X056/301

NATIONAL QUALIFICATIONS 2001

THURSDAY, 17 MAY
9.00 AM – 10.10 AM

MATHEMATICS
HIGHER
Units 1, 2 and 3
Paper 1
(Non-calculator)

Read Carefully

1 **Calculators may <u>NOT</u> be used in this paper.**

2 Full credit will be given only where the solution contains appropriate working.

3 Answers obtained by readings from scale drawings will not receive any credit.

SCOTTISH QUALIFICATIONS AUTHORITY

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product: $\boldsymbol{a}.\boldsymbol{b} = |\boldsymbol{a}|\,|\boldsymbol{b}| \cos\theta$, where θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$

or $\boldsymbol{a}.\boldsymbol{b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\begin{aligned}
\sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\
\cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\
\sin 2A &= 2\sin A \cos A \\
\cos 2A &= \cos^2 A - \sin^2 A \\
&= 2\cos^2 A - 1 \\
&= 1 - 2\sin^2 A
\end{aligned}$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)\,dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

ALL questions should be attempted.

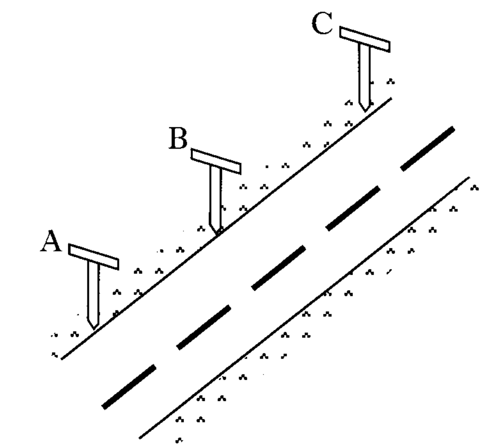
Marks

1. Find the equation of the straight line which is parallel to the line with equation $2x + 3y = 5$ and which passes through the point $(2, -1)$. **3**

2. For what value of k does the equation $x^2 - 5x + (k + 6) = 0$ have equal roots? **3**

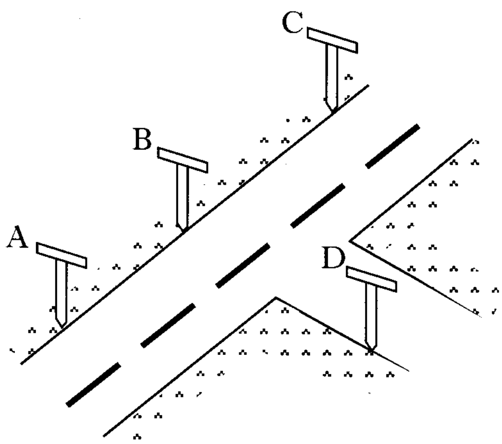
3. (*a*) Roadmakers look along the tops of a set of T-rods to ensure that straight sections of road are being created. Relative to suitable axes the top left corners of the T-rods are the points A(–8, –10, –2), B(–2, –1, 1) and C(6, 11, 5).

Determine whether or not the section of road ABC has been built in a straight line. **3**



(*b*) A further T-rod is placed such that D has coordinates (1, –4, 4).

Show that DB is perpendicular to AB. **3**



4. Given $f(x) = x^2 + 2x - 8$, express $f(x)$ in the form $(x + a)^2 - b$. **2**

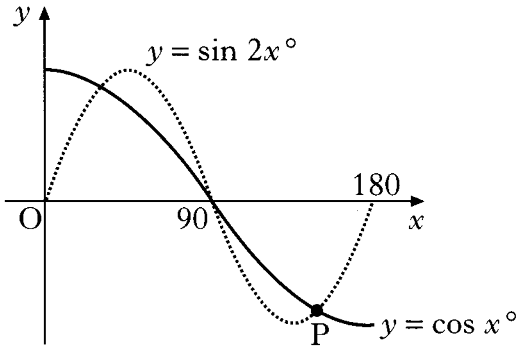
[Turn over

Marks

5. (*a*) Solve the equation $\sin 2x° - \cos x° = 0$ in the interval $0 \leq x \leq 180$. **4**

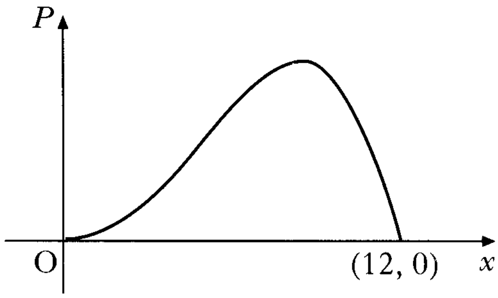
(*b*) The diagram shows parts of two trigonometric graphs, $y = \sin 2x°$ and $y = \cos x°$.

Use your solutions in (*a*) to write down the coordinates of the point P. **1**



6. A company spends x thousand pounds a year on advertising and this results in a profit of P thousand pounds. A mathematical model, illustrated in the diagram, suggests that P and x are related by $P = 12x^3 - x^4$ for $0 \leq x \leq 12$.

