Higher Maths Leckie & Leckie

5A

① (a)(i)

$$3\underline{m} - 2\underline{n}$$

$$= 3\begin{pmatrix}1\\2\\-1\end{pmatrix} - 2\begin{pmatrix}3\\0\\-4\end{pmatrix}$$

$$= \begin{pmatrix}3\\6\\-3\end{pmatrix} - \begin{pmatrix}6\\0\\-8\end{pmatrix}$$

$$= \begin{pmatrix}-3\\6\\5\end{pmatrix}$$

(ii) $|3\underline{m} - 2\underline{n}|$

$$= \sqrt{(-3)^2 + 6^2 + 5^2}$$

$$= \sqrt{70}$$

(b)(i) $5\underline{m} - \underline{n} + 2\underline{p}$

$$= 5\begin{pmatrix}1\\2\\-1\end{pmatrix} - \begin{pmatrix}3\\0\\-4\end{pmatrix} + 2\begin{pmatrix}-1\\-3\\-2\end{pmatrix}$$

$$= \begin{pmatrix}5\\10\\-5\end{pmatrix} - \begin{pmatrix}3\\0\\-4\end{pmatrix} + \begin{pmatrix}-2\\-6\\-4\end{pmatrix}$$

$$= \begin{pmatrix}0\\4\\-5\end{pmatrix}$$

(ii) $|5\underline{m} - 4\underline{n} + 2\underline{p}|$

$$= \sqrt{0^2 + 4^2 + (-5)^2}$$

$$= \sqrt{41}$$

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1 (c) (i)

$$2(\underline{n} - 3\underline{p})$$

$$= 2\begin{pmatrix} 3 \\ 0 \\ -4 \end{pmatrix} - 6\begin{pmatrix} -1 \\ -3 \\ -2 \end{pmatrix}$$

$$= \begin{pmatrix} 6 \\ 0 \\ -8 \end{pmatrix} - \begin{pmatrix} -6 \\ -18 \\ -12 \end{pmatrix}$$

$$= \begin{pmatrix} 12 \\ 18 \\ 4 \end{pmatrix}$$

(ii) $|2(\underline{n} - 3\underline{p})|$

$$= \sqrt{12^2 + 18^2 + 4^2}$$

$$= \sqrt{484}$$

$$= \underline{\underline{22}}$$

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2(a) $\begin{pmatrix} 3 \\ 5 \end{pmatrix}$

$= 3\underline{i} + 5\underline{j}$

(b) $\begin{pmatrix} 7 \\ -9 \end{pmatrix}$

$= 7\underline{i} - 9\underline{j}$

(c) $\begin{pmatrix} 3 \\ -2 \\ -6 \end{pmatrix}$

$= 3\underline{i} - 2\underline{j} - 6\underline{k}$

(d) $\begin{pmatrix} 1 \\ 0 \\ -4 \end{pmatrix}$ a

$= \underline{i} \quad - 4\underline{j}$

(e) $\begin{pmatrix} 8 \\ 2 \\ -1 \end{pmatrix}$

$= 8\underline{i} + 2\underline{j} - \underline{k}$

3(a) $2\underline{i} + 5\underline{j} - \underline{k}$

$= \begin{pmatrix} 2 \\ 5 \\ -1 \end{pmatrix}$

(b) $7\underline{i} - 3\underline{j} + 9\underline{k}$

$= \begin{pmatrix} 7 \\ -3 \\ 9 \end{pmatrix}$

(c) $6\underline{i} - 5\underline{k}$

$= \begin{pmatrix} 6 \\ 0 \\ -5 \end{pmatrix}$ (no $\underline{j}$ term)

(d) $8\underline{j} + 5\underline{k}$

$\begin{pmatrix} 0 \\ 8 \\ 5 \end{pmatrix}$ no $\underline{i}$ term

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4(a) (i) $2\underline{p} - \underline{q}$

$$= 2\begin{pmatrix}3\\-2\\4\end{pmatrix} - \begin{pmatrix}2\\0\\-3\end{pmatrix}$$

$$= \begin{pmatrix}6\\-4\\8\end{pmatrix} - \begin{pmatrix}2\\0\\-3\end{pmatrix}$$

$$= \begin{pmatrix}4\\-4\\11\end{pmatrix}$$

$$= 4\underline{i} - 4\underline{j} + 11\underline{k}$$

(ii) $|2\underline{p} - \underline{q}|$

$$= \sqrt{4^2 + (-4)^2 + (11)^2}$$

$$= \sqrt{153}$$

$$= \sqrt{9}\sqrt{17}$$

$$= 3\sqrt{17}$$

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4(b)(i) $2\underline{p} - 5\underline{q} + \underline{r}$

$$= 2\begin{pmatrix} 3 \\ -2 \\ 4 \end{pmatrix} - 5\begin{pmatrix} 2 \\ 0 \\ -3 \end{pmatrix} + \begin{pmatrix} -2 \\ 3 \\ -1 \end{pmatrix}$$

$$= \begin{pmatrix} 6 \\ -4 \\ 8 \end{pmatrix} - \begin{pmatrix} 10 \\ 0 \\ -15 \end{pmatrix} + \begin{pmatrix} -2 \\ 3 \\ -1 \end{pmatrix}$$

$$= \begin{pmatrix} -6 \\ -1 \\ 22 \end{pmatrix}$$

$$= -6\underline{i} - \underline{j} + 22\underline{k}$$

(ii) $|2\underline{p} - 5\underline{q} + \underline{r}|$

$$= \sqrt{(-6)^2 + (-1)^2 + (22)^2}$$

$$= \sqrt{521}$$

5A

4(c) (i) $3(\underline{r} - \underline{q}) + \underline{p}$

$$= 3\left(\begin{pmatrix}-2\\3\\-1\end{pmatrix} - \begin{pmatrix}2\\0\\-3\end{pmatrix}\right) + \begin{pmatrix}3\\-2\\4\end{pmatrix}$$

$$= 3\begin{pmatrix}-4\\3\\2\end{pmatrix} + \begin{pmatrix}3\\-2\\4\end{pmatrix}$$

$$= \begin{pmatrix}-12\\9\\6\end{pmatrix} + \begin{pmatrix}3\\-2\\4\end{pmatrix}$$

$$= \begin{pmatrix}-9\\7\\10\end{pmatrix}$$

$$= \underline{\underline{-9\underline{i} + 7\underline{j} + 10\underline{k}}}$$

(ii) $|3(\underline{r} - \underline{q}) + \underline{p}|$

$$= \sqrt{(-9)^2 + 7^2 + 10^2}$$

$$= \underline{\underline{\sqrt{230}}}$$

5A

(5) $\underline{u} = \underline{v}$

$$\begin{pmatrix} 3x \\ y \end{pmatrix} = \begin{pmatrix} 15 \\ 6-2y \end{pmatrix}$$

