Homework 10 solutions

1) a) $x = t - \sin t$ $\qquad$ $y = 1 - \cos t$

$$\frac{dx}{dt} = 1 - \cos t \qquad \frac{dy}{dt} = \sin t$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\sin t}{1 - \cos t}$$

b)

$$v = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$$

at $t = \pi/3$

$$v = \sqrt{\left(1 - \cos\frac{\pi}{3}\right)^2 + \left(\sin \pi/3\right)^2}$$

$$v = \sqrt{\frac{1}{4} + 3/4}$$

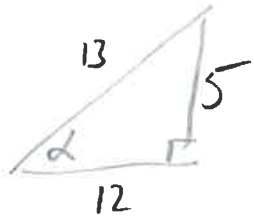
$$v = \sqrt{1} = 1\,\text{ms}^{-1}$$

2)

$T\cos\alpha = mg$ ✓

$T\sin\alpha = m\omega^2 r$

$\tan\alpha = 5/12$



$\cos\alpha = \frac{12}{13}$

a)

$T\cos\alpha = mg$

$T \times \frac{12}{13} = mg$

$T = \frac{13mg}{12}$ ✓

$\sin\alpha = \frac{r}{L}$ ✓

b)

$T\sin\alpha = m\omega^2 r$ ✓

$T \times \frac{\cancel{r}}{L} = m\omega^2 \cancel{r}$

$T = m\omega^2 L$

$T = \frac{13mg}{12}$

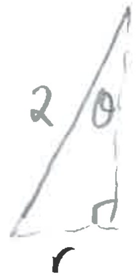
$\Rightarrow \quad \cancel{m}\omega^2 L = \frac{13\cancel{m}g}{12}$

$\omega^2 = \frac{13g}{12L}$

$\omega = \sqrt{\frac{13g}{12L}}$ ✓

(3)

$m = 2kg$
$L = 2 \text{ metres}$
$w = 5$



$\sin\theta = \frac{r}{2}$

$r = 2\sin\theta$

$T\cos\theta = mg$

$T\sin\theta = mw^2 r$

$\Rightarrow$ $T\sin\theta = mw^2 \times 2\sin\theta$

$T = 2mw^2$

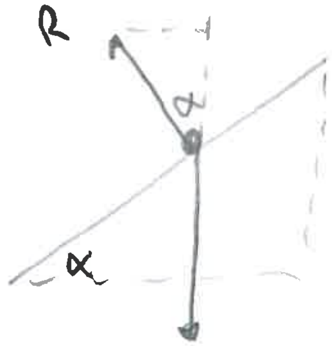
$T = 100N$

$T\cos\theta = mg$

$\cos\theta = \frac{mg}{T}$

$\theta = 78.7°$

4) a)



vertical

$R\cos\alpha = mg$

horizontal

$\Sigma F = ma$

$ma = R\sin\alpha$

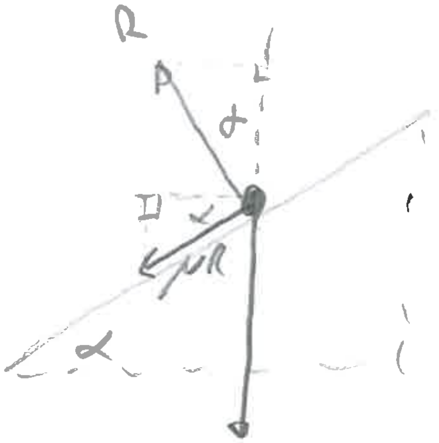
$R\sin\alpha = \frac{mv^2}{r}$

$R\cos\alpha = mg$

$\Rightarrow \tan\alpha = \frac{v^2}{gr}$

$v = \sqrt{gr\tan\alpha}$

b)



vertical

$R\cos\alpha = \mu R\sin\alpha + mg$

horizontal

$\Sigma F = ma$

$R\sin\alpha + \mu R\cos\alpha = \frac{mv^2}{r}$

$R\cos\alpha - \mu R\sin\alpha = mg$

$\Rightarrow \frac{\sin\alpha + \mu\cos\alpha \ (\div\cos\alpha)}{\cos\alpha - \mu\sin\alpha \ (\div\cos\alpha)} = \frac{v^2}{gr}$

$\Rightarrow \frac{\tan\alpha + \mu}{1 - \mu\tan\alpha} = \frac{v^2}{gr}$

$v^2 = \frac{gr(\tan\alpha + \mu)}{1 - \mu\tan\alpha}$

c) $v = 2V$

$\Rightarrow \quad v^2 = 4V^2$

$$\frac{g(\tan\alpha + \mu)}{1 - \mu\tan\alpha} = 4gr\tan\alpha$$

$$\tan\alpha + \mu = 4\tan\alpha(1 - \mu\tan\alpha) \qquad \mu = 3/4$$

$$\tan\alpha + \frac{3}{4} = 4\tan\alpha - 3\tan^2\alpha$$

$$3\tan^2\alpha - 3\tan\alpha + \frac{3}{4} = 0 \qquad (\times 4)$$

$$12\tan^2\alpha - 12\tan\alpha + 3 = 0$$

$$4\tan^2\alpha - 4\tan\alpha + 1 = 0$$

$$(2\tan\alpha - 1)(2\tan\alpha - 1) = 0$$

$$\tan\alpha = \frac{1}{2}$$

$$\underline{\alpha = 26.6°}$$

d) on the point of slipping down so $v^2 = \dfrac{gr(\tan\alpha - \mu')}{1 + \mu'\tan\alpha}$

↑ it would be fine to just state this having already found the "skidding up" equation

$v = 0 \Rightarrow$

$$\frac{gr(\tan\alpha - \mu')}{1 + \mu'\tan\alpha} = 0$$

$$gr(\tan\alpha - \mu') = 0$$

$$\tan\alpha - \mu' = 0$$

$$\mu' = \tan 26.6$$

$$\underline{\mu' = 1/2}$$

e) <u>in dry conditions</u>

$\tan\alpha = \frac{1}{2}$

$v = 2V$

$= 2\sqrt{gr\tan\alpha} = \underline{4.43\sqrt{r}}$

<u>in wet conditions</u>

$\mu' = \frac{1}{2} \quad \tan\alpha = \frac{1}{2}$

$$v = \sqrt{\frac{gr(\tan\alpha + \mu')}{1 - \mu'\tan\alpha}}$$

$$v = \sqrt{\frac{gr}{0.75}}$$

$\underline{v = 3.61\sqrt{r}}$

$\frac{3.61\sqrt{r}}{4.43\sqrt{r}} \times 100 = 81.6\% \simeq 82\%$ as required.