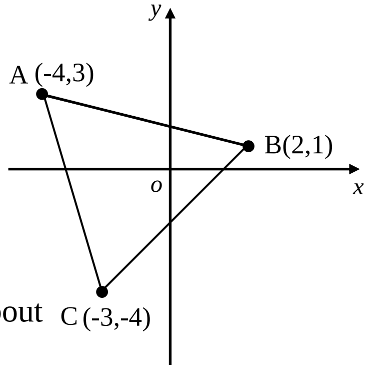
TEST A

All questions should be attempted

1. Triangle ABC has vertices A(-4,3), B(2, 1) and C(-3, -4)



(a) Find the equation of the perpendicular bisector of AB. **(4)**

(b) Establish the equation of the median through C. **(2)**

(c) From your answers to (a) and (b), what can you say about triangle ABC, giving a reason for your answer? **(2)**

2. Given that $f(x) = \dfrac{x^3 - 5}{\sqrt{x}}$, $x > 0$, evaluate $f'(4)$. **(4)**

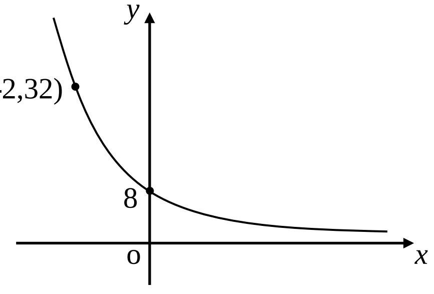
3. A sequence of numbers is defined by the recurrence relation $U_{n+1} = aU_n - 5$, where a is a constant.

(a) Given that $U_0 = 30$, show that, in terms of a, $U_2 = 30a^2 - 5a - 5$ **(2)**

(b) Hence find a, where $a > 0$, given that $U_2 = 0$. **(2)**

(c) Establish the limit of this sequence as $n \to \infty$. **(2)**

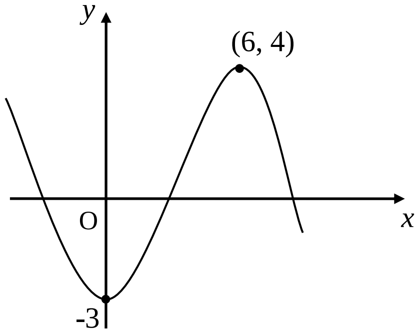
4. The diagram shows part of the graph of $y = k(a^{-x})$.



(a) State the value of k. **(1)**

(b) By considering point P, establish the value of a, where $a > 0$. **(3)**

5. The diagram below shows part of the graph of $y = h(x)$.
The function has stationary points at (0, -3) and (6, 4).



Sketch the graph of the derived function $y = h'(x)$. **(3)**

6. A curve has as its equation $y = 8x^3 - 2x^4$.

(a) Find the points at which this curve cuts the x and y axes. **(2)**

(b) Establish the coordinates and nature of the stationary point(s) of this curve. **(7)**

(c) Sketch the graph of $y = 8x^3 - 2x^4$ showing clearly the points found in (a) and (b) above. **(2)**

7. The function $g(x) = \dfrac{x-8}{x}$ is defined on a suitable domain.

(a) Evaluate $g(g(4))$. **(1)**

(b) Given that $k(x) = g(g(x))$, show that $k(x) = \dfrac{-7x-8}{x-8}$. **(2)**

(c) For what value of x is $k(x)$ undefined? **(1)**

8. **(a)** A function is given as $f(x) = \sin 3x + \cos^2 x$.

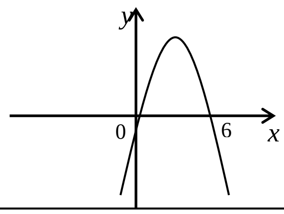
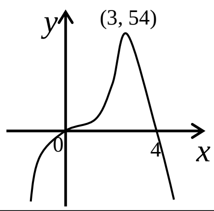
By finding the rate of change of this function when $x = \dfrac{\pi}{2}$, make a statement about the function at this point. **(5)**

