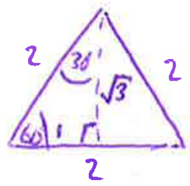
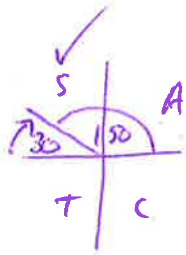
Higher Maths L&L

1(a) $\cos 30$



$= \dfrac{\sqrt{3}}{2}$

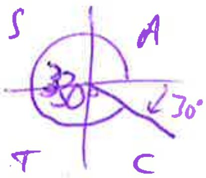
(b) $\sin 150$



$= \sin 30$

$= \dfrac{1}{2}$

(c) $\cos 330$

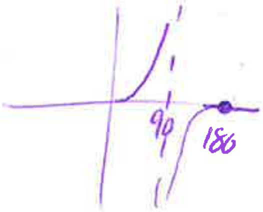


$= \cos 30$

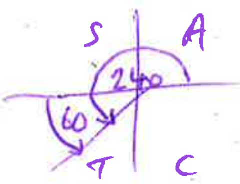
$= \dfrac{\sqrt{3}}{2}$

(d) $\tan 180$

$= 0$



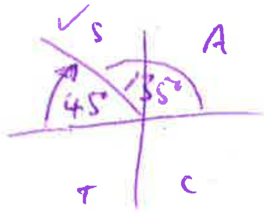
(e) $\sin 240$



$= -\sin 60$

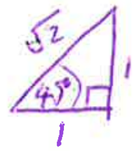
$= -\dfrac{\sqrt{3}}{2}$

(f) $\cos 135$

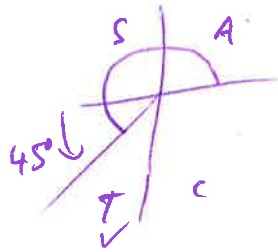


$= -\cos 45$

$= -\dfrac{1}{\sqrt{2}}$



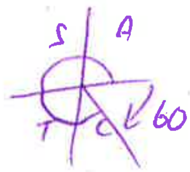
(g) $\tan 225$



$= +\tan 45$

$= +1$

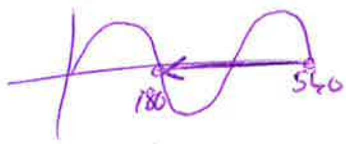
(h) $\cos 300$



$= +\cos 60$

$= +\dfrac{1}{2}$

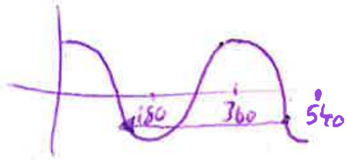
(i) $\sin 540$



$= \sin 180$

$= 0$

(j) $\cos 510$



$= \cos(510 - 360)$

$= \cos 150$

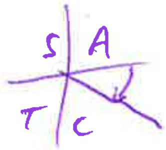
$= -\cos 30$

$= -\dfrac{\sqrt{3}}{2}$

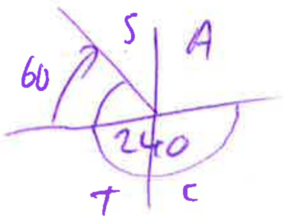
1 (k) $\tan(-30)$ 2A

$= -\tan 30$

$= -\dfrac{1}{\sqrt{3}}$



(l) $\sin(-240)$

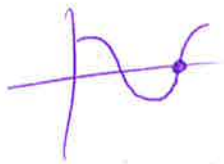


$= \sin 60$

$= \dfrac{\sqrt{3}}{2}$

(m) $\cos 270$

$= 0$



(n) $\tan 750$

$= \tan(750-360)$

$= \tan 390$

$= \tan(390-180)$

$= \tan 30$

$= \dfrac{1}{\sqrt{3}}$

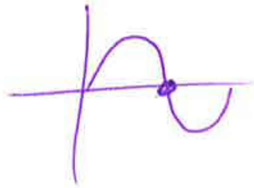
(o) $\sin(-330)$

$= \sin 30$

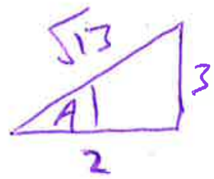
$= \dfrac{1}{2}$

(p) $\sin 180$

$= 0$



(2)