Find the value of x which gives the maximum profit. **5**



7. Functions $f(x) = \sin x$, $g(x) = \cos x$ and $h(x) = x + \frac{\pi}{4}$ are defined on a suitable set of real numbers.

(*a*) Find expressions for:

(i) $f(h(x))$;

(ii) $g(h(x))$. **2**

(*b*) (i) Show that $f(h(x)) = \frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x$.

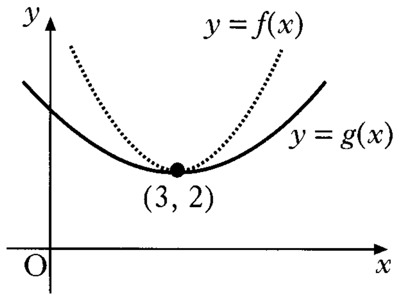
(ii) Find a similar expression for $g(h(x))$ and hence solve the equation $f(h(x)) - g(h(x)) = 1$ for $0 \leq x \leq 2\pi$. **5**

8. Find x if $4\log_x 6 - 2\log_x 4 = 1$. **3**

Marks

9. The diagram shows the graphs of two quadratic functions $y = f(x)$ and $y = g(x)$. Both graphs have a minimum turning point at (3, 2).

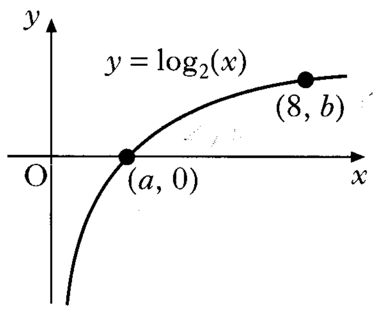
Sketch the graph of $y = f'(x)$ and on the same diagram sketch the graph of $y = g'(x)$. **2**



10. The diagram shows a sketch of part of the graph of $y = \log_2(x)$.

(*a*) State the values of a and b. **1**

(*b*) Sketch the graph of $y = \log_2(x + 1) - 3$. **3**



11. Circle P has equation $x^2 + y^2 - 8x - 10y + 9 = 0$. Circle Q has centre (–2, –1) and radius $2\sqrt{2}$.

(*a*) (i) Show that the radius of circle P is $4\sqrt{2}$.

(ii) Hence show that circles P and Q touch. **4**

(*b*) Find the equation of the tangent to circle Q at the point (–4, 1). **3**

(*c*) The tangent in (*b*) intersects circle P in two points. Find the x-coordinates of the points of intersection, expressing your answers in the form $a \pm b\sqrt{3}$. **3**

[*END OF QUESTION PAPER*]

X056/303

NATIONAL QUALIFICATIONS 2001

THURSDAY, 17 MAY
10.30 AM – 12.00 NOON

MATHEMATICS HIGHER

Units 1, 2 and 3
Paper 2

Read Carefully

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ALL questions should be attempted.

Marks

1. (*a*) Given that $x + 2$ is a factor of $2x^3 + x^2 + kx + 2$, find the value of k. **3**

(*b*) Hence solve the equation $2x^3 + x^2 + kx + 2 = 0$ when k takes this value. **2**

2. A curve has equation $y = x - \dfrac{16}{\sqrt{x}}$, $\quad x > 0$.

Find the equation of the tangent at the point where $x = 4$. **6**

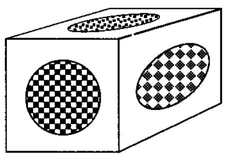
3. On the first day of March, a bank loans a man £2500 at a fixed rate of interest of 1·5% per month. This interest is added on the last day of each month and is calculated on the amount due on the first day of the month. He agrees to make repayments on the first day of each subsequent month. Each repayment is £300 except for the smaller final amount which will pay off the loan.

(*a*) The amount that he owes at the start of each month is taken to be the amount still owing just after the monthly repayment has been made.

Let u_n and u_{n+1} represent the amounts that he owes at the starts of two successive months. Write down a recurrence relation involving u_{n+1} and u_n. **2**

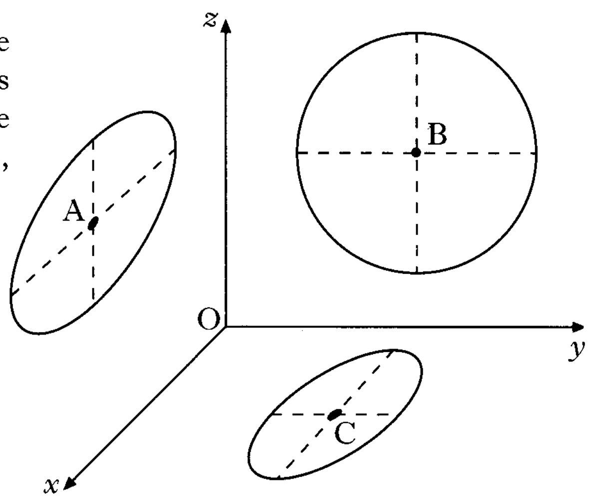
(*b*) Find the date and the amount of the final payment. **4**

4. A box in the shape of a cuboid is designed with **circles** of different sizes on each face.



The diagram shows three of the circles, where the origin represents one of the corners of the cuboid. The centres of the circles are A(6, 0, 7), B(0, 5, 6) and C(4, 5, 0).