(b) A function, defined on a suitable domain is given as

$$f(x) = \frac{3}{(1+3x)^2}\,, \qquad \text{find } f'(1)$$ **(5)**

SET B **Marking Scheme - UNIT 1**

	Give 1 mark for each •	Illustration(s) for awarding each mark
1(a)	**ans: $y = 3x + 5$ (4 marks)**	
	•[1] knows to find midpoint •[2] finds gradient of AB •[3] finds perpendicular gradient •[4] substitutes in $y - b = m(x - a)$	•[1] Midpoint of AB is (-1, 2) •[2] $m_{AB} = -\frac{1}{3}$ •[3] $m_{PERP} = 3$ •[4] $y - 2 = 3(x + 1)$
(b)	**ans: $y = 3x + 5$ (2 marks)**	
	•[1] finds gradient of median •[2] substitutes in equation	•[1] $m_{median} = 3$ •[2] $y - 2 = 3(x + 1)$ (or equivalent)
(c)	**ans: ABC is isosceles with reason (2 marks)**	
	•[1] identifies type of triangle •[2] valid reason	•[1] triangle ABC is isosceles •[2] the median is also the perp. bisector
2	**ans: $20\frac{5}{16}$ (4 marks)**	
	•[1] prepare to differentiate •[2] differentiates •[3] substitutes •[4] evaluates to answer	•[1] $x^{\frac{5}{2}} - 5x^{-\frac{1}{2}}$ •[2] $\frac{5}{2}x^{\frac{3}{2}} + \frac{5}{2}x^{-\frac{3}{2}}$ •[3] $\frac{5}{2}(4)^{\frac{3}{2}} + \frac{5}{2(4)^{\frac{3}{2}}}$ (or equivalent) •[4] $20\frac{5}{16}$
3(a)	**ans: proof (2 marks)**	
	•[1] finds expression for U_1 •[2] substitutes and rearranges to answer	•[1] $30a - 5$ •[2] $a(30a - 5) - 5$
(b)	**ans: $a = \frac{1}{2}$ (2 marks)**	
	•[1] equates to 0 and factorises •[2] solves quadratic and discards	•[1] $30a^2 - 5a - 5 = 0$; $5(3a + 1)(2a - 1) = 0$. •[2] $\cancel{a = -\frac{1}{3}}$ or $a = \frac{1}{2}$
(c)	**ans: -10 (2 marks)**	
	•[1] knows how to find limit •[2] answer	•[1] $L = \frac{-5}{1 - \frac{1}{2}}$ •[2] -10

	Give 1 mark for each •	Illustration(s) for awarding each mark
4(a)	**ans: *k* = 8** **(1 mark)** ●[1] answer	 ●[1] $8 = k(a^0)$, $8 = k(1)$ $\therefore$ $k = 8$
(b)	**ans: a = 2** **(3 marks)** ●[1] for substituting in point & *k* ●[2] simplifying ●[3] answer	 ●[1] $y = 8(a^{-x}) \Rightarrow 8(a^{-(-2)}) = 32$ ●[2] $8a^2 = 32$ ●[3] $a^2 = 4$ $\therefore$ $a = 2$
5	**ans: graph drawn** **(3 marks)** ●[1] intercepts with *x*-axis correct ●[2] parabola drawn ●[3] correct parabola drawn	●[1] ●[2] ●[3] 
6(a)	**ans: (0, 0), (4, 0); (0, 0)** **(2 marks)** ●[1] correct *x*-intercepts ●[2] correct *y*-intercept	 ●[1] $8x^3 - 2x^4 = 0$; $2x^3(4 - x) = 0$; (0,0),(4,0) ●[2] (0, 0)
(b)	**ans: (0, 0) pt.of inflection;** **(3, 54) maximum.** **(7 marks)** ●[1] knows to differentiate ●[2] differentiates correctly ●[3] knows $\frac{dy}{dx} = 0$ ●[4] finds *x*-coordinates of SPs ●[5] finds *y*-coordinates of SPs ●[6] uses relevant method ●[7] establishes nature of stationary points	 ●[1] $\frac{dy}{dx} = \ldots\ldots$ ●[2] ….. $24x^2 - 8x^3$ ●[3] ….. = 0 ●[4] 0 and 3 ●[5] 0 and 54 ●[6] any acceptable method ●[7] pt. of inflexion at (0, 0) maximum at (3, 54)
(c)	**ans: graph drawn** **(2 marks)** ●[1] shape of graph correct ●[2] relevant points marked	●[1] ●[2] 
7(a)	**ans: 9** **(1 mark)** ●[1] knows how to evaluate	 ●[1] 9
(b)	**ans: proof** **(2 marks)** ●[1] substitutes correctly ●[2] rearranges to answer	 ●[1] $g(g(x)) = \dfrac{\dfrac{x-8}{x} - 8}{\dfrac{x-8}{x}}$ ●[2] any relevant method
(c)	**ans: *x* = 8** **(1 mark)**	

●[1]	answer	●[1]	$x = 8$

8(a) ans: function is stationary (5 marks)

●[1]	knows to differentiate	●[1]	$f'(x)$......
●[2]	differentiates 1st term correctly	●[2]	$3\cos 3x$……….
●[3]	differentiates 2nd term correctly	●[3]	………. $-2\cos x \sin x$
●[4]	knows to substitute $\frac{\pi}{2}$ and evaluate	●[4]	$3\cos 3(\frac{\pi}{2}) - 2\cos\frac{\pi}{2}\sin\frac{\pi}{2}$
●[5]	correct statement	●[5]	function is stationary when $x = \frac{\pi}{2}$

(b) ans: $-\frac{9}{32}$ (5 marks)

●[1]	prepares to differentiate	●[1]	$f(x) = 3(1+3x)^{-2}$
●[2]	starts to differentiate	●[2]	$f'(x) = -6(1+3x)^{-3}$.....
●[3]	completes differentiating	●[3]	…….. $\times 3 = \dfrac{-18}{(1+3x)^3}$
●[4]	substitutes into derivative	●[4]	$f'(1) = \dfrac{-18}{(1+3(1))^3}$
●[5]	evaluates	●[5]	$= \dfrac{-18}{64} = -\dfrac{9}{32}$

Total: 50 marks