$\sqrt{3^2+2^2}$

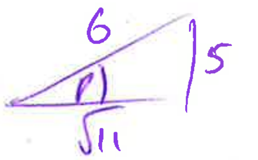
$= \sqrt{13}$

$\tan A = \dfrac{3}{2} = \dfrac{Opp}{Adj}$

$\sin A = \dfrac{3}{\sqrt{13}}$

$\cos A = \dfrac{2}{\sqrt{13}}$

(3) $\sin P = \dfrac{5}{6} = \dfrac{O}{H}$



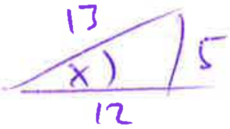
$\sqrt{6^2-5^2}$

$= \sqrt{11}$

$\cos P = \dfrac{\sqrt{11}}{6}$

$\tan P = \dfrac{5}{\sqrt{11}}$

(4) $\sin x = \frac{5}{13}$ $\underline{\underline{2A}}$

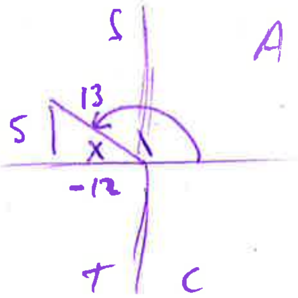


$\sqrt{13^2 - 5^2}$

$= 12$

$\cos x = \frac{5}{13}$

$\tan x = \frac{5}{12}$



$\tan x = -\frac{5}{12}$

$\cos x = \frac{-12}{13}$

Higher Maths 2B

1(a) $30°$

$= \frac{30\pi}{180}$

$= \pi/6$ rads

(b) $\frac{150\pi}{180}$

$= \frac{5\pi}{6}$ rads

(c) $\frac{300\pi}{180}$

$= \frac{5\pi}{3}$ rads

(d) $\frac{135\pi}{180}$

$= \frac{3\pi}{4}$ rad

(e) $\frac{315\pi}{180}$

$= \frac{7\pi}{4}$ rads

(f) $\frac{120\pi}{180}$

$= \frac{2\pi}{3}$ rad

(g) $\frac{240\pi}{180}$

$= \frac{4\pi}{3}$ rads

(h) $\frac{450\pi}{180}$

$= \frac{5\pi}{2}$ rads

(i) $\frac{-60\pi}{180}$

$= -\frac{\pi}{3}$ rads

(j) $= \frac{720\pi}{180}$

$= \underline{\underline{4\pi \text{ rads}}}$

2(a) $\pi/4$ rads

$= \frac{180°}{4}$

$= \underline{\underline{45°}}$

(b) $\frac{\pi}{12}$

$= \frac{180°}{12}$

$= \underline{\underline{15°}}$

(c) $\pi/5$

$= \frac{180°}{5}$

$= 36°$

(d) $\frac{3\pi}{2}$

$= \frac{3(180)}{2}$

$= 270°$

(e) $\frac{5(180)}{4}$

$= \underline{\underline{225°}}$

(f) $\frac{3(180)}{4}$

$= 135°$

(g) $\frac{7(180)}{6}$

$= 210°$

(h) $\frac{7(180)}{3}$

$= 420°$

(i) $-\pi/6$

$= -180/6$

$= \underline{-30°}$

(j) 3π

$= 3(180) = \underline{\underline{540°}}$

3 (a) $\frac{47\pi}{180}$

$= \underline{\underline{0.82 \text{ rads}}}$

(ii) $\frac{324\pi}{180} = \frac{162\pi}{90} = \frac{81\pi}{45} = \frac{9\pi}{5}$

$= \underline{\underline{5.65}} \text{ rads}$

(b) (i) $4.36 \times \left(\frac{180}{\pi}\right)^\circ$

$= 249.8^\circ$

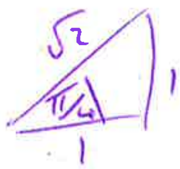
(ii) $1.57 \times \left(\frac{180}{\pi}\right)$

$= 90.0^\circ$ (3 s.f.)

4 (a) $\frac{\pi}{4}$ =x

$= \frac{180}{4}$

$= 45^\circ$



$\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

(b) $\frac{3\pi}{4}$

$= 3 \times 45$

$= 135^\circ$

$\cos \frac{3\pi}{4}$

$= -\cos \frac{\pi}{4}$

$= \underline{\underline{-\frac{1}{\sqrt{2}}}}$

(c) $\sin\left(\frac{5\pi}{3}\right)$ rads

$= \sin 300^\circ$

$= -\sin 60^\circ$

$= -\frac{\sqrt{3}}{2}$

(d) $\tan \frac{11\pi}{6}$

$= \tan 330^\circ$

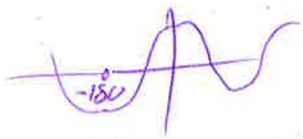
$= -\tan 30^\circ$

$= \underline{\underline{-\frac{1}{\sqrt{3}}}}$

(e) $\cos(-\pi)$

$= \cos(-180^\circ)$

$= -1$



(f) $\tan \frac{7\pi}{3}$

$= \tan 420°$

$= \tan 60°$

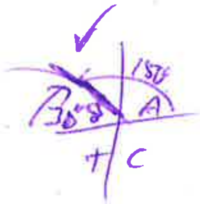
$= \underline{\underline{\sqrt{3}}}$

(g) $\sin\left(\frac{5\pi}{6}\right)$

$= \sin(150°)$

$= \sin(30°)$

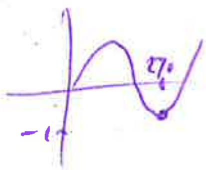
$= \frac{1}{2}$



(h) $\sin \frac{3\pi}{2}$

$= \sin(270°)$

$= \underline{\underline{-1}}$



Higher Maths LoL

2C

(a) $\overset{A}{\sin P}\overset{B}{\cos Q} + \overset{A}{\cos P}\overset{B}{\sin Q}$

$= \underline{\underline{\sin(P+Q)}}$

(b) $\sin \overset{A}{M} \cos \overset{B}{N} - \cos \overset{A}{M} \sin \overset{B}{N}$

$= \sin(M-N)$

(c) $\cos \overset{A}{75} \cos \overset{B}{30} - \sin \overset{A}{75} \sin \overset{B}{30}$

$= \cos(75+30)$

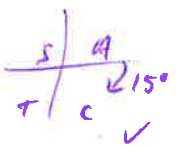
$= \cos$ ~~45~~ 105

(d) $\sin 25 \cos 40 - \cos 25 \sin 40$

$= \sin(25-40)$

$= \sin(-15^\circ)$

$= -\sin(15^\circ)$



(e) $\sin \frac{\pi}{3} \cos \frac{\pi}{5} - \cos\frac{\pi}{3} \sin \frac{\pi}{5}$

$= \sin\left(\frac{\pi}{3} - \frac{\pi}{5}\right)$ $\quad \frac{\pi \times 5}{3\times 5} - \frac{\pi \times 3}{5 \times 3}$

$= \sin\left(\frac{2\pi}{15}\right)$ $\quad = \frac{5\pi - 3\pi}{15}$

(f) $\cos \overset{A}{140} \cos \overset{B}{65} - \sin \overset{A}{140} \sin \overset{B}{65}$

$= \cos(140+65)$

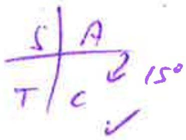
$= \cos(205^\circ)$

(g) $\cos 70 \cos 85 + \sin 70 \sin 85$

$= \cos(70-85)$

$= \cos(-15^\circ)$

$= \cos(15^\circ)$



(h) $\cos \frac{2\pi}{3} \cos \frac{\pi}{4} - \sin \frac{2\pi}{3} \sin \frac{\pi}{4}$

