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1) $2x + 3y = 5$

$\therefore \; y = -\frac{2}{3}x + \frac{5}{3}$ $\qquad \therefore \; M = -\frac{2}{3}$

$y - b = M(x-a)$ $\qquad (a,b) = (2,-1)$

$y + 1 = \frac{-2}{3}(x-2)$ $\times 3$

$3y + 3 = -2(x-2)$

$3y + 3 = -2x + 4$

$\underline{\underline{2x + 3y - 1 = 0}}$

---

2) $b^2 - 4ac = 0$ $\qquad a = 1, \; b = -5 \quad c = k+6$

$25 - 4(k+6) = 0$

$25 - 4k - 24 = 0$

$1 = 4k$

$\underline{\underline{k = 1/4}}$

---

3) a) $\overrightarrow{AB} = \underline{b} - \underline{a} = \begin{pmatrix} 6 \\ 9 \\ 3 \end{pmatrix}$ $\qquad \overrightarrow{BC} = \underline{c} - \underline{b} = \begin{pmatrix} 8 \\ 12 \\ 4 \end{pmatrix}$

$\overrightarrow{AB} = \frac{3}{4}\overrightarrow{BC}$ B is a common point

$\therefore$ Straight line

b) $\overrightarrow{BA} = \underline{a} - \underline{b} = \begin{pmatrix} -6 \\ -9 \\ -3 \end{pmatrix}$ $\qquad \overrightarrow{BD} = \underline{d} - \underline{b} = \begin{pmatrix} 3 \\ -3 \\ 3 \end{pmatrix}$

$\overrightarrow{BA} \cdot \overrightarrow{BD} = -18 + 27 - 9 = 0$

$\therefore$ Perpendicular.

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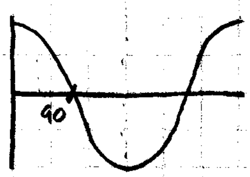
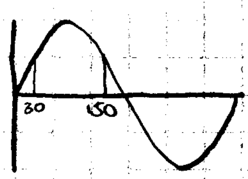
4)
$$f(x) = x^2 + 2x - 8$$
$$= (x^2 + 2x + 1^2) - 8 - 1^2$$
$$= (x^2 + 2x + 1) - 9$$
$$= (x+1)(x+1) - 9$$
$$= \underline{\underline{(x+1)^2 - 9}}$$

---

5) a)
$$\sin 2x - \cos x = 0$$
$$2\sin x\cos x - \cos x = 0$$
$$\cos x(2\sin x - 1) = 0$$
$$\cos x = 0 \quad \text{OR} \quad 2\sin x - 1 = 0$$
$$\cos x = 0 \quad \text{OR} \quad \sin x = \tfrac{1}{2}$$



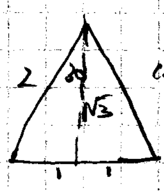


$$x = 90° \quad \text{OR} \quad x = 30°, 150°$$

$$x = 30°, 90°, 150°$$

b) At P:
$$\sin 2x = \cos x$$
$$2\sin x\cos x = \cos x$$
$$2\sin x = 1$$
$$\sin x = \tfrac{1}{2}$$
$$x = 30° \text{ OR } 150°$$
At P $x = 150°$

$$y = \cos x = \cos 150°$$
$$y = \cos 150°$$
$$y = \frac{-\sqrt{3}}{2}$$



$\cos 30 = \frac{\sqrt{3}}{2}$

$$\therefore P\left(150°, \frac{-\sqrt{3}}{2}\right)$$

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6) Find stationary points. $f'(x) = 0$

$\therefore \quad 36x^2 - 4x^3 = 0$

$4x^2(9 - x) = 0$

$4x^2 = 0$ OR $9 - x = 0$

$x = 0$ OR $x = 9$

by inspection Max T.P. $x = 9$

$\therefore \; P = 12 \times 9^3 - 9^4$

$= 9^2(12 \times 9 - 9^2)$

$= 81(108 - 81)$

$= 81 \times 27$

$= 2187$

$$\begin{array}{r} 27 \\ 81 \\ \hline 27 \\ 2160 \\ \hline 2187 \end{array}$$

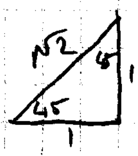
$\therefore$ Max P = £ 2187000

7) a) i) $f(h(x)) = \sin\left(x + \frac{\pi}{4}\right)$

ii) $g(h(x)) = \cos\left(x + \frac{\pi}{4}\right)$

b) $f(h(x)) = \sin\left(x + \frac{\pi}{4}\right) = \sin x \cos\frac{\pi}{4} + \cos x \sin\frac{\pi}{4}$