$3x = 15$

$\underline{\underline{x = 5}}$

$y = 6-2y$

$3y = 6$

$\underline{\underline{y = 2}}$

(6) $p = q$

$(11-2x)\underline{i} + 14\underline{j} = 5\underline{i} - (3y+1)\underline{j}$

$11-2x = 5$

$6 = 2x$

$\underline{\underline{x = 3}}$

$14 = -(3y+1)$

$14 = -3y-1$

$15 = -3y$

$\underline{\underline{y = -5}}$

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(7) $\underline{u} = \underline{v}$

$$\begin{pmatrix} 3x \\ y \\ 5 \end{pmatrix} = \begin{pmatrix} 12 \\ 9-2y \\ z+3 \end{pmatrix}$$

$3x = 12$

$\underline{\underline{x = 4}}$

$y = 9 - 2y$

$3y = 9$

$\underline{\underline{y = 3}}$

$5 = z + 3$

$\underline{\underline{z = 2}}$

(8) $p = q$

$$\begin{pmatrix} 7-x \\ 13 \\ 2z-3 \end{pmatrix} = \begin{pmatrix} 11 \\ 2x+y \\ 12 \end{pmatrix}$$

$7 - x = 11$

$\underline{\underline{x = -4}}$

$2x + y = 13$

$2(-4) + y = 13$

$-8 + y = 13$

$\underline{\underline{y = 21}}$

$2z - 3 = -7$

$2z = -4$

$\underline{\underline{z = -2}}$

(9) $\underset{\sim}{m} = \underset{\sim}{n}$

$$\begin{pmatrix} 3x+2y \\ 4 \\ 2z \end{pmatrix} = \begin{pmatrix} 5 \\ 2x+y \\ 12 \end{pmatrix}$$

$3x + 2y = 5$ (1)

$2x + y = 4$ (2)

$4x + 2y = 8$ (3) ((2) × 2)

$\underline{\underline{x = 3}}$ (3) − (1)

sub $x=3$ in (1)

$3(3) + 2y = 5$

$9 + 2y = 5$

$2y = -4$

$\underline{\underline{y = -2}}$

$2z = 12$

$\underline{\underline{z = 6}}$

5A

10 (a) $\vec{AB} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$

$|\vec{AB}| = \sqrt{4^2 + 3^2}$

$= 5$

unit vector $\underline{u} = \frac{1}{5}\begin{pmatrix} 4 \\ 3 \end{pmatrix}$

(b) $\underline{u} = \begin{pmatrix} -6 \\ 8 \end{pmatrix}$

$|\underline{u}| = \sqrt{(-6)^2 + 8^2}$

$|\underline{u}| = 10$

unit vector $\underline{v} = \frac{1}{10}\begin{pmatrix} -6 \\ 8 \end{pmatrix}$

(c) $\underline{v} = \begin{pmatrix} \sqrt{2} \\ \sqrt{2} \end{pmatrix}$

$|\underline{v}| = \sqrt{(\sqrt{2})^2 + (\sqrt{2})^2}$

$= \sqrt{4}$

$= 2$

unit vector $\underline{w} = \frac{1}{2}\begin{pmatrix} \sqrt{2} \\ \sqrt{2} \end{pmatrix}$

5A

(11) $|\underline{p}|$

$= \sqrt{3^2 + (-1)^2 + 2^2}$

$= \sqrt{14}$

$\underline{u}_1 = \frac{1}{\sqrt{14}}\begin{pmatrix}3\\-1\\2\end{pmatrix}$ $\quad \underline{u}_2 = -\frac{1}{\sqrt{14}}\begin{pmatrix}3\\-1\\2\end{pmatrix}$

(note $\underline{u}_2$ is heading in the opposite direction to $\underline{p}$ but will still be parallel to it. Think North is parallel to South.)

(12) $\left(\frac{3}{4}\right)^2 + \left(\frac{1}{2}\right)^2 + z^2 = 1$ (as unit vector has magnitude of 1)

$\frac{9}{16} + \frac{1}{4} + z^2 = 1$

$z^2 = \frac{3}{16}$

$z = \pm\frac{\sqrt{3}}{4}$

$z = \frac{\sqrt{3}}{4}$ as $z > 0$

5A

(13)

$$\left(\frac{1}{3}\right)^2 + y^2 + \left(\frac{1}{2}\right)^2 = 1$$

$$\frac{1}{9} + y^2 + \frac{1}{4} = 1$$

$$\frac{4}{36} + y^2 + \frac{9}{36} = 1$$

$$y^2 = \frac{25}{36}$$

$$y = \frac{\pm 5}{6}$$

(14)

$$|\underline{u}| = |\underline{v}|$$

$$\sqrt{2^2 + (-2)^2 + 4^2} = \sqrt{1^2 + 3^2 + a^2}$$

$$24 = 10 + a^2$$

$$a^2 = 14$$

$$a = \pm\sqrt{14}$$

5B

1 (a) $\vec{CB} = \vec{CO} + \vec{OB}$

$= \underline{b} - \underline{c}$

$= \begin{pmatrix} 3 \\ 4 \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

$= \underline{\underline{\begin{pmatrix} 2 \\ 5 \end{pmatrix}}}$

$\vec{AB}$

$= \vec{AO} + \vec{OB}$

$= \underline{b} - \underline{a}$

$= \begin{pmatrix} 3 \\ 4 \end{pmatrix} - \begin{pmatrix} -2 \\ 5 \end{pmatrix}$

$= \underline{\underline{\begin{pmatrix} 5 \\ -1 \end{pmatrix}}}$

$\vec{AC}$

$= \vec{AO} + \vec{OC}$

$= \underline{c} - \underline{a}$

$= \begin{pmatrix} 1 \\ -1 \end{pmatrix} - \begin{pmatrix} -2 \\ 5 \end{pmatrix}$

$= \begin{pmatrix} 3 \\ -6 \end{pmatrix}$

1(b) $|\vec{CB}|$

$= \sqrt{2^2 + 5^2}$

$= \underline{\underline{\sqrt{29}}}$

$|\vec{AB}|$

$= \sqrt{5^2 + (-1)^2}$

$= \underline{\underline{\sqrt{26}}}$

$|\vec{AC}|$

$= \sqrt{3^2 + (-6)^2}$

$= \sqrt{45}$

$= \sqrt{9}\sqrt{5}$

$= \underline{\underline{3\sqrt{5}}}$

5B

(2) (a)

