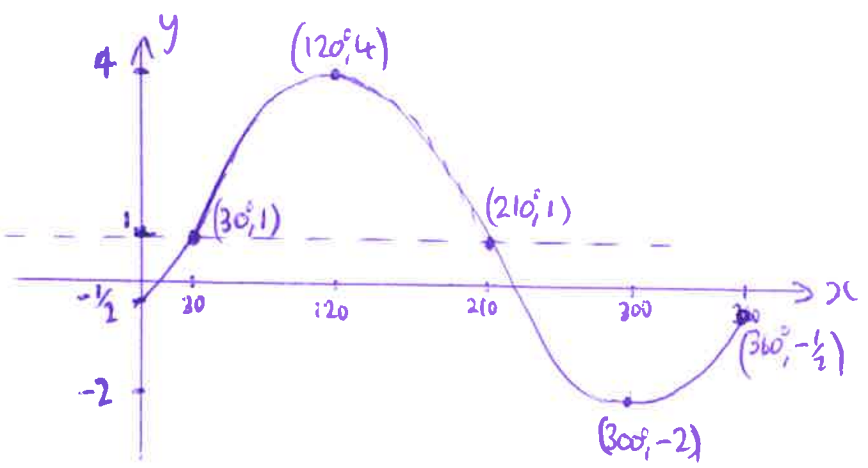
$x = 0, \; y = 3\sin(0-30) + 1$

$= 3\left(-\frac{1}{2}\right) + 1$

$= -\frac{3}{2} + 1$

$= -\frac{1}{2}$

y-int $= \left(0, -\frac{1}{2}\right)$ note as period $= 360$ curve will also pass $\left(360, -\frac{1}{2}\right)$.



3F

1(c) $y = 5 - 3\cos(2x+60)^\circ$

$= -3\cos(2x+60)^\circ + 5$

$= -3\cos(2(x+30))^\circ + 5$

$\updownarrow -3$ (negative sign reflects in x-axis). 2 waves $\leftarrow 30$ $\uparrow 5$