$\frac{\pi}{4} = 45°$



$\therefore \; \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

$\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

so: $\underline{\frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x}$

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7) b) ii) $g(h(x)) = \cos\left(x + \frac{\pi}{4}\right)$

$$= \cos x \cos \frac{\pi}{4} - \sin x \sin \frac{\pi}{4}$$

$$= \frac{1}{\sqrt{2}} \cos x - \frac{1}{\sqrt{2}} \sin x$$

$$f(h(x)) - g(h(x)) = \frac{1}{\sqrt{2}} \sin x + \frac{1}{\sqrt{2}} \cos x - \frac{1}{\sqrt{2}} \cos x + \frac{1}{\sqrt{2}} \sin x$$

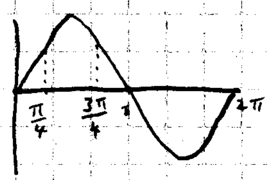
$$= \frac{2}{\sqrt{2}} \sin x = 1$$

$$= \sin x = \frac{\sqrt{2}}{2} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$\sin x = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$\sin x = \frac{1}{\sqrt{2}}$$

$$\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$$



$$\therefore \; \underline{\underline{x = \frac{\pi}{4}, \frac{3\pi}{4}}}$$

---

8) $4 \log_x 6 - 2 \log_x 4 = 1$

$$\log_x 6^4 - \log_x 4^2 = 1$$

$$\log_x \frac{6^4}{16} = 1$$

$$\log_x \frac{1296}{16} = 1$$

$$\log_x 81 = 1$$

$Y = a^x \quad x = \log_a Y \;\therefore\; 81 = x^1 \;\therefore\; \underline{\underline{x = 81}}$

$36 \times 6 = 216$, $216 \times 6 = 1296$ $\therefore 6^4 = 1296$

$1296 \div 16 = 81$

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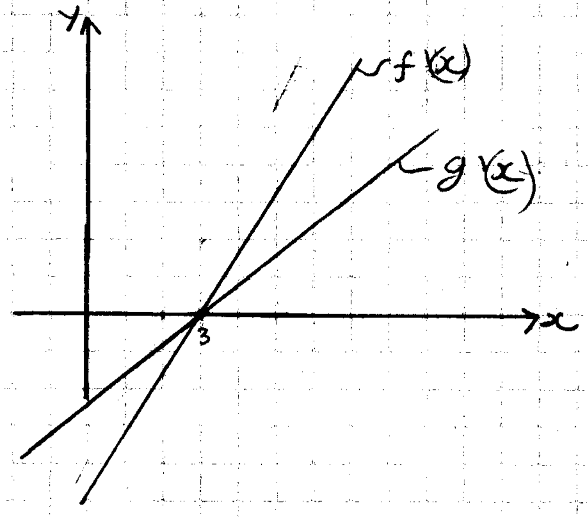
9)