$\overrightarrow{PR}$

$= \overrightarrow{PO} + \overrightarrow{OR}$

$= \underline{r} - \underline{p}$

$= \begin{pmatrix} -2 \\ 2 \end{pmatrix} - \begin{pmatrix} 1 \\ 7 \end{pmatrix}$

$= \begin{pmatrix} -3 \\ -5 \end{pmatrix}$

$= -3\underline{i} - 5\underline{j}$

(b) $\overrightarrow{RQ}$

$= \overrightarrow{RO} + \overrightarrow{OQ}$

$= \underline{q} - \underline{r}$

$= \begin{pmatrix} -4 \\ -5 \end{pmatrix} - \begin{pmatrix} -2 \\ 2 \end{pmatrix}$

$= \begin{pmatrix} -2 \\ -7 \end{pmatrix}$

$= -2\underline{i} - 7\underline{j}$

(c) $\overrightarrow{QP}$

$= \overrightarrow{QO} + \overrightarrow{OP}$

$= \underline{p} - \underline{q}$

$= \begin{pmatrix} 1 \\ 7 \end{pmatrix} - \begin{pmatrix} -4 \\ -5 \end{pmatrix}$

$= \begin{pmatrix} 5 \\ 12 \end{pmatrix}$

$= 5\underline{i} + 12\underline{j}$

5B

(3) $\overrightarrow{AB}$

$= \overrightarrow{AO} + \overrightarrow{OB}$

$= \underline{b} - \underline{a}$

$= \begin{pmatrix} 3 \\ 4 \\ -1 \end{pmatrix} - \begin{pmatrix} -2 \\ 5 \\ 3 \end{pmatrix}$

$= \underline{\underline{\begin{pmatrix} 5 \\ -1 \\ -4 \end{pmatrix}}}$

$\overrightarrow{BC}$

$= \overrightarrow{BO} + \overrightarrow{OC}$

$= \underline{c} - \underline{b}$

$= \begin{pmatrix} 1 \\ -4 \\ 9 \end{pmatrix} - \begin{pmatrix} 3 \\ 4 \\ -1 \end{pmatrix}$

$= \begin{pmatrix} -2 \\ -8 \\ 10 \end{pmatrix}$

$\overrightarrow{CA}$

$= \overrightarrow{CO} + \overrightarrow{OA}$

$= \underline{a} - \underline{c}$

$= \begin{pmatrix} -2 \\ 5 \\ 3 \end{pmatrix} - \begin{pmatrix} 1 \\ -4 \\ 9 \end{pmatrix}$

$= \begin{pmatrix} -3 \\ 9 \\ \cancel{12}-6 \end{pmatrix}$

(b) $|\overrightarrow{AB}|$

$= \sqrt{5^2 + (-1)^2 + (-4)^2}$

$= \underline{\underline{\sqrt{42}}}$

$|\overrightarrow{BC}|$

$= \sqrt{(-2)^2 + (-8)^2 + 10^2}$

$= \sqrt{168}$

$= \sqrt{4}\sqrt{42}$

$= \underline{\underline{2\sqrt{42}}}$

$|\overrightarrow{CA}|$

$= \sqrt{(-3)^2 + 9^2 + (-6)^2}$

$= \sqrt{126}$

$= \sqrt{9}\sqrt{14}$

$= \underline{\underline{3\sqrt{14}}}$

5B

4

$\overrightarrow{PR}$

$= \overrightarrow{PO} + \overrightarrow{OR}$

$= \underline{r} - \underline{p}$

$= \begin{pmatrix} -5 \\ 3 \\ 2 \end{pmatrix} - \begin{pmatrix} 3 \\ 1 \\ 7 \end{pmatrix}$

$= \begin{pmatrix} -8 \\ 4 \\ -5 \end{pmatrix}$

$= \underline{\underline{-8\underline{i} + 4\underline{j} - 5\underline{k}}}$

$\overrightarrow{RQ}$

$= \overrightarrow{RO} + \overrightarrow{OQ}$

$= \underline{q} - \underline{r}$

$= \begin{pmatrix} 4 \\ -2 \\ 5 \end{pmatrix} - \begin{pmatrix} -5 \\ 3 \\ 2 \end{pmatrix}$

$= \begin{pmatrix} 9 \\ -5 \\ 3 \end{pmatrix}$

$= \underline{\underline{9\underline{i} - 5\underline{j} + 3\underline{k}}}$

$\overrightarrow{QP}$

$= \overrightarrow{QO} + \overrightarrow{OP}$

$= \underline{p} - \underline{q}$

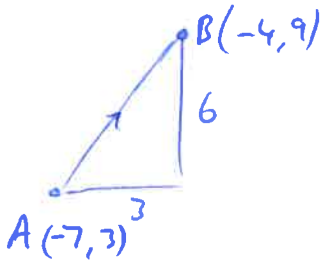
$= \begin{pmatrix} 3 \\ 1 \\ 7 \end{pmatrix} - \begin{pmatrix} 4 \\ -2 \\ 5 \end{pmatrix}$

$= \begin{pmatrix} -1 \\ 3 \\ 2 \end{pmatrix}$

$= -\underline{i} + 3\underline{j} + 2\underline{k}$

5B

(5) (a) $A(-7,3)$



$\vec{AB} = \begin{pmatrix} 3 \\ 6 \end{pmatrix}$

$B(-7+3, 3+6)$

$= \underline{\underline{B(-4, 9)}}$

(b) $B(-3+2, -2+5)$

$= \underline{\underline{B(-1, 3)}}$

(c) $B(-7+3, 3+(-3), 5+7)$

$= \underline{\underline{B(-4, 0, 12)}}$

(d) $B(2+-3, 5+-2, -4+9)$

$= \underline{\underline{B(-1, 3, 5)}}$

(e) $B(0+9, 8+-6, 4+3)$

$= \underline{\underline{B(9, 2, 7)}}$

5B

(6) $\overrightarrow{BA}$

$= \underline{a} - \underline{b}$

$= \begin{pmatrix} 2 \\ 7 \end{pmatrix} - \begin{pmatrix} -3 \\ 9 \end{pmatrix}$

$= \begin{pmatrix} 5 \\ -2 \end{pmatrix} = \overrightarrow{CD}$

$D(-1+5, 4+(-2))$

(moving from B to A uses the same vector as moving from C to D)

$= \underline{\underline{D(4, 2)}}$

(7) $\overrightarrow{QR}$

$= \underline{r} - \underline{q}$

$= \begin{pmatrix} 6 \\ 5 \\ 8 \end{pmatrix} - \begin{pmatrix} 5 \\ 3 \\ 9 \end{pmatrix}$

