Homework 3

1) A lift is initially at rest at ground level. It begins to accelerate upwards at $\frac{1}{8}g$ m s^{-2}. At the same instant, a light bulb in the ceiling of the lift begins to fall towards the lift floor. The initial distance between the lift floor and the light bulb is 2 metres.

(*a*) Measuring distances in metres relative to the ground level, show that the position of the light bulb relative to the lift floor is

$$\left(2-\frac{9}{16}gt^2\right)\mathbf{j},$$

where $\mathbf{j}$ is the unit vector in the upward vertical direction, and t is the time in seconds from the start of the motion of the lift. 3

(*b*) Calculate the distance the light bulb falls before hitting the lift floor. 3

2) The position of a model boat P, relative to a rectangular coordinate system with origin O, is given by

$$\mathbf{r}_P = t^2\mathbf{i} + 4t\mathbf{j}$$

where $\mathbf{i}$ and $\mathbf{j}$ are unit vectors in the Ox and Oy directions respectively, t is the time measured in seconds and distances are measured in metres.

The acceleration of a second boat Q is given by

$$\mathbf{a}_Q = 2\mathbf{i} + (4\pi \sin 2\pi t)\mathbf{j}.$$

Given that boat Q is initially at rest, find the first two times when the boats have the same velocity. 5

3) At 12 noon, an aircraft is above a point A and is flying due West at a uniform speed of 180 km h^{-1}. Thirty minutes later, a second aircraft, which is flying at exactly the same height as the first with a uniform speed of 240 km h^{-1}, is 60 km due south of A. The aircraft are on a collision course.

(*a*) Calculate the time when the collision would take place if no evasive action were taken. 4

(*b*) Calculate the bearing on which the second aircraft is travelling. 2

4) Relative to a rectangular coordinate system with origin O the position vector of a passenger aircraft is $-100\mathbf{i} + 250\mathbf{j}$, at 09.00 hours, where $\mathbf{i}$ and $\mathbf{j}$ are unit vectors in the Ox and Oy directions. The aircraft is travelling with uniform velocity $300\mathbf{i} + 400\mathbf{j}$.

Relative to the same coordinate system, a military aircraft travelling with uniform velocity $600\mathbf{i} + 500\mathbf{j}$, has position vector $-100\mathbf{i} + 400\mathbf{j}$ at 09.30 hours. In these expressions, the distances are measured in kilometres and speeds in kilometres per hour.

Show that the two aircraft are on a collision course. 5

5) A particle P is projected so that its position vector is given by $(t^2 + 3)\mathbf{i} + 4t\mathbf{j}$. The time is measured in seconds, distances are measured in metres and $\mathbf{i}$, $\mathbf{j}$ are the unit vectors in the directions of rectangular axes Ox and Oy respectively. A second particle Q has the same acceleration as the particle P and, at time $t = 0$, the particle Q has velocity $(-4\mathbf{i} + \mathbf{j})$ and position vector $8\mathbf{j}$.

Find:

(a) an expression, in terms of t, for the position vector of Q; 3

(b) the time taken from the start of the motion until the particles are closest to each other; 4

(c) the time at which the particles are moving at right angles to each other. 3