So $(x, y) \rightarrow \left(\frac{x}{2} - 30, -3 \times y + 5\right)$

$y = \cos x$ key points

$(0, 1) \rightarrow (-30, 2)$ height also occurs at $(150, 2)$ one wave later

$(90, 0) \rightarrow (15, 5)$

$(180, -1) \rightarrow (60, 8)$

$(270, 0) \rightarrow (105, 5)$

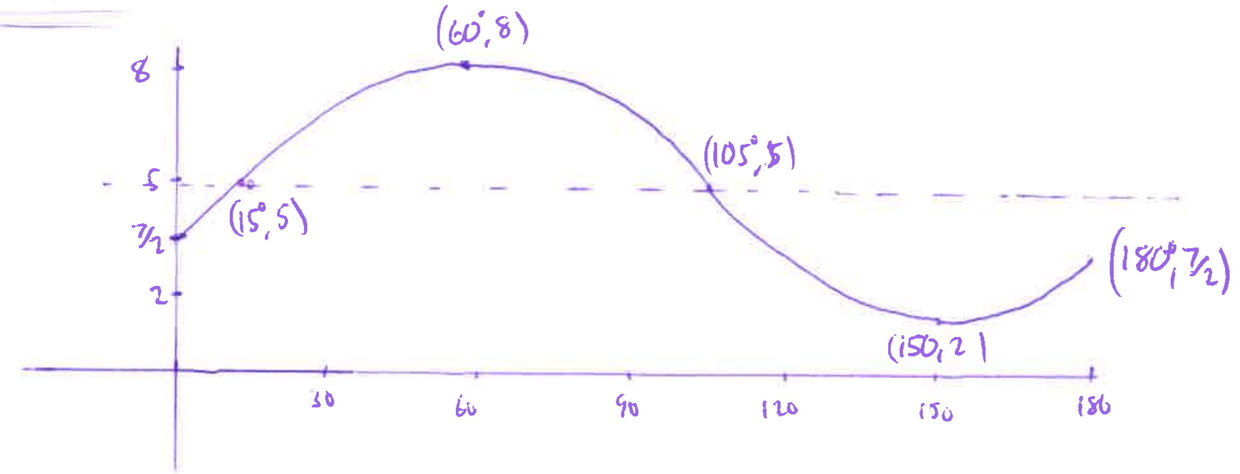
$(360, 1) \rightarrow (150, 2)$

$x = 0$ $y = 5 - 3\cos(2(0) + 60)$

$y = 5 - 3(\frac{1}{2})$

$y = \frac{7}{2}$

y-int $(0, \frac{7}{2})$



3F

1(e) $y = 2\sin(5x-6)^\circ + 1$

$= 2\sin(5(x-1.2)) + 1$

$\updownarrow 2$ 5 waves $1.2 \rightarrow$ $1\uparrow$

so $(x,y) \rightarrow \left(\frac{x}{5} + 1.2,\ 2y + 1\right)$

$y = \sin x$ key points

$(0,0) \rightarrow (1.2, 1)$

$(90, 1) \rightarrow (19.2, 3)$

$(180, 0) \rightarrow (37.2, 1)$

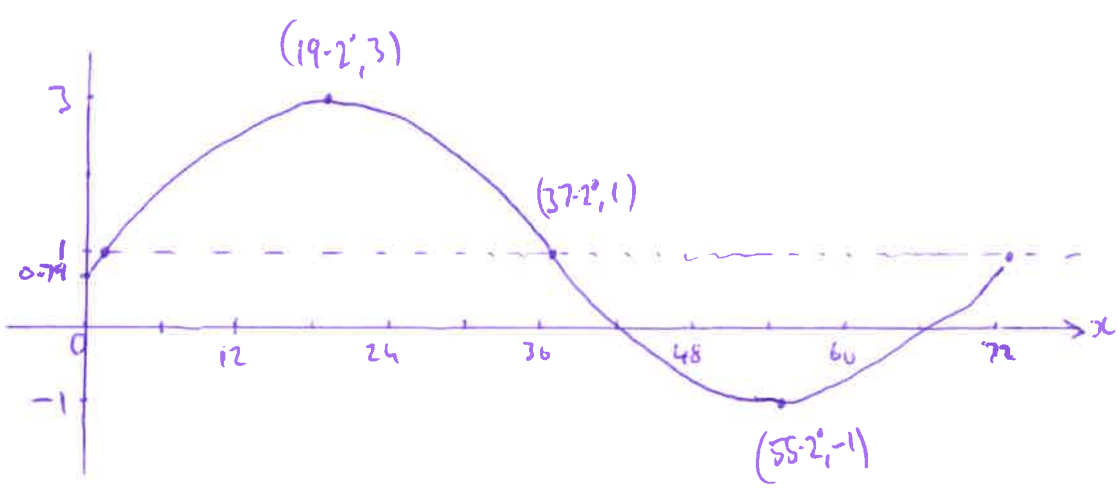
$(270, -1) \rightarrow (55.2, -1)$

$(360, 0) \rightarrow (73.2, 1)$

$x = 0$ $y = 2\sin(5(0) - 6) + 1$

$y = 0.79$

y-intercept $(0, 0.79)$



3F

(2) $d = 3\cos(200t - 60)$ $\quad 0 \le t \le 3.6$

$= 3\cos\left(200\left(t - \frac{3}{10}\right)\right)$

$\updownarrow 3$ $\quad$ 200 waves in 360° $\quad$ $\frac{3}{10} \rightarrow$

period of 1 wave $= 360 \div 200$

$= 1.8$ so 2 waves as $0 \le t \le 3.6$

so $(x, y) \rightarrow \left(\frac{x}{200} + 0.3,\ 3y\right)$

$y = \cos x$ key points

$(0, 1) \rightarrow (0.3, 3)$

$(90, 0) \rightarrow (0.75, 0)$

$(180, -1) \rightarrow (1.2, -3)$

$(270, 0) \rightarrow (1.65, 0)$

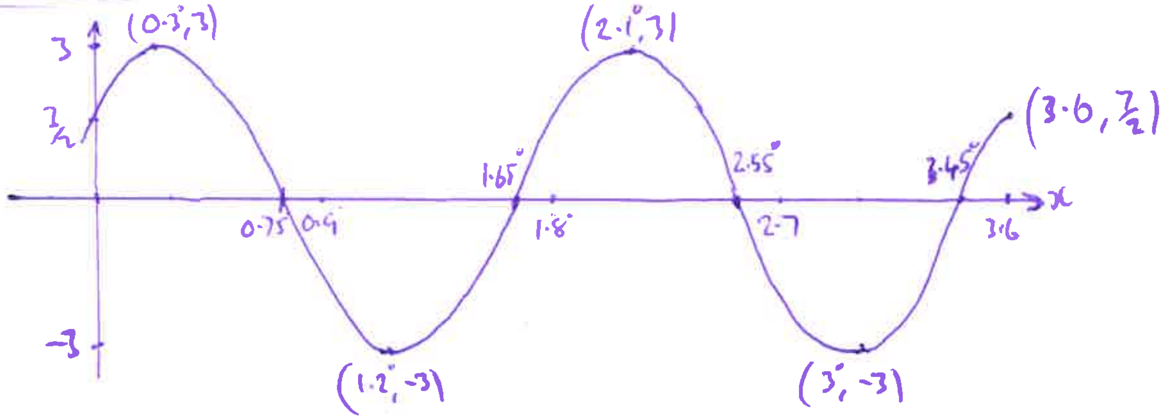
$(360, 1) \rightarrow (2.1, 3)$

note: graph is cyclical so add 1.8 to each x coordinate to get second wave.

$x = 0$ $\quad y = 3\cos(200(0) - 60)$

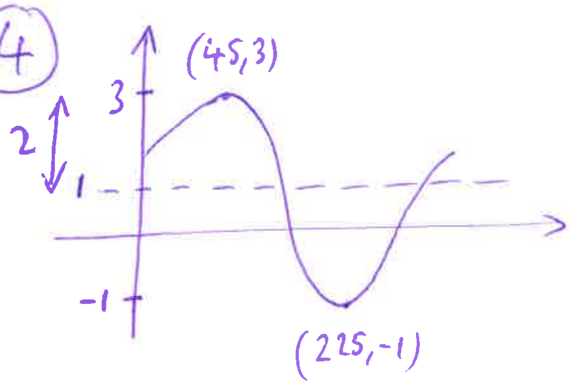
$= \frac{3}{2}$

y-intercept $\left(0, \frac{3}{2}\right)$



3F

(4)



centre line at $y=1 \Rightarrow c=1$

amplitude $=2 \Rightarrow a=2$

horizontal shift $45° \rightarrow \Rightarrow b=45$

$$y = 2\cos(x-45)° + 1$$

(5) $y = p\sin(x+q) + r$

centre line at $y=-1 \Rightarrow r=-1$

amplitude $=4 \Rightarrow p=4$

horizontal shift ~~30°~~ $\leftarrow 30°$ (min at 240° not 270°) $\Rightarrow q = 30°$

$$y = 4\sin(x+30) - 1$$

3F

Q7

(a) $3\cos(x-48)+1 \quad 0^\circ \le x \le 360^\circ$

$3 \updownarrow \quad 48 \rightarrow \quad 1 \uparrow$

so $(x,y) \rightarrow (x+48^\circ, 3y+1)$

$y = \cos x$

max $(0,1) \rightarrow \underline{(48^\circ, 4)}$ — max value of 4 at $x = 48^\circ$

min $(180,-1) \rightarrow \underline{(228^\circ, -2)}$ — min value of -2 at $x = 228^\circ$