$= \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} = \overrightarrow{PS}$

$S(-3+1, 2+2, 5+(-1))$

$= \underline{\underline{S(-2, 4, 4)}}$

5C

①

$$\begin{array}{cccc} & 1 \quad : & 2 & \\ A & B & & C \\ 2 & \begin{pmatrix} 4 \\ 4 \\ 2 \end{pmatrix} & 6 & 8 \\ 3 & & 5 & 6 \\ -1 & & 5 & 8 \end{array}$$

1:2 so split into 3 'journeys'

$\underline{\underline{B(4,4,2)}}$

2:3 → 5 'journeys'

②

$$\begin{array}{cccccc} P & 2 & : Q & 3 & & R \\ \begin{pmatrix} -2 \\ 5 \\ 7 \end{pmatrix} & \begin{matrix} 0 \\ 4 \\ 10 \end{matrix} & \begin{pmatrix} 2 \\ 3 \\ 13 \end{pmatrix} & \begin{matrix} 4 \\ 2 \\ 16 \end{matrix} & \begin{matrix} 6 \\ 1 \\ 19 \end{matrix} & \begin{pmatrix} 8 \\ 0 \\ 22 \end{pmatrix} \end{array}$$

$\underline{\underline{Q(2,3,13)}}$

③ Z divides XY in ratio 2:5

2:5 → 7 parts

$$\begin{array}{cccccccc} X & & Z & & & & & Y \\ \begin{pmatrix} -6 \\ -3 \\ 2 \end{pmatrix} & \begin{matrix} -5 \\ -2 \\ 1 \end{matrix} & \begin{pmatrix} -4 \\ -1 \\ 0 \end{pmatrix} & \begin{matrix} -3 \\ 0 \\ -1 \end{matrix} & \begin{matrix} -2 \\ 1 \\ -2 \end{matrix} & \begin{matrix} -1 \\ 2 \\ -3 \end{matrix} & \begin{matrix} 0 \\ 3 \\ -4 \end{matrix} & \begin{pmatrix} 1 \\ 4 \\ -5 \end{pmatrix} \end{array}$$

$\underline{\underline{Z(-4,-1,0)}}$

5C

2 : 1 3 parts

(4)

A		D	C
3	6	9	12
-1	-2	-3	-4
6	4	2	0

D (9, -3, 2)

(5)

A			B	C
-2	-1	0	1	2
4	3	2	1	0
3	1	-1	-3	-5

B (1, 1, -3)

(6)

A	B	C	D
3	2	1	0
5	2	-1	-4
4	-2	-8	-14

B (2, 2, -2) D (0, -4, -14)

5C

(7)

P	Q	R	S	T
2	4	6	8	10
5	+2	-1	-4	-7
1	-1	-3	-5	-7

Q (4, 2, -1)   S(8, -4, -5)   T (10, -7, -7)

(8)

2 : 3

A		B			C
3	2	1	0	-1	-2
2	3	4	5	6	7
-5	-2	1	4	7	10

C (-2, 7, 10)

(9)

3 : 2

A			Q		B
-2	-1	0	1	2	3
24	20	16	12	8	4
-20	-16	-12	-8	-4	0

A(-2, 24, -20)

SD

1 (a) $\vec{RS} + \vec{RQ}$

$= \vec{RS} + \vec{SP}$

$= \underline{\underline{\vec{RP}}}$

(b) $\vec{RS} - \vec{RQ}$

$= \vec{QP} - \vec{SP}$ $\quad (-\vec{SP} = \vec{PS})$

$= \vec{QP} + \vec{PS}$

$= \underline{\underline{\vec{QS}}}$

(c) $\vec{PQ} - \vec{QR} + \vec{QS}$

$= \vec{PQ} + \vec{QS} - \vec{QR}$

$= \vec{PS} - \vec{QR}$

$= \vec{PS} - \vec{PS}$

$= \underline{\underline{\underline{0}}}$

2 (a) $\vec{AB} + \vec{BC} + \vec{CD}$

$= \vec{AC} + \vec{CD}$

$= \underline{\underline{\vec{AD}}}$

(b) $\vec{AF} + \vec{BC} + \vec{ED}$

$= \vec{AF} + \vec{FE} + \vec{ED}$

$= \vec{AE} + \vec{ED}$

$= \underline{\underline{\vec{AD}}}$

(c) $\vec{ED} - \vec{AF}$

$= \vec{ED} - \vec{CD}$

$= \vec{ED} + \vec{DC}$

$= \underline{\underline{\vec{EC}}}$

(d) $\vec{AB} + \vec{AF} - \vec{CB}$

$= \vec{AB} + \vec{BC} + \vec{AF}$

$= \vec{AC} + \vec{AF}$

$= \vec{AC} + \vec{CD}$

$= \underline{\underline{\vec{AD}}}$

3 (a) $\vec{PV}$

$= \vec{PQ} + \vec{QR} + \vec{RV}$

$= \underline{a} + \underline{c} + \underline{b}$

$= \underline{\underline{\underline{a} + \underline{b} + \underline{c}}}$

(b) $\vec{SV}$

$= \vec{SR} + \vec{RV}$

$= \underline{\underline{\underline{a} + \underline{b}}}$

(c) $\vec{PW}$

$= \vec{PT} + \vec{TW}$

$= \underline{\underline{\underline{b} + \underline{c}}}$

5D

3(d) $\overrightarrow{WQ}$

$= \overrightarrow{WV} + \overrightarrow{VR} + \overrightarrow{RQ}$

$= \underline{a} - \underline{b} - \underline{c}$

(e) $\overrightarrow{VT}$

$= \overrightarrow{VW} + \overrightarrow{WT}$

$= -\underline{a} - \underline{c}$

(f) $\overrightarrow{RP}$

$= \overrightarrow{RQ} + \overrightarrow{QP}$

$= -\underline{c} - \underline{a}$

$= -\underline{a} - \underline{c}$

4 (a) $\overrightarrow{PV}$

$= \overrightarrow{PQ} + \overrightarrow{QU} + \overrightarrow{UV}$

$= \overrightarrow{PQ} + \overrightarrow{SW} + \overrightarrow{UV}$

$= \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ 4 \end{pmatrix} + \begin{pmatrix} 1 \\ 5 \\ 3 \end{pmatrix}$