$= \cos\left(\frac{2\pi}{3} + \frac{\pi}{4}\right)$ $\quad \frac{2\pi \times 4}{3 \times 4} + \frac{\pi \times 3}{4 \times 3}$

$= \cos\left(\frac{11\pi}{12}\right)$ $\quad = \frac{8\pi}{12} + \frac{3\pi}{12}$

2C

2 (a) $\sin P \cos Q + \cos P \sin Q$

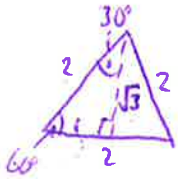
(b) $\cos R \cos S + \sin R \sin S$

(c) $\cos A \cos 48 - \sin A \sin 48$

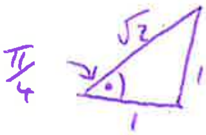
(d) $\sin B \cos 15 - \cos B \sin 15$

(e) $\cos x \cos 60 + \sin x \sin 60$

$= \frac{1}{2}\cos x + \frac{\sqrt{3}}{2}\sin x$



(f) $\sin\left(x + \frac{\pi}{4}\right)$



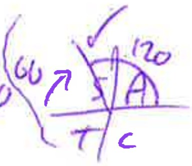
$= \sin x \cos \frac{\pi}{4} + \cos x \sin \frac{\pi}{4}$

$= \frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x$

(g) $\sin(t - 120)$

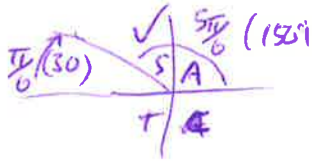
$= \sin t \cos 120 - \cos t \sin 120$

$= \sin t \times (-\cos 60) - \cos t \sin 60$



$= -\frac{1}{2}\sin t - \frac{\sqrt{3}}{2}\cos t$

(h) $\cos\left(x + \frac{5\pi}{6}\right)$



$= \cos x \cos \frac{5\pi}{6} - \sin x \sin \frac{5\pi}{6}$

$= \cos x \times (-\cos 30) - \sin x \sin 30$

$= -\frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x$

3 $\sin(30 + t) = 2\sin(30 - t)$

$\sin 30 \cos t + \cos 30 \sin t = 2(\sin 30 \cos t - \cos 30 \sin t)$

$\frac{1}{2}\cos t + \frac{\sqrt{3}}{2}\sin t = 2\left(\frac{1}{2}\cos t - \frac{\sqrt{3}}{2}\sin t\right)$

$\frac{1}{2}\cos t + \frac{\sqrt{3}}{2}\sin t = \cos t - \sqrt{3}\sin t$

$\frac{\sqrt{3}}{2}\sin t = \frac{1}{2}\cos t - \sqrt{3}\sin t$

$\sqrt{3}\sin t + \frac{\sqrt{3}}{2}\sin t = \frac{1}{2}\cos t$

$\frac{3\sqrt{3}}{2}\sin t = \frac{1}{2}\cos t$

$3\sqrt{3}\sin t = \cos t$

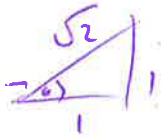
$\sin t = \frac{\cos t}{3\sqrt{3}}$

$\frac{\sin t}{\cos t} = \frac{1}{3\sqrt{3}}$

$\tan t° = \frac{1}{3\sqrt{3}}$

2C

3(b) $2\cos\left(x+\frac{\pi}{4}\right) = \cos\left(x-\frac{\pi}{4}\right)$

(45°) $\frac{\pi}{4}$ 

$$2\left(\cos x \cos \tfrac{\pi}{4} - \sin x \sin \tfrac{\pi}{4}\right) = \cos x \cos \tfrac{\pi}{4} + \sin x \sin \tfrac{\pi}{4}$$

$$2\left(\frac{1}{\sqrt{2}}\cos x - \frac{1}{\sqrt{2}}\sin x\right) = \frac{1}{\sqrt{2}}\cos x + \frac{1}{\sqrt{2}}\sin x$$

× every term by $\sqrt{2}$

$$2(\cos x - \sin x) = \cos x + \sin x$$

$$2\cos x - 2\sin x = \cos x + \sin x$$

$$\cos x = 3\sin x$$

$$1 = 3\,\frac{\sin x}{\cos x}$$

$$\frac{1}{3} = \tan x$$

$$\underline{\tan x = \frac{1}{3}}$$

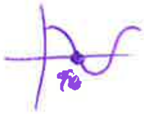
Higher Maths L&C

2D

Q(a) $\cos(140-50)$

$= \cos 90$

$= 0$

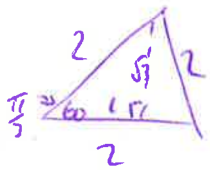


(b) $\sin\left(\frac{\pi}{5} + \frac{2\pi}{15}\right)$

$= \sin\left(\frac{3\pi}{15} + \frac{2\pi}{15}\right)$

$= \sin\left(\frac{5\pi}{15}\right)$

$= \sin\left(\frac{\pi}{3}\right)$



$= \frac{\sqrt{3}}{2}$

(c) $\cos(80-20)$

$= \cos 60$

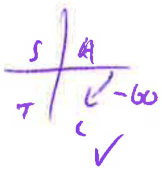
$= \frac{1}{2}$

(d) $\sin(25-85)$

$= \sin(-60)$

$= -\sin 60$

$= -\frac{\sqrt{3}}{2}$



(e) $\sin 75$

$= \sin(45+30)$

$= \sin 45 \cos 30 + \cos 45 \sin 30$

$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2}$

$= \frac{\sqrt{3}+1}{2\sqrt{2}}$

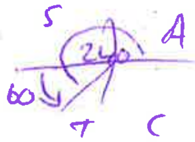
(f) $\cos(60+45)$

$= \cos 60 \cos 45 - \sin 60 \sin 45$

$= \frac{1}{2} \cdot \frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}}$

$= \frac{1-\sqrt{3}}{2\sqrt{2}}$

2D



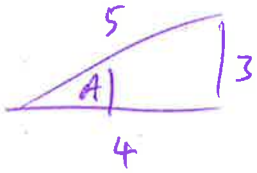
2(a) $\sin(x+60) + \sin(x+240)$

$= \sin x \cos 60 + \cos x \sin 60 + \sin x \cos 240 + \cos x \sin 240$

$= + \cancel{\sin x \cos 60} + \cancel{\cos x \sin 60} + \cancel{\sin x \times (-\cos 60)} + \cancel{\cos x \times (-\sin 60)}$

