

Homework 20

①

1)	shape	area	centre
	rectangle	60	(3, 5)
	triangle	30	$(8, \frac{10}{3}) \leftarrow R(6,0) \quad S(12,0) \quad Q(6,10)$
	circle	π	(3, 3) average

$$60 \begin{pmatrix} 3 \\ 5 \end{pmatrix} + 30 \begin{pmatrix} 8 \\ \frac{10}{3} \end{pmatrix} - \pi \begin{pmatrix} 3 \\ 3 \end{pmatrix} = (60 + 30 - \pi) \begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix}$$

$$\bar{x} = \frac{60 \times 3 + 30 \times 8 - 3\pi}{60 + 30 - \pi} = 4.73$$

$$\bar{y} = \frac{60 \times 5 + 30 \times \frac{10}{3} - 3\pi}{60 + 30 - \pi} = 4.50$$



if lamina is suspended a distance 4.50 from T then OS would hang vertically

2) $\frac{dy}{dx} - 2y = e^{3x}$

IF = $e^{\int -2dx}$
 $= e^{-2x}$ ✓

$e^{-2x} y = \int e^{-2x} \cdot e^{3x} dx$ ✓

$e^{-2x} y = \int e^x dx$ ✓

$e^{-2x} y = e^x + C$ ✓

$x=0 \quad y=2 \Rightarrow \begin{matrix} 2 = 1 + C \\ C = 1 \end{matrix}$

$e^{-2x} y = e^x + 1$
 $y = e^{2x}(e^x + 1)$ ✓

3

$$3) \quad F = 6\hat{i} + 2\hat{j} \quad m = 2\text{kg}$$

$$\Rightarrow \underline{a} = 3\hat{i} + \hat{j}$$

$$\underline{v} = 3t\hat{i} + t\hat{j} + C$$

$$\text{at } t=0 \quad v = 2\hat{i} - \hat{j} \Rightarrow v = (3t+2)\hat{i} + (t-1)\hat{j}$$

$$\text{at } t=3 \quad v = 11\hat{i} + 2\hat{j}$$

$$|\underline{v}| = \sqrt{11^2 + 2^2} \\ = \underline{11.2 \text{ms}^{-1}}$$

$$\text{or} \quad F \times t = mv - mu$$

$$(6\hat{i} + 2\hat{j}) \times 3 = 2v - 2(2\hat{i} - \hat{j})$$

$$v = 11\hat{i} + 2\hat{j}$$

$$|\underline{v}| = 11.2 \text{ms}^{-1}$$

4)

ma \rightarrow

a) $10v \leftarrow \text{---} \rightarrow F = \frac{P}{v} = \frac{6000}{v}$

$ma = \frac{6000}{v} - 10v$ ✓ $a = \frac{dv}{dt}$

$800 \frac{dv}{dt} = \frac{6000 - 10v^2}{v}$ ✓

$\int \frac{800v}{6000 - 10v^2} dv = \int dt$

b) $-40 \int_{10}^v \frac{-20v}{6000 - 10v^2} dv = \int_0^{20} dt$ ✓

$[-40 \ln |6000 - 10v^2|]_{10}^v = [t]_0^{20}$ ✓

$-40 \ln |6000 - 10v^2| - [-40 \ln 5000] = 20 - 0$ ✓

$40 \ln \frac{5000}{6000 - 10v^2} = 20$ ✓

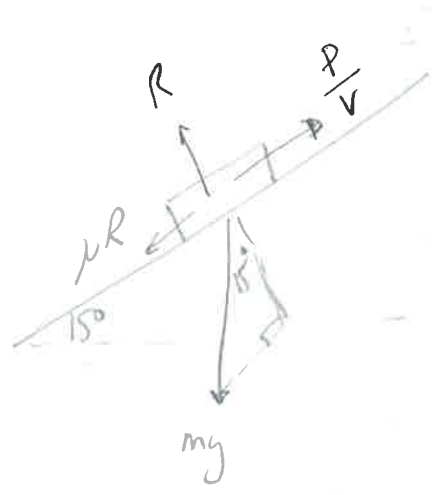
$\ln \frac{5000}{6000 - 10v^2} = 0.5$

$\frac{5000}{6000 - 10v^2} = e^{0.5}$ ✓

$\frac{5000}{e^{0.5}} = 6000 - 10v^2$

$v = 17.3 \text{ms}^{-1}$ ✓

5)



in equilibrium

$$\frac{P}{v} = mg \sin 15^\circ + \mu R \quad R = mg \cos 15^\circ$$

$$\frac{18000}{10} = mg \sin 15^\circ + \mu mg \cos 15^\circ$$

$$\underline{\mu = 0.207}$$