$= \begin{pmatrix} 5 \\ 7 \\ 7 \end{pmatrix}$

4 (b) $\overrightarrow{QW}$

$\overrightarrow{QP} + \overrightarrow{PS} + \overrightarrow{SW}$

$= -\overrightarrow{PQ} + \overrightarrow{UV} + \overrightarrow{SW}$

$= -\begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 1 \\ 5 \\ 3 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ 4 \end{pmatrix}$

$= \begin{pmatrix} -1 \\ 5 \\ 7 \end{pmatrix}$

(c) $\overrightarrow{SU}$

$= \overrightarrow{SR} + \overrightarrow{RQ} + \overrightarrow{QU}$

$= \overrightarrow{PQ} - \overrightarrow{UV} + \overrightarrow{SW}$

$= \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix} - \begin{pmatrix} 1 \\ 5 \\ 3 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ 4 \end{pmatrix}$

$= \begin{pmatrix} 3 \\ -3 \\ 7 \end{pmatrix}$

5D

(5) $\vec{EA}$

$= \vec{EC} + \vec{CD} + \vec{DA}$

$= \vec{EC} - \vec{AB} - \vec{AD}$

$= \begin{pmatrix} 1 \\ 2 \\ -5 \end{pmatrix} - \begin{pmatrix} 2 \\ 6 \\ 2 \end{pmatrix} - \begin{pmatrix} -4 \\ 1 \\ -1 \end{pmatrix}$

$= \begin{pmatrix} 3 \\ -5 \\ -6 \end{pmatrix}$

$\vec{BE}$

$= \vec{BC} + \vec{CE}$

$= \vec{AD} - \vec{EC}$

$= \begin{pmatrix} -4 \\ 1 \\ -1 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ -5 \end{pmatrix}$

$= \begin{pmatrix} -5 \\ -1 \\ 4 \end{pmatrix}$

(6) $\vec{OG}$

$= \vec{OC} + \vec{CG}$

$= \begin{pmatrix} 0 \\ 3 \\ 4 \end{pmatrix}$

$\vec{OF} = \vec{OA} + \vec{AB} + \vec{BF}$

$= \begin{pmatrix} 6 \\ 3 \\ 4 \end{pmatrix}$

point Q

G	Q	-	F
0	2	4	6
3	3	3	3
4	4	4	4

$Q(2, 3, 4)$

$\vec{OQ} = \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}$

$\vec{OP} = \vec{OA} + \frac{1}{2}\vec{AB} + \frac{1}{2}\vec{BF}$

$= \begin{pmatrix} 6 \\ 0 \\ 0 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 0 \\ 3 \\ 0 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix}$

$= \begin{pmatrix} 6 \\ 3/2 \\ 4 \end{pmatrix}$

$P\left(6, \frac{3}{2}, 4\right)$

5D

7 (a) $\vec{AG}$

$= \vec{AB} + \vec{BC} + \vec{CG}$

$= \cancel{6\underline{i} + 2\underline{j} +}$

$= \begin{pmatrix} 6 \\ 2 \\ 2 \end{pmatrix} + \begin{pmatrix} 1 \\ 5 \\ -2 \end{pmatrix} + \begin{pmatrix} 2 \\ 2 \\ 6 \end{pmatrix}$

$= \begin{pmatrix} 9 \\ 9 \\ 6 \end{pmatrix}$

$= 9\underline{i} + 9\underline{j} + 6\underline{k}$

(b) $\vec{AX}$

$\Rightarrow = \vec{AD} + \vec{DH} + \frac{1}{2}\vec{HG}$

$= \begin{pmatrix} 1 \\ 5 \\ -2 \end{pmatrix} + \begin{pmatrix} 2 \\ 2 \\ 6 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 6 \\ 2 \\ 2 \end{pmatrix}$

$= \begin{pmatrix} 6 \\ 8 \\ 5 \end{pmatrix}$

$= 6\underline{i} + 8\underline{j} + 5\underline{k}$

(c) $\vec{AY}$

$= \vec{AB} + \vec{BC} + \frac{1}{2}\vec{CG}$

$= \begin{pmatrix} 6 \\ 2 \\ 2 \end{pmatrix} + \begin{pmatrix} 1 \\ 5 \\ -2 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 2 \\ 2 \\ 6 \end{pmatrix}$

$= \begin{pmatrix} 8 \\ 8 \\ 3 \end{pmatrix}$

$= 8\underline{i} + 8\underline{j} + 3\underline{k}$

(d) $\vec{BX}$

$= \vec{BC} + \vec{CG} + \frac{1}{2}\vec{GH}$

$= \vec{BC} + \vec{CG} - \frac{1}{2}\vec{HG}$ $\quad (\vec{GH} = -\vec{HG})$

$= \begin{pmatrix} 1 \\ 5 \\ -2 \end{pmatrix} + \begin{pmatrix} 2 \\ 2 \\ 6 \end{pmatrix} - \frac{1}{2}\begin{pmatrix} 6 \\ 2 \\ 2 \end{pmatrix}$

$= \begin{pmatrix} 0 \\ 6 \\ 5 \end{pmatrix}$

$= 6\underline{j} + 5\underline{k}$

51)

7(e) $\vec{XY}$

$= \frac{1}{2}\vec{HG} + \frac{1}{2}\vec{GC}$

$= \frac{1}{2}\vec{HG} - \frac{1}{2}\vec{CG}$

$= \frac{1}{2}\begin{pmatrix}6\\2\\2\end{pmatrix} - \frac{1}{2}\begin{pmatrix}2\\2\\6\end{pmatrix}$

$= \begin{pmatrix}2\\0\\-2\end{pmatrix}$

$= \cancel{2i\ 0j} -2$

$= 2\underline{i} - 2\underline{k}$

8(a) $\vec{PA}$

$= \vec{PQ} + \frac{1}{3}\vec{QR}$

$= \begin{pmatrix}8\\4\\3\end{pmatrix} + \frac{1}{3}\begin{pmatrix}3\\-6\\3\end{pmatrix}$

$= \begin{pmatrix}9\\2\\4\end{pmatrix}$

8(b) $\vec{PB}$

$= \vec{PS} + \frac{1}{4}\vec{SR}$

$= \begin{pmatrix}3\\-6\\3\end{pmatrix} + \frac{1}{4}\begin{pmatrix}8\\4\\3\end{pmatrix}$

$= \begin{pmatrix}5\\-5\\3\frac{3}{4}\end{pmatrix} = \begin{pmatrix}5\\-5\\\frac{15}{4}\end{pmatrix}$