(c) $20\sin\left(x+\frac{\pi}{4}\right)+5 \quad 0 \le x \le 2\pi$

$20 \updownarrow \quad \leftarrow \frac{\pi}{4} \quad 5 \uparrow$

$(x,y) \rightarrow \left(x-\frac{\pi}{4}, 20y+5\right)$

$y = \sin x$

max $\left(\frac{\pi}{2}, 1\right) \rightarrow \underline{\left(\frac{\pi}{4}, 25\right)}$ — max value of 25 at $x = \frac{\pi}{4}$ rads

min $\left(\frac{3\pi}{2}, -1\right) \rightarrow \underline{\left(\frac{5\pi}{4}, -15\right)}$ — min value of -15 at $x = \frac{5\pi}{4}$ rads

3F

7(e) $-7\cos(2x-72)^\circ + 4$ $\quad 0 \le x \le 180^\circ$

$= -7\cos(2(x-36)) + 4$

$\updownarrow -7$ $\quad$ 2 waves $\quad 36 \rightarrow$ $\quad \uparrow 4$

so $(x,y) \rightarrow \left(\frac{x}{2}+36, -7y+4\right)$

note as -7 this will change the max to a min as it represents a reflection in the x-axis.

$y = \cos x$ key points

max $(0,1) \rightarrow (36^\circ, -3)$ minimum

min $(180,-1) \rightarrow (126^\circ, 11)$ maximum

min value -3 @ $x = 36^\circ$

max value $= 11$ @ $x = 126^\circ$

7(g) $-50\sin(30x-60) + 10$

$= -50\sin(30(x-2)) + 10$

$\updownarrow -50$ $\quad$ 30 waves $\quad 2 \rightarrow$ $\quad \uparrow 10$

note -50 reflects curve in x-axis

so $(x,y) \rightarrow \left(\frac{x}{30}+2, -50y+10\right)$

$y = \sin x$ key points

max $(90,1) \rightarrow (5^\circ, -40)$ minimum

min $(270,-1) \rightarrow (11, 60)$ maximum

min value -40 @ $x = 5^\circ$

max value 60 @ $x = 11^\circ$

3F

8 $d = 160\cos(30t - 60) + 200$

$= 160\cos(30(t-2)) + 200$

$\updownarrow 160$

30 waves 2 → 200

$(x, y) \to \left(\frac{x}{30} + 2, 160y + 200\right)$

$y = \cos x$

min value $(180, -1) \to (8, 40)$ note low tide occurs at min value.

at low tide depth of water = 40cm occurring at 8 am.

3F

(10) (a) $4\cos x + 3\sin x = k\cos(x-\alpha)^\circ$

$4\cos x + 3\sin x = k(\cos x\cos\alpha + \sin x\sin\alpha)$

$\underline{4\cos x} + \underline{3\sin x} = k\cos\alpha\,\underline{\cos x} + k\sin\alpha\,\underline{\sin x}$

$k\cos\alpha = 4$

$k\sin\alpha = 3$

Calculate α

$\dfrac{k\sin\alpha}{k\cos\alpha} = \dfrac{3}{4}$

$\tan\alpha = 3/4$

$\alpha = \tan^{-1}(3/4)$

$\alpha = 36.9^\circ$

S | A
I | C

Calculate k

$k^2\sin^2\alpha + k^2\cos^2\alpha = 3^2 + 4^2$

$k^2(\cancel{\sin^2\alpha + \cos^2\alpha}) = 25$

$\underline{k = 5}$

$4\cos x + 3\sin x = 5\cos(x - 36.9)^\circ$

(b) $y = 4\cos x + 3\sin x + 2$

$y = 5\cos(x-36.9)^\circ + 2$

5 ↕ → 36.9 ↑ 2

$y = \cos x$ key points

$(0, 1) \rightarrow (36.9^\circ, 7)$

$(90, 0) \rightarrow (126.9^\circ, 2)$

$(180, -1) \rightarrow (216.9^\circ, -3)$

$(270, 0) \rightarrow (306.9^\circ, 2)$

$(360, 1) \rightarrow (396.9^\circ, 7)$

P.T.O.

