

Total marks – 100
Attempt ALL questions

1. Write down the binomial expansion of $\left(\frac{2}{y^2} - 5y\right)^3$ and simplify your answer. 4
2. Express $\frac{x^2 - 6x + 20}{(x+1)(x-2)^2}$ in partial fractions. 4
3. On a suitable domain, a function is defined by $f(x) = \frac{e^{x^2-1}}{x^2-1}$.
Find $f'(x)$, simplifying your answer. 3
4. The fifth term of an arithmetic sequence is -6 and the twelfth term is -34 .
- (a) Determine the values of the first term and the common difference. 2
- (b) Obtain algebraically the value of n for which $S_n = -144$. 3
5. (a) (i) Use Gaussian elimination on the system of equations below to give an expression for z in terms of λ . 4
- $$\begin{aligned}x + 2y - z &= -3 \\4x - 2y + 3z &= 11 \\3x + y + 2\lambda z &= 8\end{aligned}$$
- (ii) For what value of λ is this system of equations inconsistent? 1
- (b) Determine the solution of this system when $\lambda = -2.5$. 1
6. Use the substitution $u = 5x^2$ to find the exact value of $\int_0^{\frac{1}{\sqrt{10}}} \frac{x}{\sqrt{1-25x^4}} dx$. 6

7. Matrices P and Q are defined by $P = \begin{pmatrix} x & 2 \\ -5 & -1 \end{pmatrix}$ and $Q = \begin{pmatrix} 2 & -3 \\ 4 & y \end{pmatrix}$, where $x, y \in \mathbb{R}$.

(a) Given the determinant of P is 2, obtain:

(i) The value of x . 1

(ii) P^{-1} . 1

(iii) $P^{-1}Q'$, where Q' is the transpose of Q . 2

(b) The matrix R is defined by $R = \begin{pmatrix} 5 & -2 \\ z & -6 \end{pmatrix}$, where $z \in \mathbb{R}$.

Determine the value of z such that R is singular. 2

8. Use the Euclidean algorithm to find integers a and b such that $1595a + 1218b = 29$. 4

9. Solve $\frac{dy}{dx} = e^{2x}(1 + y^2)$ given that when $x = 0, y = 1$.

Express y in terms of x . 5

10. S_n is defined by $\sum_{r=1}^n \left(r^2 + \frac{1}{3}r \right)$.

(a) Find an expression for S_n , fully factorising your answer. 2

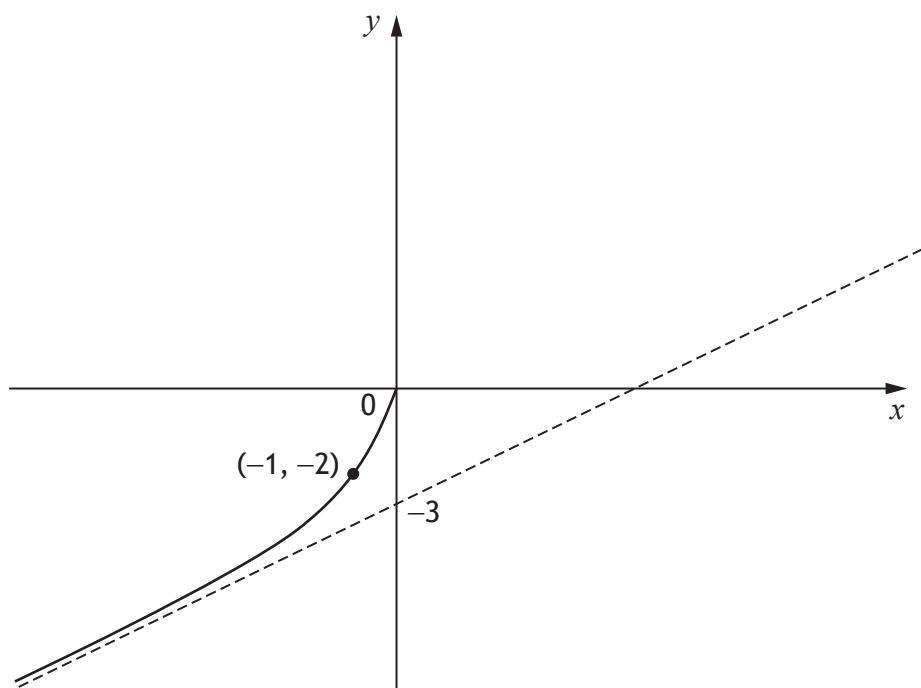
(b) Hence find an expression for $\sum_{r=10}^{2p} \left(r^2 + \frac{1}{3}r \right)$ where $p > 5$. 2