(c) $\vec{QV}$

$= \vec{QR} + \vec{RV}$

$= \begin{pmatrix}3\\-6\\3\end{pmatrix} + \begin{pmatrix}-1\\-2\\7\end{pmatrix}$

$= \begin{pmatrix}2\\-8\\10\end{pmatrix}$

(d) $\vec{PV}$

$= \vec{PQ} + \vec{QR} + \vec{RV}$

$= \begin{pmatrix}8\\4\\3\end{pmatrix} + \begin{pmatrix}3\\-6\\3\end{pmatrix} + \begin{pmatrix}-1\\-2\\7\end{pmatrix}$

$= \begin{pmatrix}10\\-8\\13\end{pmatrix}$

5D

8(e) $\vec{AB}$

$= \frac{2}{3}\vec{QR} + \frac{3}{4}\vec{RS}$

$= \frac{2}{3}\vec{QR} - \frac{3}{4}\vec{SR}$

$= \frac{2}{3}\begin{pmatrix}3\\-6\\3\end{pmatrix} - \frac{3}{4}\begin{pmatrix}8\\4\\3\end{pmatrix}$

$= \begin{pmatrix}2\\-4\\2\end{pmatrix} - \begin{pmatrix}6\\3\\9/4\end{pmatrix}$

$= \begin{pmatrix}-4\\-7\\-1/4\end{pmatrix}$

9 (a) $\vec{PA}$

$= \vec{PS} + \vec{SW} + \frac{3}{4}\vec{WV}$

$= \begin{pmatrix}-4\\2\\0\end{pmatrix} + \begin{pmatrix}-6\\-12\\30\end{pmatrix} + \frac{3}{4}\begin{pmatrix}3\\6\\3\end{pmatrix}$

$= \begin{pmatrix}-4\\2\\0\end{pmatrix} + \begin{pmatrix}-6\\-12\\30\end{pmatrix} + \begin{pmatrix}9/4\\9/2\\9/4\end{pmatrix}$

$= \begin{pmatrix}-10 + 9/4\\-10 + 9/2\\30 + 9/4\end{pmatrix}$

$= \begin{pmatrix}-31/4\\-11/2\\\frac{129}{4}\end{pmatrix}$

9(b) $\vec{PB}$

$= \vec{PQ} + \vec{QU} + \frac{2}{3}\vec{UV}$

$= \begin{pmatrix}3\\6\\3\end{pmatrix} + \begin{pmatrix}-6\\-12\\0\end{pmatrix} + \frac{2}{3}\begin{pmatrix}-4\\2\\0\end{pmatrix}$

$= \begin{pmatrix}3\\6\\3\end{pmatrix} + \begin{pmatrix}-6\\-12\\0\end{pmatrix} + \begin{pmatrix}-8/3\\4/3\\0\end{pmatrix}$

$= \begin{pmatrix}-3 - 8/3\\-6 + 4/3\\3\end{pmatrix}$

$= \begin{pmatrix}-\frac{17}{3}\\-\frac{20}{3}\\3\end{pmatrix}$

5E

1(a) $\underset{\sim}{f} = k \underset{\sim}{q}$

$$\begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} = k \begin{pmatrix} 6 \\ -3 \\ t-1 \end{pmatrix}$$

x: $2 = 6k$

$k = 2/6$

$k = 1/3$

z: $3 = \frac{1}{3}(t-1)$

$9 = t-1$

$\underline{\underline{t = 10}}$

(b) $$\begin{pmatrix} -2 \\ 3 \\ -5 \end{pmatrix} = k \begin{pmatrix} t \\ -6 \\ 10 \end{pmatrix}$$

~~$x: -2 = kt$~~

y: $3 = -6k$

$\frac{3}{-6} = k$

$k = -\frac{1}{2}$

x: $-2 = -\frac{1}{2} t$

$\underline{\underline{t = 4}}$

2(a) $k\underset{\sim}{f} = \underset{\sim}{q}$

x: $2k = 6$

$\underline{\underline{k = 3}}$

y: $3(h+1) = -3$

$3h + 3 = -3$

$3h = -6$

$\underline{\underline{h = -2}}$

z: $3\left(\frac{3}{2}\right) = \frac{k+1}{2}$

$\frac{9}{2} = \frac{k+1}{2}$

$9 = k+1$

$\underline{\underline{k = 8}}$

3 $$\begin{pmatrix} -8 \\ c-1 \\ 12 \end{pmatrix} = k \begin{pmatrix} 4 \\ -3 \\ 2d+4 \end{pmatrix}$$

x: $-8 = 4k$

$k = -2$

y: $c-1 = -2 \times -3$

$c-1 = 6$

$\underline{\underline{c = 7}}$

z: $12 = -2(2d+4)$

$12 = -4d - 8$

$20 = -4d$

$\underline{\underline{d = -5}}$

5E

4 (a) $\begin{pmatrix} 6 \\ 12 \\ 15 \end{pmatrix} = 3 \begin{pmatrix} 2 \\ 4 \\ 5 \end{pmatrix}$

$\therefore \begin{pmatrix} 6 \\ 12 \\ 15 \end{pmatrix} \parallel \begin{pmatrix} 2 \\ 4 \\ 5 \end{pmatrix}$

(b) $\begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 6 \\ -4 \\ 3 \end{pmatrix}$

$\therefore \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix} \parallel \begin{pmatrix} 6 \\ -4 \\ 3 \end{pmatrix}$

(c) $\frac{5}{2}\left(4\underline{i} + 6\underline{j} - 8\underline{k}\right) = 10\underline{i} + 15\underline{j} - 20\underline{k}$

$\therefore 4\underline{i} + 6\underline{j} - 8\underline{k} \parallel 10\underline{i} + 15\underline{j} - 20\underline{k}$

(d) $\begin{pmatrix} 15 \\ -21 \\ 3 \end{pmatrix} = -\frac{3}{2} \begin{pmatrix} -10 \\ 14 \\ -2 \end{pmatrix}$

$\therefore \begin{pmatrix} 15 \\ -21 \\ 3 \end{pmatrix} \parallel \begin{pmatrix} -10 \\ 14 \\ -2 \end{pmatrix}$

5E

(5) $\overrightarrow{AB}$

$= \underline{b} - \underline{a}$

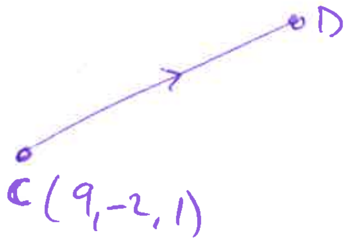
$= \begin{pmatrix} 5 \\ 3 \\ 1 \end{pmatrix} - \begin{pmatrix} -2 \\ 3 \\ 5 \end{pmatrix}$