Find the size of angle ABC. **7**



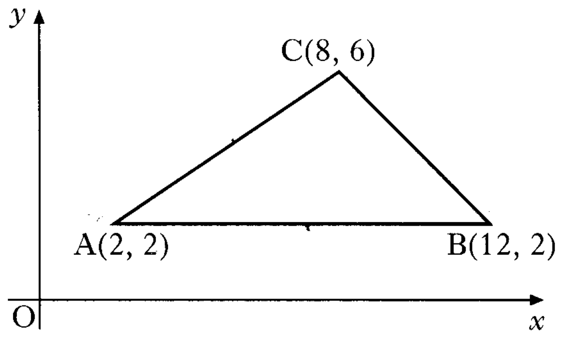
[Turn over

Marks

5. Express $8\cos x° - 6\sin x°$ in the form $k\cos(x + a)°$ where $k > 0$ and $0 < a < 360$. **4**

6. Find $\int \frac{(x^2 - 2)(x^2 + 2)}{x^2} dx, \quad x \neq 0$ **4**

7. Triangle ABC has vertices A(2, 2), B(12, 2) and C(8, 6).



(*a*) Write down the equation of l_1, the perpendicular bisector of AB. **1**

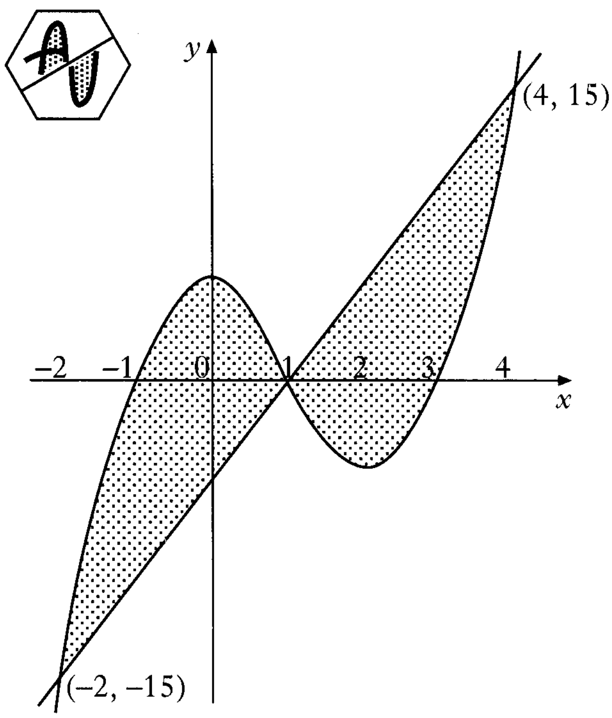
(*b*) Find the equation of l_2, the perpendicular bisector of AC. **4**

(*c*) Find the point of intersection of lines l_1 and l_2. **1**

(*d*) Hence find the equation of the circle passing through A, B and C. **2**

8. A firm asked for a logo to be designed involving the letters A and U. Their initial sketch is shown in the hexagon.

A mathematical representation of the final logo is shown in the coordinate diagram.



The curve has equation $y = (x + 1)(x - 1)(x - 3)$ and the straight line has equation $y = 5x - 5$. The point (1, 0) is the centre of half-turn symmetry.

Calculate the total shaded area. **7**

Marks

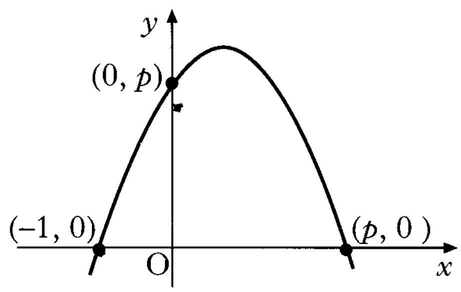
9. Before a forest fire was brought under control, the spread of the fire was described by a law of the form $A = A_0e^{kt}$ where A_0 is the area covered by the fire when it was first detected and A is the area covered by the fire t hours later.

If it takes one and half hours for the area of the forest fire to double, find the value of the constant k. **3**

10. A curve for which $\frac{dy}{dx} = 3\sin(2x)$ passes through the point $\left(\frac{5}{12}\pi, \sqrt{3}\right)$.

Find y in terms of x. **4**

11. The diagram shows a sketch of a parabola passing through $(-1, 0)$, $(0, p)$ and $(p, 0)$.



(*a*) Show that the equation of the parabola is $y = p + (p - 1)x - x^2$. **3**

(*b*) For what value of p will the line $y = x + p$ be a tangent to this curve? **3**

[*END OF QUESTION PAPER*]