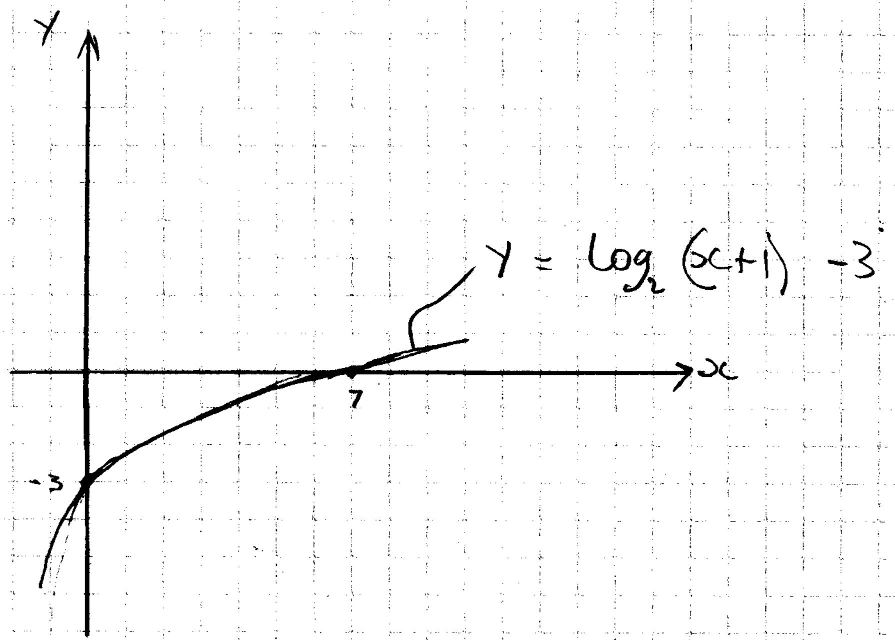


10) a) at a: $\log_2 a = 0$ at b: $\log_2 8 = b$

$\therefore a = 2^0$ $8 = 2^b$

$\underline{\underline{a = 1}}$ $\underline{\underline{b = 3}}$

b)



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11) $x^2 + y^2 + 2gx + 2fy + c = 0$ centre $= (-g, -f)$

radius $= \sqrt{g^2 + f^2 - c}$

a) i) $2g = -8$ $2f = -10$

$g = -4$ $f = -5$ $c = 9$

$r = \sqrt{16 + 25 - 9}$

$= \sqrt{32} = \sqrt{16}\sqrt{2}$

$= \underline{\underline{4\sqrt{2}}}$

ii) Centre of P = (4, 5)

$\therefore$ distance PQ $= \sqrt{6^2 + 6^2}$

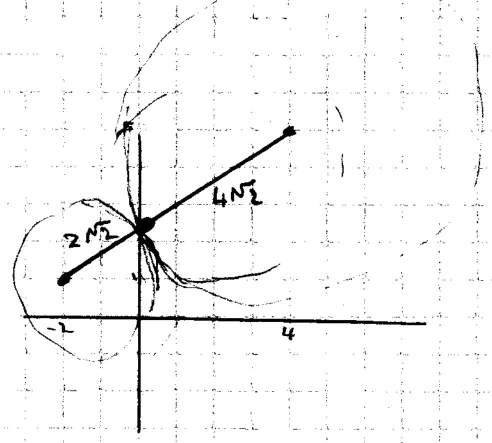
$= \sqrt{72} = \sqrt{36}\sqrt{2}$

$= \underline{\underline{6\sqrt{2}}}$

Now $r_p + r_q = 4\sqrt{2} + 2\sqrt{2}$

$= \underline{\underline{6\sqrt{2}}}$

$\therefore$ The circles must touch.



b) $M_1 = \frac{1+1}{-4+2} = \frac{2}{-2} = \underline{\underline{-1}}$

$M_1 M_2 = -1$ $\therefore$ $\underline{\underline{M_2 = 1}}$

$y - b = m(x - a)$ $m = 1,$ $(a, b) = (-4, 1)$

$y - 1 = 1(x + 4)$

$y - 1 = x + 4$

$\underline{\underline{x - y + 5 = 0}}$

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11) c) $y = x + 5$

Sub: $x^2 + (x+5)^2 - 8x - 10(x+5) + 9 = 0$

$x^2 + x^2 + 10x + 25 - 8x - 10x - 50 + 9 = 0$

$2x^2 - 8x - 16 = 0$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$a = 2,\ b = -8,\ c = -16$

$b^2 - 4ac$

$64 - (4 \times 2 \times -16)$

$64 + 128$

192

$\frac{8 \pm \sqrt{192}}{4}$

$16 \times 8 = 128$

$\frac{8 \pm \sqrt{64}\sqrt{3}}{4}$

$192 \div 3 = 64$

$\frac{8 \pm 8\sqrt{3}}{4}$

$\underline{\underline{2 \pm 2\sqrt{3}}}$

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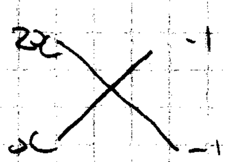
1) a)