$= \begin{pmatrix} 7 \\ 0 \\ -4 \end{pmatrix}$

$\overrightarrow{CD}$

$= 2\overrightarrow{AB}$

$= \begin{pmatrix} 14 \\ 0 \\ -8 \end{pmatrix}$



$D(9+14, -2+0, 1+(-8))$

$= D(23, -2, -7)$

$\overrightarrow{BC}$

$= \underline{c} - \underline{b}$

$= \begin{pmatrix} 9 \\ -2 \\ 1 \end{pmatrix} - \begin{pmatrix} 5 \\ 3 \\ 1 \end{pmatrix}$

$= \begin{pmatrix} 4 \\ -5 \\ 0 \end{pmatrix}$

$\overrightarrow{AB} \neq k\overrightarrow{BC}$

$\therefore$ A, B and C are not collinear

(6) (a) $\overrightarrow{AB}$

$= \underline{b} - \underline{a}$

$= \begin{pmatrix} 1 \\ 4 \\ 1 \end{pmatrix} - \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$

$= \begin{pmatrix} -1 \\ 5 \\ -2 \end{pmatrix}$

5E

6b $\vec{PQ}$

$= \underset{\sim}{q} - \underset{\sim}{p}$

$= \begin{pmatrix} 1 \\ 3 \\ 3 \end{pmatrix} - \begin{pmatrix} 3 \\ 0 \\ 2 \end{pmatrix}$

$= \begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix}$

$\vec{QR}$

$= \underset{\sim}{r} - \underset{\sim}{q}$

$= \begin{pmatrix} -5 \\ 12 \\ 5 \end{pmatrix} - \begin{pmatrix} 1 \\ 3 \\ 3 \end{pmatrix}$

$= \begin{pmatrix} -6 \\ 9 \\ 2 \end{pmatrix}$

$\vec{PQ} \neq k\vec{QR}$

$\therefore$ P, Q & R are not collinear

6(c) $\vec{XY}$

$= \underset{\sim}{y} - \underset{\sim}{x}$

$= \begin{pmatrix} 5 \\ -2 \\ 11 \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix}$

$= \begin{pmatrix} 3 \\ -5 \\ 6 \end{pmatrix}$

$\vec{YZ}$

$= \underset{\sim}{z} - \underset{\sim}{y}$

$= \begin{pmatrix} 11 \\ -10 \\ 22 \end{pmatrix} - \begin{pmatrix} 5 \\ -2 \\ 11 \end{pmatrix}$

$= \begin{pmatrix} 6 \\ -8 \\ 11 \end{pmatrix}$

$\vec{XY} \neq k\vec{YZ}$

$\therefore$ X, Y & Z are not collinear

5E

6(d) $\overrightarrow{DE}$

$= \underline{e} - \underline{d}$

$= \begin{pmatrix} 3 \\ -4 \\ 1 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix}$

$= \begin{pmatrix} 2 \\ -4 \\ -2 \end{pmatrix}$

$\overrightarrow{EF}$

$= \underline{f} - \underline{e}$

$= \begin{pmatrix} 6 \\ -10 \\ -2 \end{pmatrix} - \begin{pmatrix} 3 \\ -4 \\ 1 \end{pmatrix}$

$= \begin{pmatrix} 3 \\ -6 \\ -3 \end{pmatrix}$

$\overrightarrow{DE} = \frac{2}{3}\overrightarrow{EF}$

$\overrightarrow{DE}$ is parallel to $\overrightarrow{EF}$ and they share a common point E, therefore D, E and F are collinear

Must make statement

5E

7(a) $\vec{AB}$

$= \underline{b} - \underline{a}$

$= \begin{pmatrix} 5 \\ 3 \\ 2 \end{pmatrix} - \begin{pmatrix} 3 \\ -2 \\ 5 \end{pmatrix}$

$= \begin{pmatrix} 2 \\ 5 \\ -3 \end{pmatrix}$

$\vec{BC}$

$= \underline{c} - \underline{b}$

$= \begin{pmatrix} 9 \\ 13 \\ -4 \end{pmatrix} - \begin{pmatrix} 5 \\ 3 \\ 2 \end{pmatrix}$

$= \begin{pmatrix} 4 \\ 10 \\ -6 \end{pmatrix}$

$\underline{\underline{\vec{BC} = 2\vec{AB}}}$

$\vec{AB}$ is parallel to $\vec{BC}$ and they share a common point B, therefore they are collinear.

(b)

A		B		C
3	$+2$	5	$+4$	9
-2	$+5$	3	$+10$	13
5	-3	2	-6	-4

B divides AC in the ratio $\underline{\underline{1:2}}$

5E

8 (a) $\overrightarrow{PQ}$

$= \underset{\sim}{q} - \underset{\sim}{p}$

$= \begin{pmatrix} 3 \\ 1 \\ 8 \end{pmatrix} - \begin{pmatrix} 2 \\ 5 \\ 3 \end{pmatrix}$

$= \begin{pmatrix} 1 \\ -4 \\ 5 \end{pmatrix}$

$\overrightarrow{QR}$

$= \underset{\sim}{r} - \underset{\sim}{q}$

$= \begin{pmatrix} 6 \\ -11 \\ 23 \end{pmatrix} - \begin{pmatrix} 3 \\ 1 \\ 8 \end{pmatrix}$

$= \begin{pmatrix} 3 \\ -12 \\ 15 \end{pmatrix}$

$\overrightarrow{QR} = 3\overrightarrow{PQ}$

$\overrightarrow{PQ}$ is parallel to $\overrightarrow{QR}$ and they share a common point Q, therefore P, Q and R are collinear

(b)

P		Q		R
2	+1	3	+3	6
5	−4	1	−12	−11
3	+5	8	+15	23

Q divides ~~Q~~PR in the ratio 1:3

5E

9(a) $\overrightarrow{MN}$

$= \underline{n} - \underline{m}$

$= \begin{pmatrix} 4 \\ 1 \\ 7 \end{pmatrix} - \begin{pmatrix} 2 \\ 5 \\ 3 \end{pmatrix}$

$= \begin{pmatrix} 2 \\ -4 \\ 4 \end{pmatrix}$

$\overrightarrow{NP}$

$= \underline{p} - \underline{n}$

$= \begin{pmatrix} 5 \\ -1 \\ 9 \end{pmatrix} - \begin{pmatrix} 4 \\ 1 \\ 7 \end{pmatrix}$

$= \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$

$\overrightarrow{MN} = 2\overrightarrow{NP}$

$\overrightarrow{MN}$ is parallel to $\overrightarrow{NP}$ and they share a common point N, therefore M N and P are ~~[illegible]~~ collinear

