Homework 4

1) $x = ut\cos\theta$ $\quad y = ut\sin\theta - \frac{1}{2}gt^2$

at $t = 3$ $x = 70$ $u = 30$

$70 = 90\cos\theta$ ✓

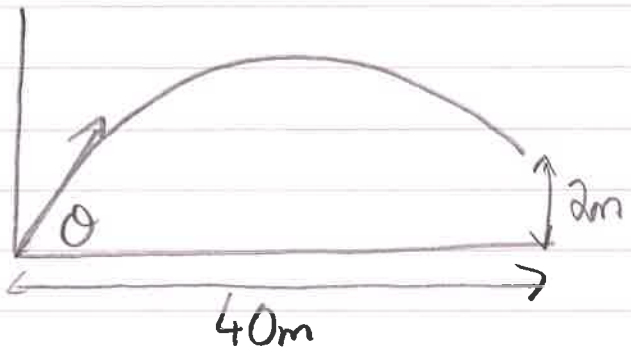
$\cos\theta = 70/90$

$\theta = 38.9°$ ✓

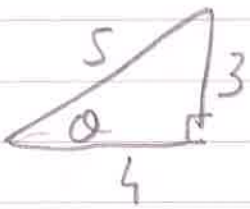
$y = 30 \times 3 \times \sin 38.9 - \frac{1}{2} \times 9.8 \times 3^2$ ✓

$y = 12.5\text{m}$ ✓ (4)

2)



$\tan\theta = 3/4$



$x = ut\cos\theta$ $\quad y = ut\sin\theta - \frac{1}{2}gt^2$

$40 = ut \times \frac{4}{5}$ ✓ $\quad 2 = ut \times \frac{3}{5} - \frac{1}{2}gt^2$

$ut = 50$ ✓ $\Rightarrow$ $2 = 50 \times \frac{3}{5} - \frac{1}{2} \times 9.8t^2$ ✓

$4.9t^2 = 28$

$t = 2.39$ ✓

$ut = 50$

$\Rightarrow u = 50/2.39$

$u = 20.9\text{ms}^{-1}$ ✓ (5)

3) $R = \frac{u^2 \sin 2\theta}{g}$ $\quad H = \frac{u^2 \sin^2 \theta}{2g}$

$R = 10 \times H$

$\Rightarrow \quad \frac{u^2 \sin 2\theta}{g} = \frac{10 u^2 \sin^2 \theta}{2g}$ ✓✓

$\sin 2\theta = 5 \sin^2 \theta$

$5 \sin^2 \theta - \sin 2\theta = 0$

$5 \sin^2 \theta - 2 \sin \theta \cos \theta = 0$

$\sin \theta (5 \sin \theta - 2 \cos \theta) = 0$ ✓

$\Rightarrow \quad \sin \theta = 0$ $\qquad 5 \sin \theta - 2 \cos \theta = 0$

$5 \sin \theta = 2 \cos \theta$

$\frac{\sin \theta}{\cos \theta} = \frac{2}{5}$ ✓

$\tan \theta = 2/5$

$\theta = 21.8°$ ✓

(5)

$$\underline{v} = u\cos\alpha\,\underline{i} + (u\sin\alpha - gt)\underline{j}$$

4) $x = ut\cos\alpha \qquad y = ut\sin\alpha - \frac{1}{2}gt^2$ ✓

at max height the vertical component of the velocity $= 0$

$$\Rightarrow \quad u\sin\alpha - gt = 0$$

$$t = \frac{u\sin\alpha}{g} \checkmark$$

so for range $T = 2 \times \frac{u\sin\alpha}{g}$

$$T = \frac{2u\sin\alpha}{g} \checkmark$$

and $R = ut\cos\alpha$ when $t = \frac{2u\sin\alpha}{g}$

$$R = u\left(\frac{2u\sin\alpha}{g}\right)\cos\alpha \checkmark$$

$$R = \frac{2u^2\sin\alpha\cos\alpha}{g}$$

$$\Rightarrow \quad R = \frac{u^2\sin 2\alpha}{g}$$

(4)

b) if ball lands between L and 2L then

$$L < R < 2L$$

at $\alpha = 15°$

$$R = \frac{v^2}{g} \sin 2x15$$

$$R = \frac{v^2}{g} \times \sin 30$$

$$R = \frac{v^2}{2g} \checkmark$$

so

$$L < \frac{v^2}{2g} < 2L \checkmark$$

$$1 < \frac{v^2}{2gL} < 2$$

$$2 < \frac{v^2}{gL} < 4 \checkmark$$

$$\sqrt{2} < \frac{v}{\sqrt{gL}} < 2$$

(3)

5) $x = ut\cos\theta$ $\quad y = ut\sin\theta - \frac{1}{2}gt^2$

$x = 3H \quad y = H \quad \theta = 30°$

$ut\cos 30 = 3H$ ✓ $\quad ut\sin 30 - \frac{1}{2}gt^2 = H$ ✓

$ut \times \frac{\sqrt{3}}{2} = 3H$ $\quad \frac{1}{2}ut - \frac{1}{2}gt^2 = H$

$\sqrt{3}ut = 6H$ $\quad$ (2) $- ut - gt^2 = 2H$

(1) $- \quad ut = 2\sqrt{3}H$

sub (1) into (2) $\quad 2\sqrt{3}H - gt^2 = 2H$

also from (1) $t = \frac{2\sqrt{3}H}{u}$ ✓

$$2\sqrt{3}H - g\left(\frac{2\sqrt{3}H}{u}\right)^2 = 2H$$

$$2\sqrt{3}H - \frac{12H^2g}{u^2} = 2H$$ ✓

$$2\sqrt{3} - \frac{12Hg}{u^2} = 2$$

$$\frac{12Hg}{u^2} = 2\sqrt{3} - 2$$

$$\frac{6Hg}{u^2} = \sqrt{3} - 1$$ ✓

$$\Rightarrow \quad u^2 = \frac{6gH}{\sqrt{3}-1}$$ ✓ $$\Rightarrow \quad u > \sqrt{\frac{6gH}{\sqrt{3}-1}}$$ ✓ to get over the wall