-2	2	1	k	2
		-4	6	$-2(6+k)$
	2	-3	$(6+k)$	$-2(6+k)+2$

$$\therefore \quad -2(6+k)+2 = 0$$
$$-12-2k+2 = 0$$
$$-2k-10 = 0$$
$$2k = -10$$
$$\underline{\underline{k = -5}}$$

b) $6+k = 1$

$$\therefore (x+2)(2x^2-3x+1) = 0$$



$$(x+2)(2x-1)(x-1) = 0$$

$$\underline{\underline{x=-2, \; x = \tfrac{1}{2}, \; x=1}}$$

---

2) $M = \frac{dy}{dx}$: $y = x - 16x^{-1/2}$

$$\frac{dy}{dx} = 1 + 8x^{-\frac{3}{2}}$$
$$= \underline{\underline{1 + \frac{8}{\sqrt{x^3}}}}$$

at $x=4$: $M = 1 + (8 \div \sqrt{4^3}) = 2$

at $x=4$: $y = 4 - (16 \div \sqrt{4}) = -4 \qquad (a,b) = (4,-4)$

$$y-b = M(x-a) = y+4 = 2(x-4)$$
$$y+4 = 2x-8$$
$$\underline{\underline{2x-y-12 = 0}}$$

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3) $U_{n+1} = 1.015\,U_n - 300$

Amount owes

April	=	$1.015 \times 2500 - 300$	=	£2237.50
May	=	$1.015 \times 2237.5 - 300$	=	£1971.00
June	=	$1.015 \times 1971 - 300$	=	£1700.60
July	=	$1.015 \times 1700.6 - 300$	=	£1426.10
Aug	=	$1.015 \times 1426.1 - 300$	=	£1147.50
Sept	=	$1.015 \times 1147.5 - 300$	=	£864.74
Oct	=	$1.015 \times 864.74 - 300$	=	£577.71
Nov	=	$1.015 \times 577.71 - 300$	=	£286.37
Dec	=	$1.015 \times 286.37 - 300$	=	£-9.32

$\therefore$ Last payment in December of

$300 - 9.32 = \underline{£290.68}$

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4) $\vec{BA} = \underline{a} - \underline{b} = \begin{pmatrix}6\\0\\7\end{pmatrix} - \begin{pmatrix}0\\5\\6\end{pmatrix} = \underline{\underline{\begin{pmatrix}6\\-5\\1\end{pmatrix}}}$

$\vec{BC} = \underline{c} - \underline{b} = \begin{pmatrix}4\\5\\0\end{pmatrix} - \begin{pmatrix}0\\5\\6\end{pmatrix} = \begin{pmatrix}4\\0\\-6\end{pmatrix}$

$$\cos\phi = \frac{a.b}{|a||b|}$$

$$a.b = (6\times4) + (-5\times0) + (1\times-6)$$
$$= 24 + 0 - 6$$
$$= \underline{\underline{18}}$$

$$|a| = \sqrt{6^2 + -5^2 + 1^2} = \sqrt{36+25+1} = \underline{\underline{\sqrt{62}}}$$

$$|b| = \sqrt{4^2 + 0^2 + -6^2} = \sqrt{16+0+36} = \sqrt{52}$$

$$\cos\phi = \frac{18}{\sqrt{62}\sqrt{52}} = 0.317$$

$$\phi = \cos^{-1} 0.317 = \underline{\underline{71.5^\circ}}$$

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5) $8\cos x^\circ - 6\sin x^\circ = k\cos(x+a)^\circ$

$= k(\cos x\cos a - \sin x\sin a)$

$= k\cos x\cos a - k\sin x\sin a$

$\therefore \quad 8 = k\cos a^\circ \quad - (1)$

$6 = k\sin a^\circ \quad - (2)$

$(1)^2 + (2)^2$

$8^2 + 6^2 = k^2\cos^2 a + k^2\sin^2 a$

$100 = k^2(\cos^2 a + \sin^2 a)$

$100 = k^2$

$k = \sqrt{100}$

$k = 10$

$(2) \div (1)$

$\frac{\cancel{k}\sin a^\circ}{\cancel{k}\cos a^\circ} = \frac{6}{8}$

$\frac{\sin a}{\cos a} = \frac{3}{4}$

$\tan a^\circ = \frac{3}{4}$

$\sin a^\circ$, $\cos a^\circ$, $\tan a^\circ$ all +ve $\therefore$ 1st quadrant

$a^\circ = \tan^{-1}\left(\frac{3}{4}\right) = 36.87^\circ$

$\therefore \quad 8\cos x^\circ - 6\sin x^\circ = 10\cos(x + 36.87)^\circ$

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6) $\int \frac{(x^2-2)(x^2+2)}{x^2}\,dx$

