# Homework 5 Solutions

(1) **The lift**

$a_L = -\frac{1}{9}g\,j$

$v_L = -\frac{1}{9}gt\,j + C$

starts from rest $\Rightarrow C = 0$

$v_L = -\frac{1}{9}gt\,j$ ✓

$s_L = -\frac{1}{18}gt^2 j + C$

take initial position $= 0 \Rightarrow C = 0$

$s_L = -\frac{1}{18}gt^2 j$ ✓

**The ball**

$a_B = -g\,j$

$v_B = -gt\,j + C$

at $t = 0$ $v = 3.5 \Rightarrow C = 3.5$

$v_B = -gt\,j + 3.5\,j$ ✓

$s_B = -\frac{1}{2}gt^2 j + 3.5t\,j + C$

initial position $= 1 \Rightarrow C = 1$

$s_B = \left(1 - \frac{1}{2}gt^2 + 3.5t\right)j$ ✓

Ball and floor of lift collide when $s_L = s_B$

$$-\frac{1}{18}gt^2 = 1 - \frac{1}{2}gt^2 + 3.5t$$ ✓

$$\frac{4}{9}gt^2 - 3.5t - 1 = 0$$

$$4.356t^2 - 3.5t - 1 = 0$$

$$t = \frac{3.5 \pm \sqrt{29.67}}{8.712}$$ ✓

$t = 1.03, -0.213$

so collision occurs at $t = 1.03$ seconds ✓

② car

$V_c = a\underline{j}$

$S_c = at\underline{j} + C$

at $t=0$ position $= -40\underline{j}$

$\Rightarrow \underline{S_c = (at-40)\underline{j}}$ ✓

motorcycle

$V_M = 20\underline{i}$

$S_M = 20t\underline{i} + C$

at $t=0$ position $= 0$

$\Rightarrow \underline{S_M = 20t\underline{i}}$ ✓

$MS_c = 20t\underline{i} - (at-40)\underline{j}$ ✓

$|MS_c|^2 = (20t)^2 + (at-40)^2$ ✓

$|MS_c|^2 = 400t^2 + a^2t^2 - 80at + 1600$

$|MS_c|^2 = 5a^2t^2 - 80at + 1600$

$\frac{d|MS_c|^2}{dt} = 10a^2t - 80a$ ✓

at minimum distance $\frac{d|MS_c|^2}{dt} = 0$

$10a^2t - 80a = 0$

$a^2t - 8a = 0$

$a^2t = 8a$

$t = 8/a$ ✓

$|MS_c|^2 = 5a^2t^2 - 80at + 1600$

$= 5a^2(8/a)^2 - 80a(8/a) + 1600$ ✓

$= 1280 \Rightarrow$ min distance $= \sqrt{1280} = \underline{35.8 \text{ metres}}$

(3)

a) $s = (vt\cos\theta)\underline{i} + (vt\sin\theta - \frac{1}{2}gt^2)\underline{j}$ ✓

$x = vt\cos\theta \qquad y = vt\sin\theta - \frac{1}{2}gt^2$

$t = \frac{x}{v\cos\theta}$ ✓

$\Rightarrow y = x\left(\frac{x}{v\cos\theta}\right)\sin\theta - \frac{1}{2}g\left(\frac{x}{v\cos\theta}\right)^2$ ✓

$y = x\tan\theta - \frac{gx^2}{2v^2\cos^2\theta}$

$\frac{1}{\cos^2\theta} = \sec^2\theta = 1 + \tan^2\theta$

$\Rightarrow y = x\tan\theta - \frac{gx^2}{2v^2}(1+\tan^2\theta)$

b) $x = 3h$ $y = h$ and $V = 3\sqrt{\frac{gh}{2}}$

$h = 3h\tan\theta - \frac{g(3h)^2}{2\times\left(3\sqrt{\frac{gh}{2}}\right)^2}(1+\tan^2\theta)$ ✓

$h = 3h\tan\theta - \frac{9h^{1}\cancel{g}}{2\times\cancel{9}\cancel{h}^{1}}(1+\tan^2\theta)$

$h = 3h\tan\theta - h(1+\tan^2\theta)$ ✓

$1 = 3\tan\theta - (1+\tan^2\theta)$

$1 = 3\tan\theta - 1 - \tan^2\theta$

$\tan^2\theta - 3\tan\theta + 2 = 0$ ✓

$(\tan\theta - 2)(\tan\theta - 1) = 0$

$\Rightarrow \tan\theta = 2$ and $\tan\theta = 1$

$\theta = 63.4°$ $\theta = 45°$ ✓

c) $y = x\tan\theta - \frac{gx^2}{2v^2}(1+\tan^2\theta)$

at range $y = 0$ $\tan\theta = 2$ $x = R$

$$0 = 2R - \frac{gR^2}{2\times\left(3\sqrt{\frac{gh}{2}}\right)^2}(1+2^2) \checkmark$$

$$0 = 2R - \frac{gR^2}{2\left(\frac{9gh}{2}\right)} \times 5$$

$$0 = 2R - \frac{5R^2}{9h} \checkmark$$

$$\frac{5R^1}{9h} = 2R$$

$$5R = 18h$$

$$R = \frac{18h}{5} \checkmark$$

(4) $a = \frac{1}{3}(13-2t)$

$a = \frac{13}{3} - \frac{2}{3}t$

$v = \frac{13}{3}t - \frac{1}{3}t^2 + C$

at $t=0$ $v=12 \Rightarrow C=12$

$v = \frac{13}{3}t - \frac{1}{3}t^2 + 12$ ✓

reaches 26 when $\frac{13}{3}t - \frac{1}{3}t^2 + 12 = 26$ $(\times 3)$

$13t - t^2 + 36 = 78$ ✓

$t^2 - 13t + 42 = 0$

$(t-6)(t-7)$

$t = 6, 7$

so first reaches 26 when $t = 6$ secs ✓

b) $v = \frac{13}{3}t - \frac{1}{3}t^2 + 12$

$s = \frac{13}{6}t^2 - \frac{1}{9}t^3 + 12t + C$

initially $s = -40$

so $s = \frac{13}{6}t^2 - \frac{1}{9}t^3 + 12t - 40$ ✓

at $t=6$ $s = 86$m outside built up area ✓