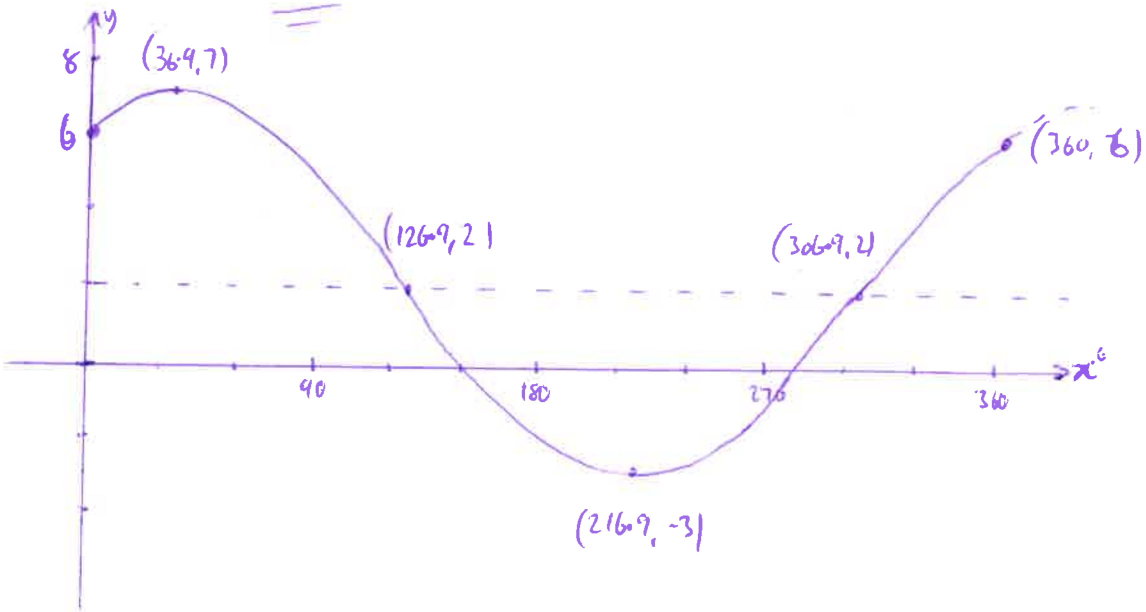
3E

1Q(b) continued

y-intercept $x = 0$, $y = 5\cos(0 - 36.9)^\circ + 2$

$= 6$

$(0, 6)$



3F Note: Question has degrees but last section has a range in radians so I have given answer in both degrees & radians.

12(a) $\cos 2x + \sqrt{3}\sin 2x = R\sin(2x+\alpha)^\circ$

$\cos 2x + \sqrt{3}\sin 2x = R(\sin 2x\cos\alpha + \cos 2x\sin\alpha)$

$\underline{\cos 2x} + \sqrt{3}\,\underline{\sin 2x} = R\cos\alpha\,\underline{\sin 2x} + R\sin\alpha\,\underline{\cos 2x}$

$R\cos\alpha = \sqrt{3}$

$R\sin\alpha = 1$

Calculate α

$\frac{R\sin\alpha}{R\cos\alpha} = \frac{1}{\sqrt{3}}$

$\tan\alpha = \frac{1}{\sqrt{3}}$

$\alpha = 30^\circ$ (or $\pi/6$ radians)

Calculate R

$R^2\sin^2\alpha + R^2\cos^2\alpha = (1)^2 + (\sqrt{3})^2$

$R^2(\cancel{\sin^2\alpha + \cos^2\alpha}) = 4$

$R = 2$

$\cos 2x + \sqrt{3}\sin 2x = 2\sin(2x+30)^\circ$

(or $2\sin(2x+\pi/6)$ radians).

(b) $3(\cos 2x + \sqrt{3}\sin 2x) - 5$

$= 3(2\sin(2x+30)) - 5$

$= 6\sin(2x+30) - 5$

$= 6\sin(2(x+15)) - 5$

6↕ 2 waves ←15 ↓5

so $(x,y) \to \left(\frac{x}{2} - 15,\ 6y - 5\right)$

$y = \sin x$

max $(90, 1) \to (30, 1)$

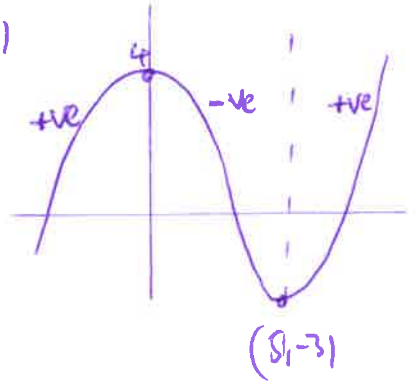
min $(270, -1) \to (120, -11)$

$\boxed{\text{max value of } 1 \text{ @ } x = 30^\circ \text{ (or } \pi/6 \text{ rads)}}$

$\boxed{\text{min value of } -11 \text{ @ } x = 120^\circ \text{ (or } \tfrac{2\pi}{3} \text{ rads)}}$

3G

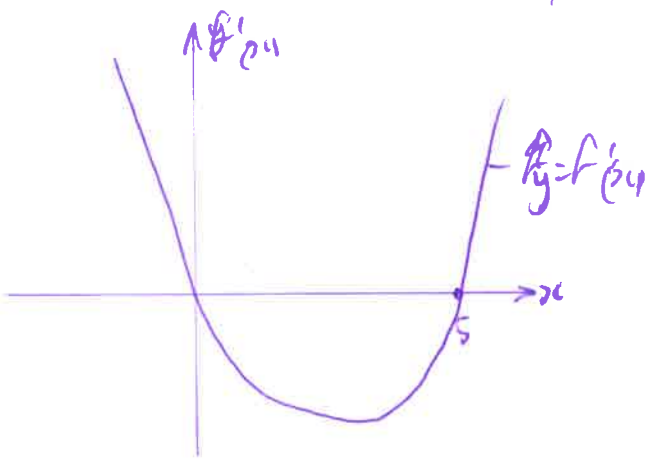
1(a)