(b)

M		N		P
2	+2	4	+1	5
5	−4	1	−2	−1
3	+4	7	+2	9

N divides MP in the ratio 2:1

5E

10(a) $\overrightarrow{AB}$

$= \underline{b} - \underline{a}$

$= \begin{pmatrix} 5 \\ 5 \\ 9 \end{pmatrix} - \begin{pmatrix} 3 \\ 1 \\ 7 \end{pmatrix}$

$= \begin{pmatrix} 2 \\ 4 \\ 2 \end{pmatrix}$

$\overrightarrow{BC}$

$= \underline{c} - \underline{b}$

$= \begin{pmatrix} 8 \\ 11 \\ 12 \end{pmatrix} - \begin{pmatrix} 5 \\ 5 \\ 9 \end{pmatrix}$

$= \begin{pmatrix} 3 \\ 6 \\ 3 \end{pmatrix}$

$\overrightarrow{BC} = \frac{3}{2}\overrightarrow{AB}$

$\overrightarrow{AB}$ is parallel to $\overrightarrow{BC}$ and they share a common point B, therefore A, B and C are ~~parall~~ collinear

2 : 3

(b)

$$\begin{array}{ccccc} A & & B & & C \\ 3 & \xrightarrow{+2} & 5 & \xrightarrow{+3} & 8 \\ 1 & \xrightarrow{+4} & 5 & \xrightarrow{+6} & 11 \\ 7 & \xrightarrow{+2} & 9 & \xrightarrow{+3} & 12 \end{array}$$

B divides $\overrightarrow{AC}$ in the ratio $\underline{\underline{2:3}}$

(11) $\overrightarrow{EF}$

$= \underline{f} - \underline{e}$

$= \begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix} - \begin{pmatrix} 5 \\ -2 \\ 7 \end{pmatrix}$

$= \begin{pmatrix} -2 \\ 6 \\ -2 \end{pmatrix}$

$\overrightarrow{FG}$

$= \underline{g} - \underline{f}$

$= \begin{pmatrix} 0 \\ 12 \\ 2 \end{pmatrix} - \begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix}$

$= \begin{pmatrix} -3 \\ 8 \\ -3 \end{pmatrix}$

$\underline{\overrightarrow{EF} \neq k\overrightarrow{FG}}$ $\therefore$ <u>E, F and G are not collinear.</u>

5E

(12) $\overrightarrow{PQ}$

$= \underset{\sim}{q} - \underset{\sim}{p}$

$= \begin{pmatrix} 1 \\ 2 \\ 5 \end{pmatrix} - \begin{pmatrix} -2 \\ 1 \\ 7 \end{pmatrix}$

$= \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix}$

$\overrightarrow{QR}$

$= \underset{\sim}{r} - \underset{\sim}{q}$

$= \begin{pmatrix} 7 \\ 4 \\ 1 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ 5 \end{pmatrix}$

$= \begin{pmatrix} 6 \\ 2 \\ -4 \end{pmatrix}$

$\overrightarrow{QR} = 2\overrightarrow{PQ}$

$\overrightarrow{PQ}$ is parallel to $\overrightarrow{QR}$ and they share a common point Q, therefore P, Q and R are parallel

(b)

P		Q
−2	+3	1
1	+1	2
7	−2	5

$\overrightarrow{PS} = 4\overrightarrow{PQ}$

P		S
−2	+12	10
1	+4	5
7	−8	−1

S(10, 5, −1)

5E

(a) $\overrightarrow{PA}$

$= \underline{a} - \underline{p}$

$= \begin{pmatrix} 6 \\ 15 \end{pmatrix} - \begin{pmatrix} 1 \\ 3 \end{pmatrix}$

$= \begin{pmatrix} 5 \\ 12 \end{pmatrix}$

$\overrightarrow{AB}$

$= \underline{b} - \underline{a}$

$= \begin{pmatrix} 21 \\ 51 \end{pmatrix} - \begin{pmatrix} 6 \\ 15 \end{pmatrix}$

$= \begin{pmatrix} 15 \\ 36 \end{pmatrix}$

$3\overrightarrow{PA} = \overrightarrow{AB}$

$\overrightarrow{PA}$ is parallel to $\overrightarrow{AB}$ and they share a common point A, therefore P, A and B are collinear. ~~and~~ Meaning the boat is heading towards the lighthouse.

(b)

P		A		B
1	+5	6	+15	21
3	+12	15	+36	51

journey $\overrightarrow{AB} = 3\overrightarrow{AP}$ so another 3×20 minutes is needed.

It will reach the lighthouse at

13:25 + 80mins

= 14:45

5E

(14) $\overrightarrow{AB}$

$= \underline{b} - \underline{a}$

$= \begin{pmatrix} 1 \\ -9 \\ 4 \end{pmatrix} - \begin{pmatrix} -2 \\ -6 \\ -2 \end{pmatrix}$

$= \begin{pmatrix} 3 \\ -3 \\ 6 \end{pmatrix}$

$\overrightarrow{BC}$

$= \underline{c} - \underline{b}$

$= \begin{pmatrix} 3 \\ -11 \\ 8 \end{pmatrix} - \begin{pmatrix} 1 \\ -9 \\ 4 \end{pmatrix}$

$= \begin{pmatrix} 2 \\ -2 \\ 4 \end{pmatrix}$

$\overrightarrow{AB} = \frac{3}{2} \overrightarrow{BC}$

$\overrightarrow{AB}$ is parallel to $\overrightarrow{BC}$ and they share a common point B, therefore A, B and C are ~~parallel~~ collinear.

5E

15(a) 1 minute 5 minutes

A		B		C
-3	$+2$	-1	$+10$	9
-5	-6	-11	-30	-41
2	$+5$	7	$+25$	32

$C(9, -41, 32)$

(b) $\vec{AB}$

$= \underline{b} - \underline{a}$

$= \begin{pmatrix} -1 \\ -11 \\ 7 \end{pmatrix} - \begin{pmatrix} -3 \\ -5 \\ 2 \end{pmatrix}$

$= \begin{pmatrix} 2 \\ -6 \\ 5 \end{pmatrix}$

$\vec{BD}$

$= \underline{d} - \underline{b}$

$= \begin{pmatrix} 5 \\ -19 \\ 22 \end{pmatrix} - \begin{pmatrix} -1 \\ -11 \\ 7 \end{pmatrix}$

$= \begin{pmatrix} 6 \\ -8 \\ 15 \end{pmatrix}$

$\vec{AB} \neq k\vec{BD}$ $\therefore$ A, B and D are not collinear. The plane will not pass point D.