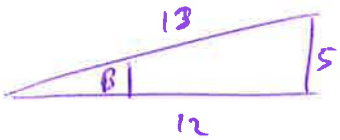
$= 0$

(3)



$\sin A = \frac{3}{5}$

$\cos A = \frac{4}{5}$



$\sin B = \frac{5}{13}$

$\cos B = \frac{12}{13}$

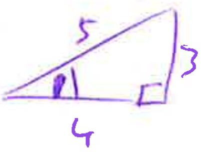
$\sin(A-B) = \sin A \cos B - \cos A \sin B$

$= \frac{3}{5} \times \frac{12}{13} - \frac{4}{5} \times \frac{5}{13}$

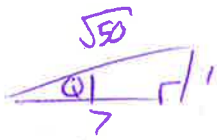
$= \frac{36}{65} - \frac{20}{65}$

$= \frac{16}{65}$

(4) $\tan P = \frac{3}{4}$



$\tan Q = \frac{1}{7}$



$\cos P = \frac{4}{5}$ $\sin P = \frac{3}{5}$ $\sin Q = \frac{1}{\sqrt{50}}$ $\cos Q = \frac{7}{\sqrt{50}}$

$\cos(P+Q) = \cos P \cos Q - \sin P \sin Q$

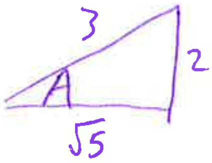
$= \frac{4}{5} \times \frac{7}{\sqrt{50}} - \frac{3}{5} \times \frac{1}{\sqrt{50}}$

$= \frac{28}{5\sqrt{50}} - \frac{3}{5\sqrt{50}} = \frac{25}{5\sqrt{50}} = \frac{25}{5\sqrt{25}\sqrt{2}} = \frac{25}{25\sqrt{2}} = \frac{1}{\sqrt{2}}$

2D

(6) $\sin A = \frac{2}{3}$

$\sqrt{3^2 - 2^2} = \sqrt{5}$



$\cos A = \frac{\sqrt{5}}{3}$

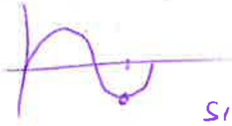
$\tan A = \frac{2}{\sqrt{5}}$

$\cos\left(A + \frac{3\pi}{2}\right)$

$= \cos A \cancel{\cos\frac{3\pi}{2}}^{\,0} - \sin A \cancel{\sin\frac{3\pi}{2}}^{\,-1}$

$\cos\left(\frac{3\pi}{2}\right) = 0$

$\frac{3\pi}{2}$ rads $= 270°$



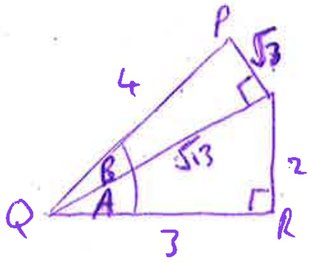
$\sin\frac{3\pi}{2} = -1$

$= -\sin A \times (-1)$

$= \sin A$

$= \frac{2}{3}$

(8)



$\sqrt{2^2 + 3^2} = \sqrt{13}$

$\sqrt{4^2 - (\sqrt{13})^2} = \sqrt{3}$

$\sin(PQR)$

$= \sin(A + B)$

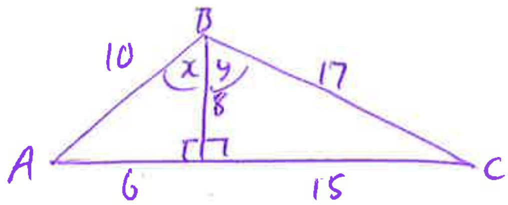
$= \sin A \cos B + \cos A \sin B$

$= \frac{2}{\sqrt{13}} \times \frac{\sqrt{13}}{4} + \frac{3}{\sqrt{13}} \times \frac{\sqrt{3}}{4}$

$= \frac{2\sqrt{13} + 3\sqrt{3}}{4\sqrt{13}}$

2D

9



$\cos(A\hat{B}C)$

$= \cos(x+y)$

$= \cos x \cos y - \sin x \sin y$

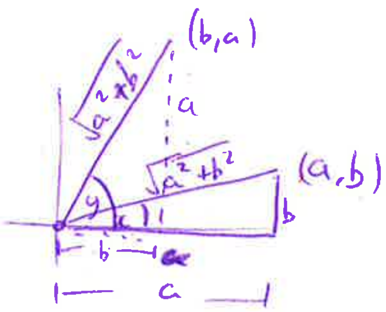
$= \frac{8}{10} \times \frac{8}{17} - \frac{6}{10} \times \frac{15}{17}$

$= \frac{64 - 90}{170}$

$= \frac{-26}{170}$

$= \frac{-13}{85}$

10



$\cos(A O B)$

$= \cos(y-x)$

$= \cos y \cos x + \sin y \sin x$

$= \frac{b}{\sqrt{a^2+b^2}} \times \frac{a}{\sqrt{a^2+b^2}} + \frac{a}{\sqrt{a^2+b^2}} \times \frac{b}{\sqrt{a^2+b^2}} = \frac{2ab}{a^2+b^2}$