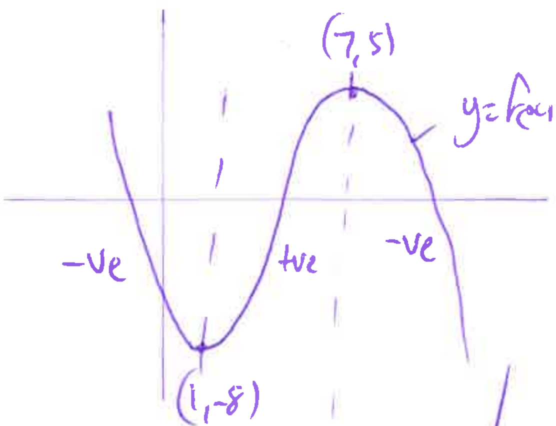
$\downarrow$

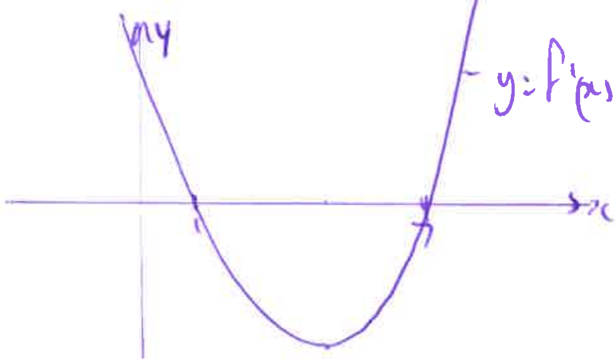
gradient at $(0,4)$ and $(5,-3)$ $= 0$

$\therefore$ x-intercepts on $y = f'(x)$ at $x = 0$, $x = 5$

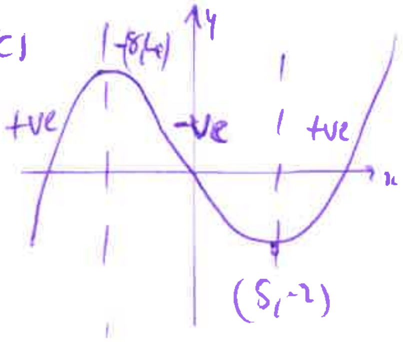


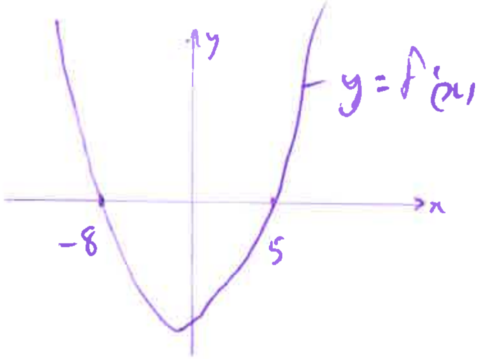
(b)



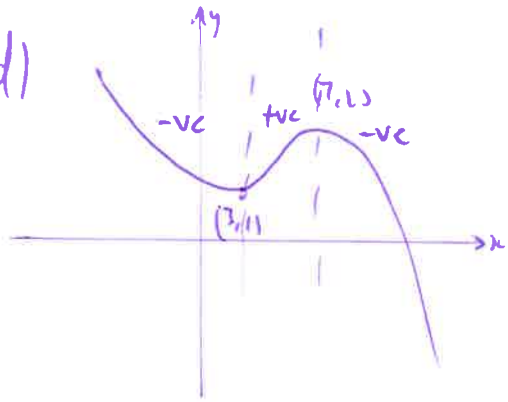


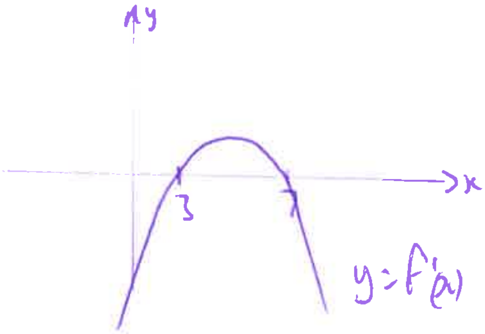
(c)



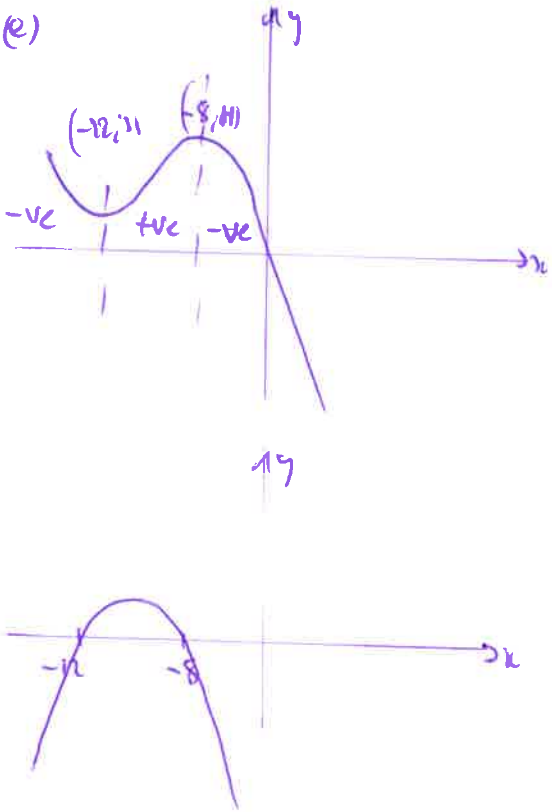


(d)

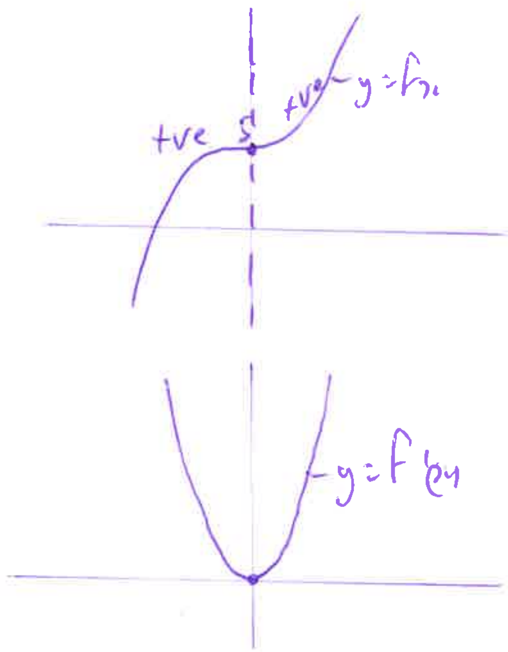




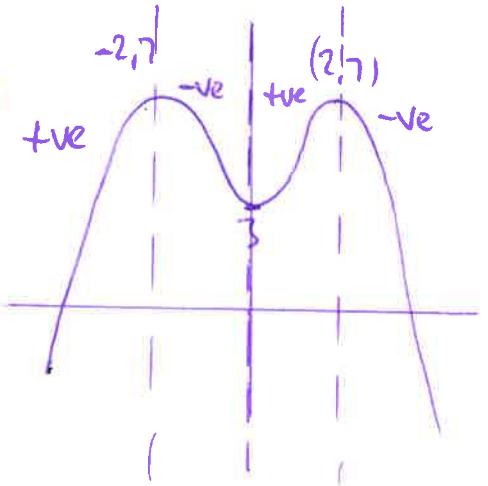
1(e)

