

Homework 10 solutions

$$1) \quad x = a \sin \omega t$$

$$\dot{x} = a\omega \cos \omega t \Rightarrow v_{\max} = a\omega$$

$$\ddot{x} = -a\omega^2 \sin \omega t \Rightarrow \ddot{x}_{\max} = a\omega^2$$

$$v_{\max} = a\omega$$

$$4 = a\omega$$

$$\ddot{x}_{\max} = a\omega^2$$

$$1 = a\omega^2$$

$$\frac{a\omega}{a\omega^2} = \frac{4}{1}$$

$$\frac{1}{\omega} = 4$$

$$\omega = \frac{1}{4}$$

$$\omega = \frac{2\pi}{T}$$

$$T = \frac{2\pi}{\omega}$$

$$T = 8\pi \text{ sec}$$

$$(T = 25.1 \text{ sec})$$

$$2) \quad a = 0.25 \text{ m}$$

$$\frac{1}{2} v_{\max} = \frac{1}{8} \omega$$

$$v_{\max} = \omega a$$

$$v_{\max} = 0.25\omega$$

$$v^2 = \omega^2 (a^2 - x^2)$$

$$\left(\frac{1}{8}\omega\right)^2 = \omega^2 (0.25^2 - x^2)$$

$$\frac{1}{64} = \frac{1}{16} - x^2$$

$$x^2 = \frac{3}{64}$$

$$x = \pm \frac{\sqrt{3}}{8} \text{ m} \quad (x = \pm 0.217 \text{ m})$$

so distance is 0.217 m



$$T = 0.6 \text{ seconds}$$

projected with speed $\frac{\pi}{5} \text{ ms}^{-1} \Rightarrow v_{\text{max}} = \frac{\pi}{5} \text{ ms}^{-1}$ ✓

$$v_{\text{max}} = \omega a$$

$$\omega = \frac{2\pi}{T}$$

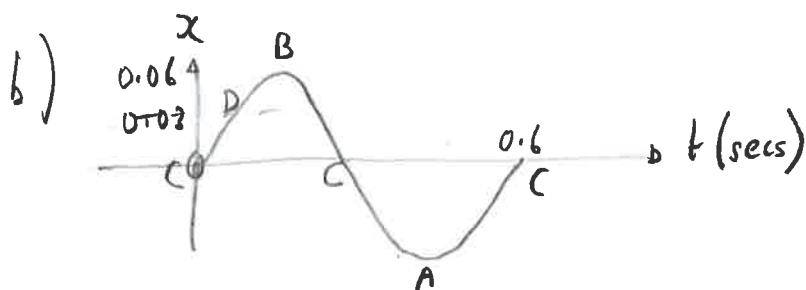
$$\frac{\pi}{5} = \frac{10\pi}{3} a$$

$$\omega = \frac{2\pi}{0.6}$$

$$a = 0.06 \text{ m}$$

$$\omega = \frac{10\pi}{3} \text{ ✓}$$

$$\Rightarrow \text{length of AB} = 2 \times 0.06 \text{ m} = \underline{\underline{0.12 \text{ m}}} \text{ ✓}$$



$$x = 0.06 \sin \frac{10\pi}{3} t$$

at D $x = 0.03 \Rightarrow$

$$0.06 \sin \frac{10\pi}{3} t = 0.03$$

$$\sin \frac{10\pi}{3} t = 0.5 \text{ ✓}$$

$$\frac{10\pi}{3} t = \frac{\pi}{6}$$

$$t = \underline{\underline{0.05 \text{ seconds}}} \text{ ✓}$$

$$4) \quad v_{\max} = 0.13 = \omega a \quad \checkmark \quad \omega a = 0.13$$

$$\Rightarrow \omega^2 a^2 = 0.13^2$$

$$\text{When } x = 0.15 \text{ m, } v = 0.12 \text{ ms}^{-1}$$

$$v^2 = \omega^2 (a^2 - x^2)$$

$$v^2 = \omega^2 a^2 - \omega^2 x^2$$

$$0.12^2 = 0.13^2 - \omega^2 \times 0.15^2 \quad \checkmark$$

$$0.15^2 \omega^2 = 0.0025$$

$$\underline{\omega = \frac{1}{3}} \quad \checkmark$$

$$T = \frac{2\pi}{\omega}$$

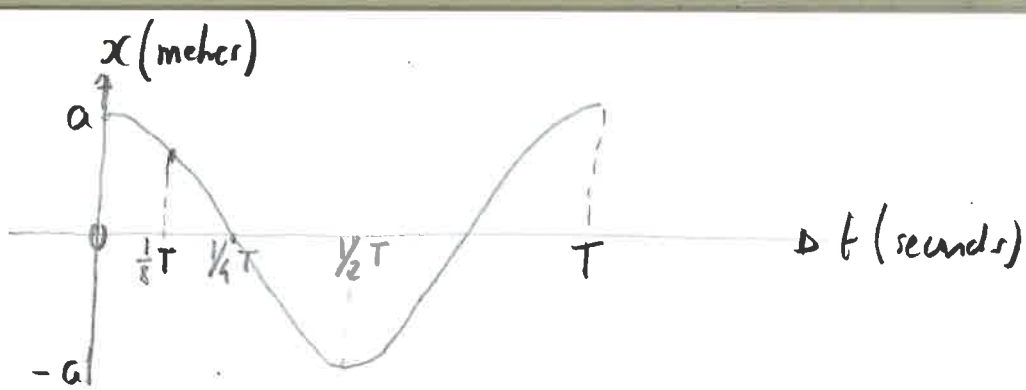
$$T = \frac{2\pi}{1/3} = \underline{6\pi \text{ sec}} \quad \checkmark \quad (T = 18.8 \text{ sec})$$

$$v_{\max} = \omega a$$

$$\frac{1}{3} a = 0.13$$

$$\underline{a = 0.39 \text{ m}} \quad \checkmark$$

5)



$$x = a \cos \omega t$$

$$\omega = \frac{2\pi}{T}$$

$$\text{and } t = \frac{1}{8}T$$

$$x = a \cos \left[\left(\frac{2\pi}{T} \right) \times \frac{1}{8}T \right]$$

$$x = a \cos \left(\frac{\pi}{4} \right)$$

$$\underline{\underline{x = \frac{\sqrt{2}}{2} a}}$$

$$v = -a\omega \sin \omega t$$

$$\ddot{x} = -a\omega^2 \cos \omega t$$

$$2 = -a\omega \sin \left(\frac{\pi}{4} \right) \quad \checkmark$$

$$8 = -a\omega^2 \cos \left(\frac{\pi}{4} \right) \quad \checkmark$$

$$a\omega \sin \left(\frac{\pi}{4} \right) = -2 \quad \checkmark$$

$$a\omega^2 \cos \left(\frac{\pi}{4} \right) = -8 \quad \checkmark$$

$$\frac{\tan \left(\frac{\pi}{4} \right)}{\omega} = \frac{1}{4}$$

$$\omega = 4 \quad \checkmark$$

$$T = \frac{2\pi}{\omega}$$

$$T = \frac{\pi}{2} \text{ seconds} \quad \checkmark$$

$$(T = 1.57 \text{ seconds})$$

$$a\omega \sin \left(\frac{\pi}{4} \right) = -2$$

$$4a \sin \left(\frac{\pi}{4} \right) = -2 \Rightarrow 2.828a = -2$$

$$a = -0.707$$

$$\text{so amplitude} = \underline{\underline{0.707 \text{ m}}} \quad \checkmark$$