[Turn over

11. Given $y = x^{2x^3+1}$, use logarithmic differentiation to find $\frac{dy}{dx}$.
Write your answer in terms of x .

5

12. In the diagram below part of the graph of $y = f(x)$ has been omitted.
The point $(-1, -2)$ lies on the graph and the line $y = \frac{1}{2}x - 3$ is an asymptote.



Given that $f(x)$ is an odd function:

- (a) Copy and complete the diagram, including any asymptotes and any points you know to be on the graph. 2
- (b) $g(x) = |f(x)|$. On a separate diagram, sketch $g(x)$.
Include known asymptotes and points. 2
- (c) State the range of values of $f'(x)$ given that $f'(0) = 2$. 1
13. Let n be an integer.
Using proof by contrapositive, show that if n^2 is even, then n is even. 4

14. Find the particular solution of the differential equation

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = 8\sin x + 19\cos x$$

given that $y=7$ and $\frac{dy}{dx} = \frac{1}{2}$ when $x=0$.

10

15. (a) A beam of light passes through the points $B(7, 8, 1)$ and $T(-3, -22, 6)$.

Obtain parametric equations of the line representing the beam of light.

2

- (b) A sheet of metal is represented by a plane containing the points $P(2, 1, 9)$, $Q(1, 2, 7)$ and $R(-3, 7, 1)$.

Find the Cartesian equation of the plane.

4

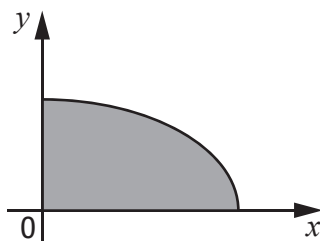
- (c) The beam of light passes through a hole in the metal at point H.

Find the coordinates of H.

3

16. On a suitable domain, a curve is defined by the equation $4x^2 + 9y^2 = 36$.

A section of the curve in the first quadrant, illustrated in the diagram below, is rotated 360° about the y -axis.



Calculate the exact value of the volume generated.

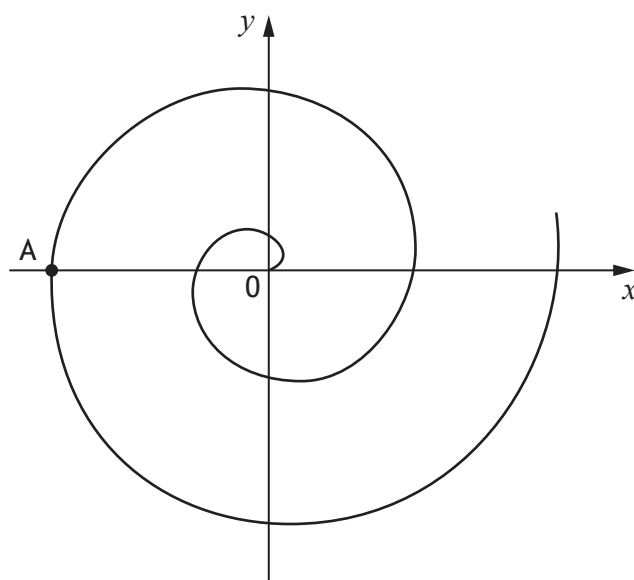
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17. The complex number $z = 2 + i$ is a root of the polynomial equation $z^4 - 6z^3 + 16z^2 - 22z + q = 0$, where $q \in \mathbb{Z}$.
- (a) State a second root of the equation. 1
- (b) Find the value of q and the remaining roots. 6
- (c) Show the solutions to $z^4 - 6z^3 + 16z^2 - 22z + q = 0$ on an Argand diagram. 1

18. The position of a particle at time t is given by the parametric equations $x = t \cos t$, $y = t \sin t$, $t \geq 0$.
- (a) Find an expression for the instantaneous speed of the particle. 5

The diagram below shows the path that the particle takes.



- (b) Calculate the instantaneous speed of the particle at point A. 2

[END OF QUESTION PAPER]