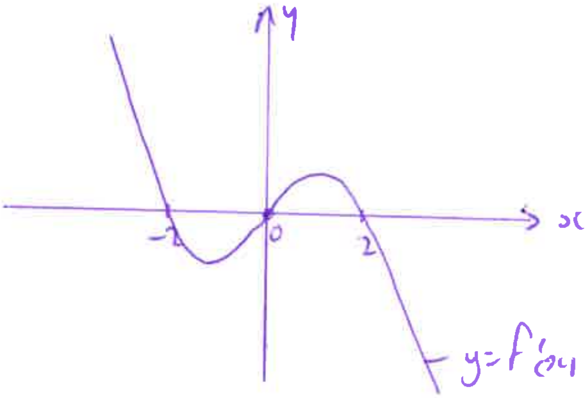


1(f)

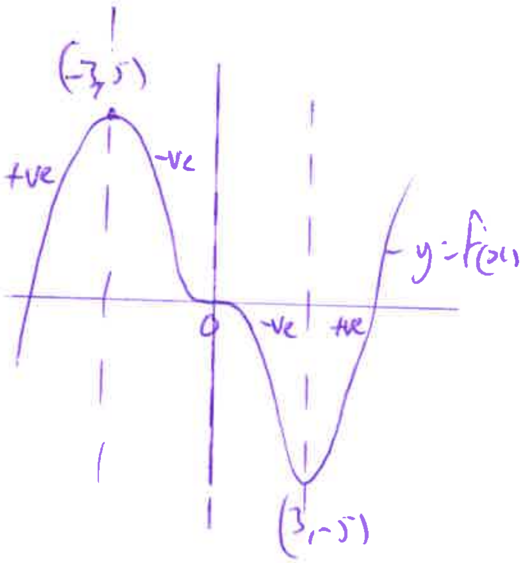


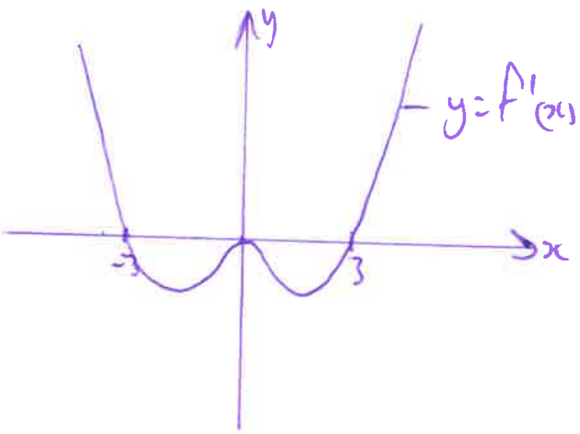
2(c)





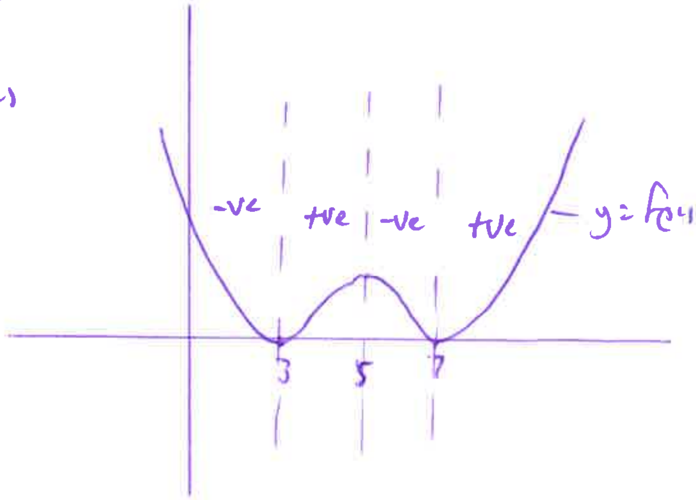
2(b)