$= \int \frac{x^4-4}{x^2}\,dx$

$= \int x^2 - 4x^{-2}\,dx$

$= \frac{x^3}{3} + 4x^{-1} + c$

$= \frac{x^3}{3} + \frac{4}{x} + c$

7) a) Mid point of AB = $(7, 2)$

Equation of perpendicular bisector is $x = 7$

b) Mid point of AC = $(5, 4)$; $M_{AC} = \frac{6-2}{8-2} = \frac{4}{6} = \frac{2}{3}$

$M_1 M_2 = -1$ $\therefore$ $M_2 = \frac{-3}{2}$

$Y - b = M(x - a)$ $M = \frac{-3}{2}$ $(a, b) = (5, 4)$

$Y - 4 = \frac{-3}{2}(x - 5)$ $(\times 2)$

$2Y - 8 = -3x + 15$

$3x + 2Y - 23 = 0$

c) Sub $x = 7$ into $3x + 2Y - 23 = 0$

$= 21 + 2Y - 23 = 0$

$2Y = 2$

$Y = 1$ $\therefore$ $(7, 1)$

d) Centre $(7, 1)$ radius $= \sqrt{(7-2)^2 + (1-2)^2} = \sqrt{26}$

$\therefore$ $(x-7)^2 + (Y-1)^2 = 26$

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8) $(x^2-1)(x-3)$

x^3-3x^2-x+3

$$\int_1^4 (5x-5)-(x^3-3x^2-x+3)\,dx$$

$$=\int_1^4 -x^3+3x^2+6x-8\quad dx$$

$$=\left[-\frac{x^4}{4}+x^3+3x^2-8x\right]_1^4$$

$$=\left(-\frac{4^4}{4}+4^3+3\times4^2-8\times4\right)-\left(-\frac{1^4}{4}+1^3+3\times1^2-8\times1\right)$$

$$=(-64+64+48-32)-(-\tfrac{1}{4}+1+3-8)$$

$$=(16)-(-4\tfrac{1}{4})$$

$$=\underline{\underline{20\tfrac{1}{4}}}$$

$\therefore$ Total Area $= 2\times 20\frac{1}{4}$

$= \underline{\underline{40\frac{1}{2}\text{ units}^2}}$

---

9) $2A_0 = A_0e^{1.5k}$

$2 = e^{1.5k}$

$\ln 2 = 1.5k$

$0.693 = 1.5k$

$\therefore k = 0.693 \div 1.5$

$= \underline{\underline{0.462}}$

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10)

$$\frac{dy}{dx} = 3\sin(2x)$$

$$\int 3\sin(2x)\,dx = -\frac{3}{2}\cos 2x + c$$

$$\therefore y = -\frac{3}{2}\cos 2x + c$$

at $\left(\frac{5}{12}\pi, \sqrt{3}\right)$

$$\sqrt{3} = -\frac{3}{2}\cos\left(2\times\frac{5}{12}\pi\right) + c$$

$$\Rightarrow \sqrt{3} = -\frac{3}{2} \times \frac{-\sqrt{3}}{2} + c$$

$$\Rightarrow \sqrt{3} = \frac{3\sqrt{3}}{4} + c \Rightarrow c = \frac{\sqrt{3}}{4}$$

$$\therefore y = -\frac{3}{2}\cos 2x + \frac{\sqrt{3}}{4}$$

11) a) $y = (x+1)(x-p)$

$y = k[(x+1)(x-p)]$ where k is a constant

at $(0, p)$: $p = k(1 \times -p)$

$p = -kp$

$\therefore k = -1$

so: $-1[(x+1)(x-p)]$

$= -1(x^2 - px + x - p)$

$= -x^2 + px - x + p$

$\therefore y = p + (p-1)x - x^2$

b) For tangency discriminant $= 0$

$p + (p-1)x - x^2 = x + p$

$\cancel{p} + px - x - x^2 - x - \cancel{p} = 0$

$-x^2\ px - 2x = 0$

$-x^2 + (p-2)x = 0$

$b^2 - 4ac = 0$

$(p-2)^2 - (4 \times -1 \times 0) = 0$

$(p-2)(p-2) = 0$

$\therefore p = 2$