Higher Maths L & L

2E

1(a) $\cos^2(15) - \sin^2(15)$

$= \cos(2(15))$

$= \cos 30$

$= \frac{\sqrt{3}}{2}$

(b) $2\cos^2\left(\frac{\pi}{8}\right) - 1$

$= \cos\left(2\left(\frac{\pi}{8}\right)\right)$

$= \cos\left(\frac{\pi}{4}\right)$

$= \frac{1}{\sqrt{2}}$

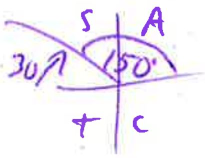
(c) $2\sin 75 \cos 75$

$= \sin(2(75))$

$= \sin 150$

$= +\sin 30$

$= +\frac{1}{2}$



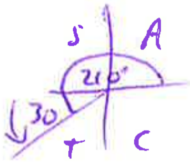
(d) $2\sin 105 \cos 105$

$= \sin(2(105))$

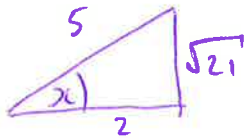
$= \sin(210)$

$= -\sin 30$

$= -\frac{1}{2}$



2 (a)



$\sqrt{5^2 - 2^2} = \sqrt{21}$

(a) $\sin 2x$

$= 2\sin x \cos x$

$= 2\,\frac{\sqrt{21}}{5} \times \frac{2}{5}$

$= \frac{4\sqrt{21}}{25}$

(b) $\cos 2x$

$= \cos^2 x - \sin^2 x$

$= \left(\frac{2}{5}\right)^2 - \left(\frac{\sqrt{21}}{5}\right)^2$

$= \frac{4}{25} - \frac{21}{25}$

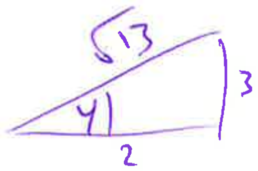
$= \frac{-17}{25}$

(c) $\tan 2x$

$= \frac{\sin 2x}{\cos 2x}$

$= \frac{\frac{4\sqrt{21}}{25}}{\frac{-17}{25}} = \frac{4\sqrt{21}}{\cancel{25}} \times \frac{\cancel{25}}{-17} = \frac{-4\sqrt{21}}{17}$

③ $\tan Y = \frac{3}{2}$



$\sin Y = \frac{3}{\sqrt{13}}$

$\cos Y = \frac{2}{\sqrt{13}}$

$\sin 2Y = 2\sin Y \cos Y$

$= 2\ \frac{3}{\sqrt{13}} \times \frac{2}{\sqrt{13}}$

$= \frac{12}{13}$

$\cos 2Y = \cos^2 Y - \sin^2 Y$

$= \left(\frac{2}{\sqrt{13}}\right)^2 - \left(\frac{3}{\sqrt{13}}\right)^2$

$= \frac{-5}{13}$

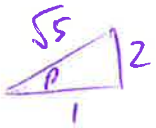
$\sin 4Y$ (double angle formula - $4Y$ is double $2Y$).

$= 2\sin 2Y \cos 2Y$

$= 2\left(\frac{12}{13}\right)\left(\frac{-5}{13}\right)$

$= \frac{-120}{169}$

④ $\sin P = \frac{2}{\sqrt{5}}$



$\cos P = \frac{1}{\sqrt{5}}$

$\sin 2P = 2\sin P \cos P$

$= 2\left(\frac{2}{\sqrt{5}}\right)\left(\frac{1}{\sqrt{5}}\right)$

$= \frac{4}{5}$

$\cos 2P = \cos^2 P - \sin^2 P$

$= \left(\frac{1}{\sqrt{5}}\right)^2 - \left(\frac{2}{\sqrt{5}}\right)^2$

$= \frac{-3}{5}$

$\cos(3P)$

$= \cos(2P + P)$

$= \cos(2P)\cos(P) - \sin 2P \sin P$

$= \frac{-3}{5} \times \frac{1}{\sqrt{5}} - \frac{4}{5} \times \frac{2}{\sqrt{5}}$

$= \frac{-3-8}{5\sqrt{5}} = \frac{-11}{5\sqrt{5}}$

(6) $\cos 2A = \frac{3}{5}$

$2\cos^2 A - 1 = \frac{3}{5}$

$2\cos^2 A = \frac{8}{5}$

$\cos^2 A = \frac{4}{5}$

$\cos A = \pm\sqrt{\frac{4}{5}}$

$\cos A = \pm\frac{2}{\sqrt{5}}$

'A' is acute so +ve value of cos A only.

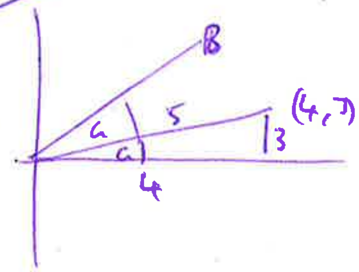
$\cos A = \frac{2}{\sqrt{5}}$

$= \frac{2\sqrt{5}}{\sqrt{5}\sqrt{5}}$

$= \underline{\underline{\frac{2\sqrt{5}}{5}}}$

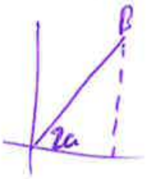
2E

(8)



$\sin a = \frac{3}{5}$

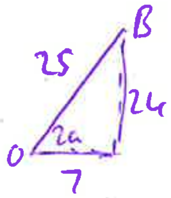
$\cos a = \frac{4}{5}$



$$\sin 2a = 2 \sin a \cos a$$

$$= 2\left(\frac{3}{5}\right)\left(\frac{4}{5}\right)$$

$$= \frac{24}{25}$$

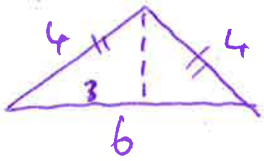


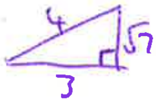
$$\text{gradient} = \frac{\Delta y}{\Delta x} \left(= \frac{\text{up}}{\text{along}}\right)$$

$$= \frac{24}{7}$$

2E

10





$A = \frac{1}{2} bh$

$= \frac{1}{2}(6)(\sqrt{7})$

$\cancel{= 3\sqrt{7}\,m^2}$

$= 3\sqrt{7}\ m^2$

2G

1(a) $\sin(x+45) + \cos(x+45)$

$= \sin x \cos 45 + \cos x \sin 45 + \cos x \cos 45 - \sin x \sin 45$

$= \cancel{\frac{1}{\sqrt{2}} \sin x} + \frac{1}{\sqrt{2}} \cos x + \frac{1}{\sqrt{2}} \cos x - \cancel{\frac{1}{\sqrt{2}} \sin x}$