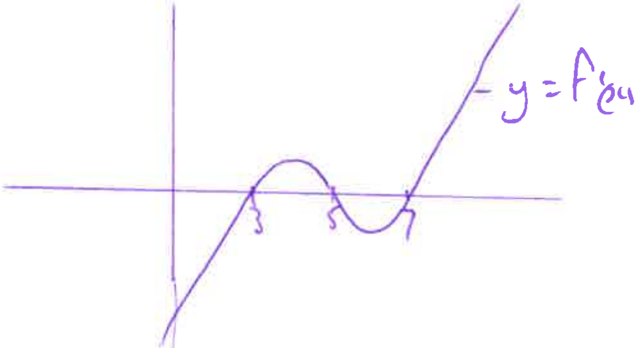




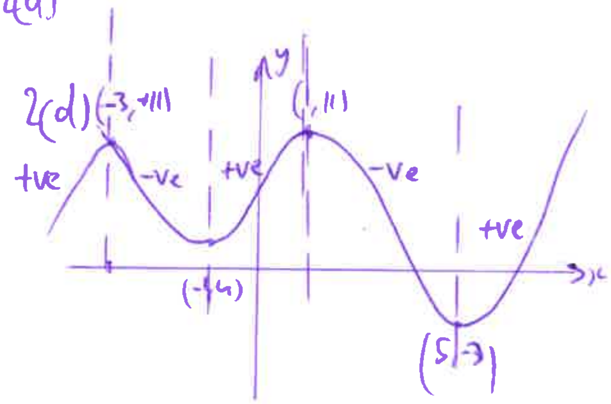
3G

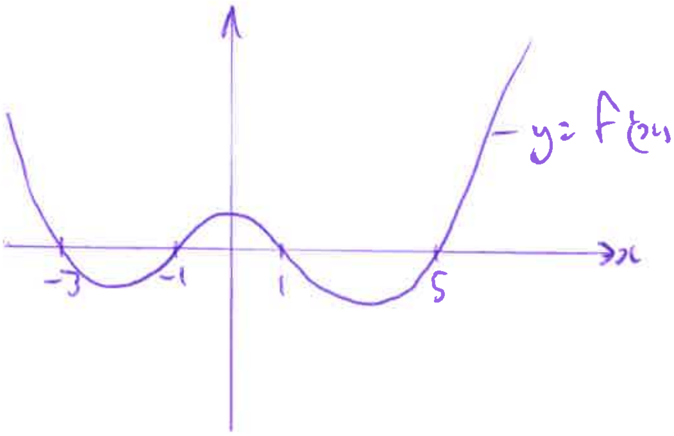
2(c)


-ve
+ve
-ve
+ve
y = f(x)
3
5
7


y = f'(x)
3
5
7

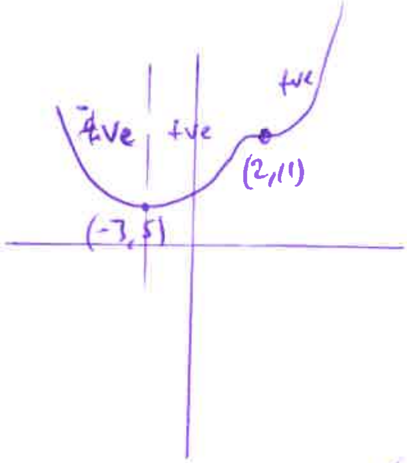
2(d)

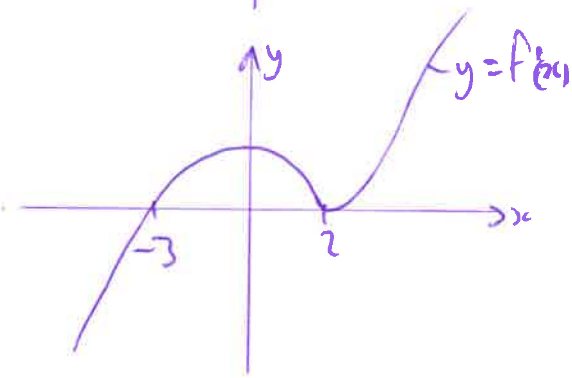

y
2(d)(-3, +11)
(1, 11)
+ve
-ve
+ve
-ve
+ve
(-1, 4)
(5, -3)
x


y = f'(x)
-3
-1
1
5
x

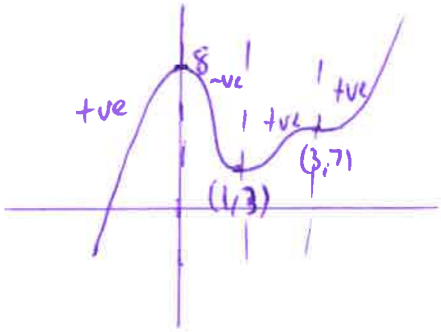
3G

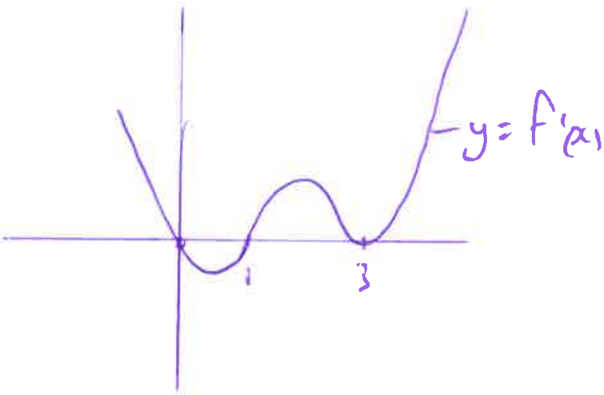
2(e)



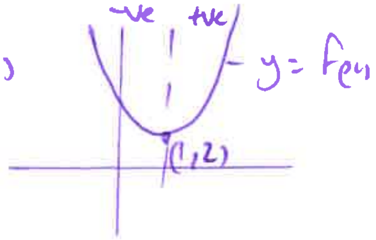


2(f)



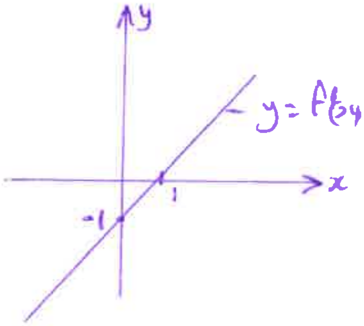


3 (a)

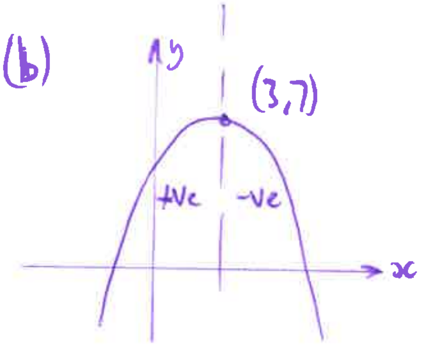


gradient at (0,7) = -1

remember f'(x) gives the gradient so for x = 0 f'(0) = -1 so (0,-1) lies on y = f'(x)



