Homework 3 solutions

1a) $a_{Lift} = \frac{1}{8} g\,j$

$V_L = \frac{1}{8} g t\,j + C$ at $t=0$ $V_L = 0 \Rightarrow C = 0$

$V_L = \frac{1}{8} g t\,j$

$r_L = \frac{1}{16} g t^2 j + C$ at $t=0$ $r_L = 0 \Rightarrow C = 0$

$r_L = \frac{1}{16} g t^2 j$

$a_{bulb} = -g\,j$

$V_b = -g t\,j + C$ at $t=0$ $V_b = 0 \Rightarrow C = 0$

$V_b = -g t\,j$

$r_b = -\frac{1}{2} g t^2 j + C$ at $t=0$ $r_b = 2j \Rightarrow C = 2j$

$r_b = (2 - \frac{1}{2} g t^2) j$

$br_L = (2 - \frac{1}{2} g t^2) j - \frac{1}{16} g t^2 j$

$br_L = (2 - \frac{9}{16} g t^2) j$

b) when the bulb hits the floor $br_L = 0$

$$2 - \frac{9}{16} g t^2 = 0$$

$$\frac{9}{16} g t^2 = 2$$

$$t^2 = 0.36$$

$$t = 0.60$$

at $t = 0.6$ $r_b = (2 - \frac{1}{2} g \times 0.36) j$

$= 0.22\text{m}$

so $2 - 0.22$

$= 1.78\text{m}$ fallen

2) $r_p = t^2\underline{i} + 4t\underline{j}$

$\underline{v_p = 2t\underline{i} + 4\underline{j}}$

$$a_Q = 2\underline{i} + (4\pi \sin 2\pi t)\underline{j}$$

$$v_Q = 2t\underline{i} - \frac{4\pi}{2\pi}\cos 2\pi t\,\underline{j} + C$$

$$v_Q = 2t\underline{i} - 2\cos 2\pi t\,\underline{j} + C$$

at $t=0$ $v_Q = 0$

$$0 = -2\underline{j} + C$$

$$C = 2\underline{j}$$

$$\underline{v_Q = 2t\underline{i} + (2 - 2\cos 2\pi t)\underline{j}}$$

velocities will be the same when

$$4 = 2 - 2\cos 2\pi t$$

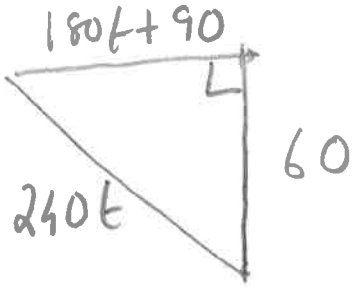
$$2\cos 2\pi t = -2$$

$$\cos 2\pi t = -1$$

$$2\pi t = \pi, 3\pi$$

$$\underline{t = 1/2, 3/2 \text{ secs}}$$

(3)



collision when $(180t+90)^2 + 60^2 = (240t)^2$

$32400t^2 + 32400t + 8100 + 3600 = 57600t^2$

$25200t^2 - 32400t - 11700 = 0 \quad \div 300$

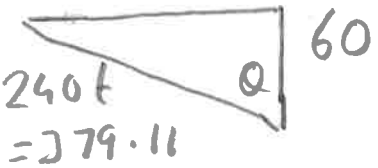
$84t^2 - 108t - 39 = 0$

$t = \dfrac{108 \pm \sqrt{24768}}{168}$

$t = 1.58$ hrs

collision 1hr 35mins so at <u>1.35pm</u>

b)



$\cos\theta = \dfrac{60}{379.11}$

$\theta = 80.9°$

bearing $360 - 80.9 = $ <u>279.1°</u>

4) $V_p = 300\underline{i} + 400\underline{j}$ ← moved half of this velocity

at 9am position is $-100\underline{i} + 250\underline{j}$

so at 9.30am position is $(-100\underline{i} + 150\underline{i}) + (250\underline{j} + 200\underline{j})$

initial position $r_p = 50\underline{i} + 450\underline{j}$

$V_p = 300\underline{i} + 400\underline{j}$

$r_p = 300t\underline{i} + 400t\underline{j} + C$

at $t=0$ $r_p = 50\underline{i} + 450\underline{j} = C$

$r_p = (300t + 50)\underline{i} + (400t + 450)\underline{j}$

$V_m = 600\underline{i} + 500\underline{j}$

$r_m = 600t\underline{i} + 500t\underline{j} + C$

at $t=0$ $r_m = -100\underline{i} + 400\underline{j} = C$

$r_m = (600t - 100)\underline{i} + (500t + 400)\underline{j}$

collision if $\underline{i}$ and $\underline{j}$ components are equal at the same time

equate $\underline{i}$

$300t + 50 = 600t - 100$

$300t = 150$

$t = 0.5$

equate $\underline{j}$

$400t + 450 = 500t + 400$

$100t = 50$

$t = 0.5$

$\therefore$ collide

5) $\underline{r_P = (t^2+3)\underline{i} + 4t\underline{j}}$

$v_P = 2t\underline{i} + 4\underline{j}$

$a_P = 2\underline{i}$ $\quad \therefore \quad a_Q = 2\underline{i}$

$v_Q = 2t\underline{i} + C$

at $t=0$ $v_Q = -4\underline{i} + \underline{j} = C$

$v_Q = (2t-4)\underline{i} + \underline{j}$

$r_Q = (t^2-4t)\underline{i} + t\underline{j} + C$

at $t=0$ $r_Q = 8\underline{j} = C$

$\underline{r_Q = (t^2-4t)\underline{i} + (t+8)\underline{j}}$

b) $_P r_Q = (t^2+3-(t^2-4t))\underline{i} + (4t-(t+8))\underline{j}$

$_P r_Q = (3+4t)\underline{i} + (3t-8)\underline{j}$

$|_P r_Q|^2 = (3+4t)^2 + (3t-8)^2$

$|_P r_Q|^2 = 9+24t+16t^2+9t^2-48t+64$

$= 25t^2-24t+73$

$\frac{d|_P r_Q|^2}{dt} = 50t-24 = 0$ at min

$\underline{t = 0.48 \text{ secs}}$

c) $v_P = 2t\underline{i} + 4\underline{j}$ $\quad v_Q = (2t-4)\underline{i} + \underline{j}$

moving at right angles when $v_P \cdot v_Q = 0$

$$2t(2t-4) + 4 \times 1 = 0$$
$$4t^2 - 8t + 4 = 0$$
$$t^2 - 2t + 1 = 0$$
$$(t-1)(t-1) = 0$$
$$\underline{t = 1 \text{ second}}$$