$= 2\left(\frac{1}{\sqrt{2}} \cos x\right)$

$= \frac{2}{\sqrt{2}} \cos x$

$= \underline{\underline{\sqrt{2} \cos x}}$

(c) $\sin(x-60) + \cos(x+30)$

$= \sin x \cos 60 - \cos x \sin 60 + \cos x \cos 30 - \sin x \sin 30$

$= \cancel{\frac{1}{2} \sin x} - \cancel{\frac{\sqrt{3}}{2} \cos x} + \cancel{\frac{\sqrt{3}}{2} \cos x} - \cancel{\frac{1}{2} \sin x}$

$= \underline{\underline{0}}$

(e) $\sin\theta - \sin\left(\theta + \frac{\pi}{3}\right) + \cos\left(\theta + \frac{\pi}{6}\right)$

$= \sin\theta - \left(\sin\theta \cos\frac{\pi}{3} + \cos\theta \sin\frac{\pi}{3}\right) + \cos\theta \cos\frac{\pi}{6} - \sin\theta \sin\frac{\pi}{6}$

$= \cancel{\sin\theta} - \cancel{\frac{1}{2} \sin\theta} - \cancel{\frac{\sqrt{3}}{2} \cos\theta} + \cancel{\frac{\sqrt{3}}{2} \cos\theta} - \cancel{\frac{1}{2} \sin\theta}$

$= \underline{\underline{0}}$

2G

2(a) $\cos(\alpha+\beta) + \cos(\alpha-\beta)$

$= \cos\alpha\cos\beta - \cancel{\sin\alpha\sin\beta} + \cos\alpha\cos\beta + \cancel{\sin\alpha\sin\beta}$

$= 2\cos\alpha\cos\beta$

(b) $\sin(\alpha+\beta) - \sin(\alpha-\beta)$

$= \cancel{\sin\alpha\cos\beta} + \cos\alpha\sin\beta - (\cancel{\sin\alpha\cos\beta} - \cos\alpha\sin\beta)$

$= 2\cos\alpha\sin\beta$

4. $\dfrac{\cos(x+y)}{\cos x\cos y}$

$= \dfrac{\cos x\cos y - \sin x\sin y}{\cos x\cos y}$

$= \dfrac{\cancel{\cos x}\,\cancel{\cos y}}{1\cancel{\cos x}\,\cancel{\cos y}} - \dfrac{\sin x\sin y}{\cos x\cos y}$ $\qquad \dfrac{\sin x}{\cos x} = \tan x \qquad \dfrac{\sin y}{\cos y} = \tan y.$