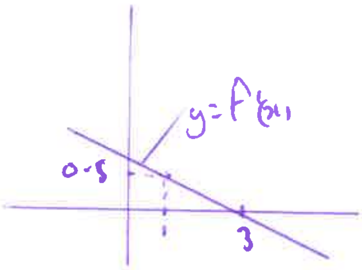
3 (b)



gradient at (1,5) = $\frac{1}{2}$

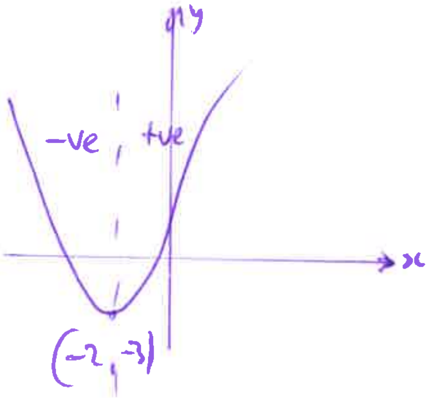
i.e. f'(1) = $\frac{1}{2}$

so (1, $\frac{1}{2}$) lies on y = f'(x)



3G

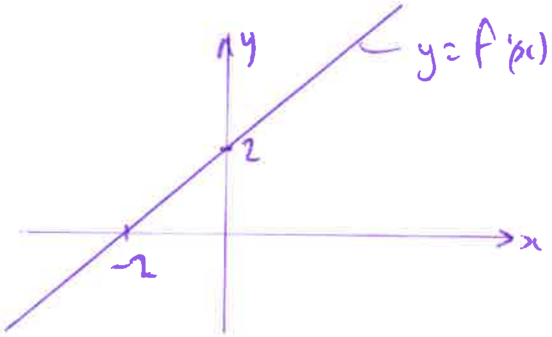
3(c)



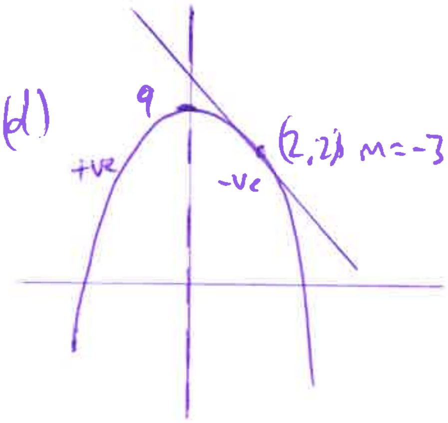
at $(0,0)$ $m = 2$

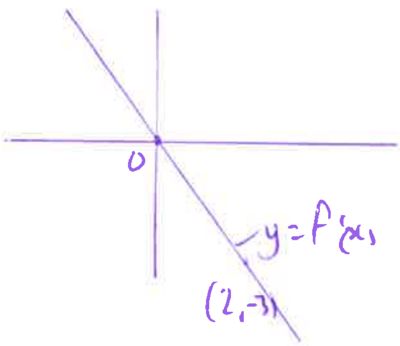
$\therefore f'(0) = 2$

$(0,2)$ lies on $y = f'(x)$



3(d)





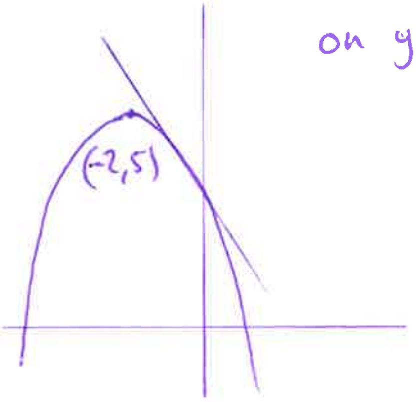
at $(2,2)$ $m = -3$

$\therefore f'(2) = -3$

$\Rightarrow (2,-3)$ lies on $y = f'(x)$

3G

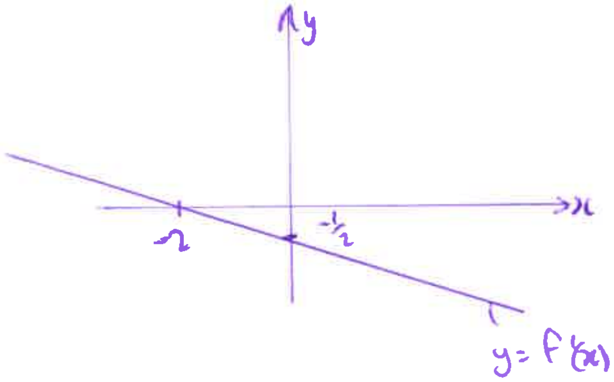
3(e)



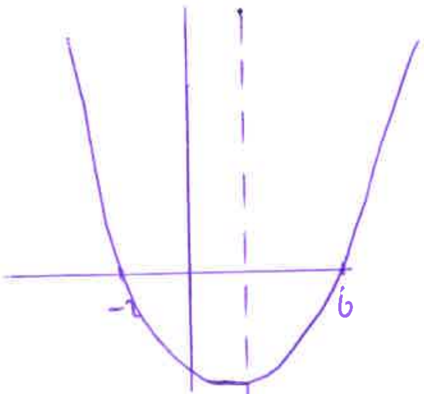
on y-axis $(x=0)$ $m=-\frac{1}{2}$

i.e. $f'(0) = -\frac{1}{2}$

$\Rightarrow$ $(0, -\frac{1}{2})$ lies on $y=f'(x)$



3(f)



at $x=6$ $m=4$

i.e. $(6,4)$ lies on $y=f'(x)$

From symmetry t.p. at $x=2$