$= 1 - \tan x\tan y$

2G

6 $2\cos 2x - \cos^2 x$

$\cos^2 x + \sin^2 x = 1$

$\cos^2 x = 1 - \sin^2 x$

$\cos 2x = 1 - 2\sin^2 x$

$= 2(1 - 2\sin^2 x) - (1 - \sin^2 x)$

$= 2 - 4\sin^2 x - 1 + \sin^2 x$

$= 1 - 3\sin^2 x$

7 $\sin 3A$

$= \sin(2A + A)$

$= \sin 2A \cos A + \cos 2A \sin A$

$= 2\sin A \cos A \cos A + (1 - 2\sin^2 A)\sin A$

$= 2\sin A \cos^2 A + \sin A - 2\sin^3 A$

$= 2\sin A(1 - \sin^2 A) + \sin A - 2\sin^3 A$

$= 2\sin A - 2\sin^3 A + \sin A - 2\sin^3 A$

$= 3\sin A - 4\sin^3 A$

(8) $\cos 4\theta$ $\quad$ 4θ is double 2θ

$= \cos(2\theta + 2\theta)$

$= \cos 2\theta \cos 2\theta - \sin 2\theta \sin 2\theta$

$= (2\cos^2\theta - 1)^2 - (2\sin\theta\cos\theta)^2$

$= (2\cos^2\theta - 1)^2 - 4\sin^2\theta\cos^2\theta$

$= (2\cos^2\theta - 1)(2\cos^2\theta - 1) - 4(1 - \cos^2\theta)\cos^2\theta$

$= 4\cos^4\theta - 4\cos^2\theta + 1 - 4(\cos^2\theta - \cos^4\theta)$

$= 4\cos^4\theta - 4\cos^2\theta + 1 - 4\cos^2\theta + 4\cos^4\theta$

$= 8\cos^4\theta - 8\cos^2\theta + 1$

2H

1(a) $K = \sqrt{4^2 + 3^2}$

$= \sqrt{25}$

$= \underline{\underline{5}}$

$\tan\alpha = \dfrac{\cancel{K}\sin\alpha}{\cancel{K}\cos\alpha}$

$\tan\alpha = \dfrac{3}{4}$

$\alpha = \tan^{-1}\left(\dfrac{3}{4}\right)$

$\underline{\underline{\alpha = 36.9°}}$

1(b) $K = \sqrt{5^2 + (-3)^2}$

$= \underline{\underline{\sqrt{34}}}$

$\tan\alpha = \dfrac{K\sin\alpha}{K\cos\alpha}$

$\tan\alpha = \dfrac{-3}{5}$

$\tan^{-1}\left(\dfrac{3}{5}\right) = 31.0°$

$\alpha = 360 - 31.0°$

$\underline{\underline{= 329.0°}}$

sinα -ve

cosα +ve

tanα -ve

2H

1(c) $K = \sqrt{(-1)^2 + 2^2}$

$K = \underline{\underline{\sqrt{5}}}$

$\tan\alpha = \dfrac{K\sin\alpha}{K\cos\alpha}$

$= \dfrac{2}{-1}$

$= -2$

$\tan^{-1}(2) = 63.4°$

$\alpha = 180° - 63.4°$

$= \underline{\underline{116.6°}}$

2(a) $K = \sqrt{1^2 + 1^2}$

$K = \underline{\underline{\sqrt{2}}}$

$\tan\alpha = \dfrac{\cancel{K}\sin\alpha}{\cancel{K}\cos\alpha}$

$= \dfrac{1}{1}$

$= 1$

$\alpha = \tan^{-1}(1)$

$= \underline{\underline{\pi/4}}$ radians

2H

2(b) $K\cos\alpha = -\sqrt{3}$ $\quad K\sin\alpha = 1$

$$K = \sqrt{(-\sqrt{3})^2 + (1)^2}$$

$$= \sqrt{3+1}$$

$$= \underline{\underline{2}}$$

$$\tan\alpha = \frac{K\sin\alpha}{K\cos\alpha}$$

$$= \frac{1}{-\sqrt{3}}$$

$$= \frac{-1}{\sqrt{3}}$$

$$\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \pi/6$$

$$\alpha = 2\pi - \frac{\pi}{6}$$

$$= \underline{\underline{\frac{5\pi}{6}}} \text{ radians}$$

2(c) $K = \sqrt{(-4)^2 + (5)^2}$

$$= \sqrt{41}$$

$$\tan\alpha = \frac{K\sin\alpha}{K\cos\alpha}$$

$$= 5/-4$$

$$= -4/5$$

$$\tan^{-1}(4/5) = 0.896\ldots$$

$$\alpha = \pi - 0.896\ldots = \underline{\underline{2.25 \text{ radians}}}$$

2I Higher Maths LoL

1 (a) $\cos x + 2\sin x = k\cos(x - \alpha)$

$= k(\cos x \cos\alpha + \sin x \sin\alpha)$

$= k\cos x \cos\alpha + k\sin x \sin\alpha$

$\underline{\cos x} + \underline{2\sin x} = k\cos\alpha\, \underline{\cos x} + k\sin\alpha\, \underline{\sin x}$

$k\cos\alpha = 1$

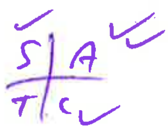
$k\sin\alpha = 2$

$k = \sqrt{2^2 + 1^2}$

$k = \underline{\underline{\sqrt{5}}}$

$\tan\alpha = \dfrac{k\sin\alpha}{k\cos\alpha}$

$= \dfrac{2}{1}$



$\alpha = \tan^{-1}(2)$

$\alpha = \underline{\underline{63.4^\circ}}$

$\cos x + 2\sin x = \underline{\underline{\sqrt{5}\cos(x - 63.4)^\circ}}$

2I

1(b) $\cos x + 2\sin x = K\cos(x+\alpha)$

$= K(\cos x \cos\alpha - \sin x \sin\alpha)$

$= K\cos x \cos\alpha - K\sin x \sin\alpha$

$\underline{\cos x} + 2\sin x = K\cos\alpha\, \underline{\cos x} - K\sin\alpha \sin x$

$\underline{\underline{K\cos\alpha = 1}}$

$-K\sin\alpha = 2$

$\underline{\underline{K\sin\alpha = -2}}$

$K = \sqrt{(-2)^2 + (1)^2}$

$= \underline{\underline{\sqrt{5}}}$

$\tan\alpha = \dfrac{K\sin\alpha}{K\cos\alpha}$

$= \dfrac{-2}{1}$

$= \underline{\underline{-2}}$

S | A ✓

✓T | C ✓✓

$\tan^{-1}(2) = 63.4°$

$\alpha = 360 - 63.4$

$\underline{\underline{\alpha = 296.6°}}$

$\underline{\underline{\underline{\cos x + 2\sin x = \sqrt{5}\cos(x - 296.6)°}}}$

2I

1(a) $\cos x + 2\sin x = K\sin(x+\alpha)$

$= K(\sin x \cos\alpha + \cos x \sin\alpha)$

$= K\sin x\cos\alpha + K\cos x\sin\alpha)$

$\cos x + 2\sin x = K\cos\alpha\sin x + K\sin\alpha\cos x$

$K\sin\alpha = 1$

$K\cos\alpha = 2$

$K = \sqrt{1^2+2^2}$

$K = \sqrt{5}$

$\tan\alpha = \dfrac{K\sin\alpha}{K\cos\alpha}$

$= \dfrac{1}{2}$

$\begin{array}{c|c} S\checkmark & A\checkmark\checkmark \\ \hline T & C\checkmark \end{array}$

$\alpha = \tan^{-1}\left(\frac{1}{2}\right)$

$\alpha = 26.6^\circ$

$\cos x + 2\sin x = \sqrt{5}\sin(x+26.6)^\circ$

2I

1(d) $\cos x + 2\sin x = K\sin(x-\alpha)$

$= K(\sin x\cos\alpha - \cos x\sin\alpha)$

$= K\sin x\cos\alpha - K\cos x\sin\alpha$

$\underline{\cos x} + 2\underline{\underline{\sin x}} = K\cos\alpha\,\underline{\underline{\sin x}} - K\sin\alpha\,\underline{\cos x}$

$1 = -K\sin\alpha$

$\underline{K\sin\alpha = -1}$

$\underline{K\cos\alpha = 2}$

$K = \sqrt{(-1)^2 + 2^2}$

$\underline{\underline{K = \sqrt{5}}}$

$\tan\alpha = \dfrac{K\sin\alpha}{K\cos\alpha}$

$= \dfrac{-1}{2}$

$$\begin{array}{c|c} S & A\checkmark \\ \hline T & C\checkmark\checkmark \end{array}$$

$\tan^{-1}\left(\frac{1}{2}\right) = 26.6°$

$\alpha = 360 - 26.6$

$= 333.4°$

$$\underline{\underline{\cos x + 2\sin x = \sqrt{5}\,\sin(x - 333.4)°}}$$

2I

(3) $\sqrt{2}\cos\theta + \sqrt{2}\sin\theta = K\sin(\theta - \alpha)$

$= K(\sin\theta\cos\alpha - \cos\theta\sin\alpha)$

$= K\sin\theta\cos\alpha - K\cos\theta\sin\alpha$

$\sqrt{2}\cos\theta + \sqrt{2}\sin\theta = K\cos\alpha\sin\theta - K\sin\alpha\cos\theta$

$-K\sin\alpha = \sqrt{2}$

$K\sin\alpha = -\sqrt{2}$

$K\cos\alpha = \sqrt{2}$

$\tan\alpha = \dfrac{K\sin\alpha}{K\cos\alpha}$

$= \dfrac{-\sqrt{2}}{\sqrt{2}}$

$= -1$

S | A ✓
T | C ✓✓
✓

$K = \sqrt{(-\sqrt{2})^2 + (\sqrt{2})^2}$

$= \sqrt{2+2}$

$= \sqrt{4}$

$= 2$

$\tan^{-1}(1) = \pi/4$

$\alpha = 2\pi - \pi/4$

$\alpha = \dfrac{7\pi}{4}$

$\sqrt{2}\cos\theta + \sqrt{2}\sin\theta = 2\sin\left(\theta - \dfrac{7\pi}{4}\right)$ radians

2I

5 $\sqrt{3}\cos x - \sin x = K\cos(x+\alpha)$

$= K(\cos x\cos\alpha - \sin x\sin\alpha)$

$= K\cos x\cos\alpha - K\sin x\sin\alpha$

$\sqrt{3}\,\underline{\cos x} - \underline{\sin x} = K\cos\alpha\,\underline{\cos x} - K\sin\alpha\,\underline{\sin x}$

$K\cos\alpha = \sqrt{3}$

$K\sin\alpha = 1$

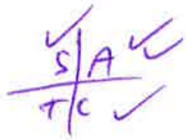
$K = \sqrt{(\sqrt{3})^2 + (1)^2}$

$= \sqrt{4}$

$= 2$

$\tan\alpha = \dfrac{K\sin\alpha}{K\cos\alpha}$

$= \dfrac{1}{\sqrt{3}}$



$\alpha = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$\alpha = 30°$

$\sqrt{3}\cos x - \sin x = 2\cos(x+30)°$

2I

7 $4\sin 2x - 3\cos 2x = K\sin(2x - \alpha)$

$= K(\sin 2x\cos\alpha - \cos 2x\sin\alpha)$

$= K\sin 2x\cos\alpha - K\cos 2x\sin\alpha$

$4\underline{\sin 2x} - 3\underline{\cos 2x} = K\cos\alpha\,\underline{\sin 2x} - K\sin\alpha\,\underline{\cos 2x}$

$K\cos\alpha = 4$

$+K\sin\alpha = +3$

$K = \sqrt{4^2 + 3^2}$

$= 5$

$\tan\alpha = \dfrac{K\sin\alpha}{K\cos\alpha}$

$= \dfrac{3}{4}$

S | A ✓✓
T | C ✓

$\alpha = \tan^{-1}\left(\frac{3}{4}\right)$

$\underline{\underline{\alpha = 36.9°}}$

$\underline{\underline{4\sin 2x - 3\cos 2x = K(\sin 2x - 36.9)°}}$

2I

(8) $2\sin 3x + 3\cos 3x = K\cos(3x - \alpha)$

$= K(\cos 3x \cos\alpha + \sin 3x \sin\alpha)$

$= K\cos 3x \cos\alpha + K\sin 3x \sin\alpha$

$2\underline{\sin 3x} + 3\underline{\cos 3x} = K\cos\alpha \underline{\cos 3x} + K\sin\alpha \underline{\sin 3x}$

$K\cos\alpha = 3$

$K\sin\alpha = 2$

$\tan\alpha = \dfrac{K\sin\alpha}{K\cos\alpha}$

$= \dfrac{2}{3}$

$K = \sqrt{2^2 + 3^2}$

$= \sqrt{13}$

S ✓ | A ✓✓

T | C ✓

$\alpha = \tan^{-1}\left(\frac{2}{3}\right)$

$\alpha = 33.7°$

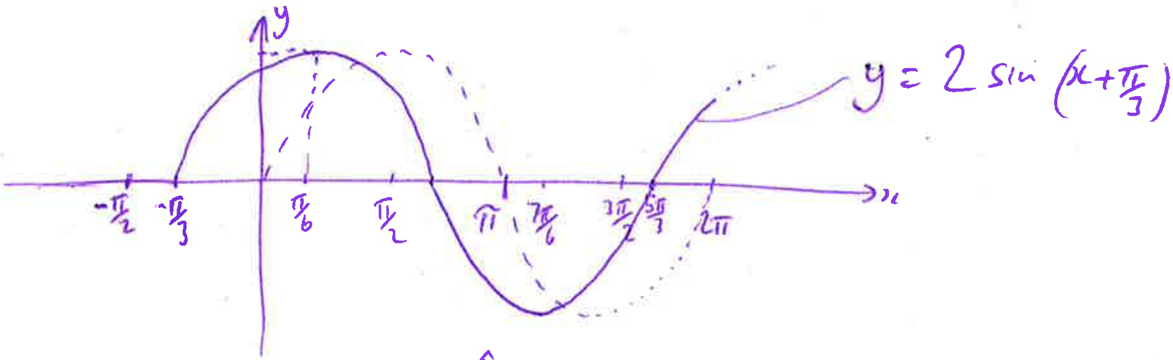
$2\sin 3x + 3\cos 3x = \sqrt{13}\cos(3x - 33.7°)$

2I

(9) $2\sin\left(x+\frac{\pi}{3}\right)$

amplitude of 2

shift $\frac{\pi}{3}$ to left



This is same as $y=2\cos x$ being moved $\frac{\pi}{6}$ to the right

$$2\sin\left(x+\frac{\pi}{3}\right) = 2\cos\left(x-\frac{\pi}{6}\right)$$

More about this shifting of graphs in chapter 3!