2003 Advanced Higher

Section E (Mechanics 1)

Marks

ONLY candidates doing the course Mathematics 1, 2 and Mechanics 1 should attempt this Section.

Answer all the questions.

Answer these questions in a separate answer book, showing clearly the section chosen.

Where appropriate, candidates should take the magnitude of the acceleration due to gravity as $9{\cdot}8\ \mathrm{m\,s^{-2}}$.

E1. (*a*) A particle moves on a straight line from the origin with initial velocity $U\mathbf{i}\ \mathrm{m\,s^{-1}}$ and uniform acceleration $a\mathbf{i}\ \mathrm{m\,s^{-2}}$, where $\mathbf{i}$ is the unit vector in the direction of motion.

Show, using calculus, that the distance $s(t)$ metres travelled by the particle in time t seconds is given by

$$s(t) = Ut + \tfrac{1}{2}at^2,$$

where t is measured from the start of the motion. **2**

(*b*) A ball is dropped from the top of a building of height H metres. The ball falls vertically from rest to the ground in 6 seconds.

Ignoring the effect of air resistance, calculate the time taken for the ball to reach a point halfway down the building. **2**

E2. An aircraft travels at 210 km/h in still air. The aircraft takes off from airfield A and lands at airfield B, where B is on a bearing of 050° from A.

Find the course the pilot must set in order to reach B if there is a steady wind blowing from the west at 30 km/h. **4**

E3. A car of mass m kg is travelling along a straight road at a constant velocity of $12\mathbf{i}\ \mathrm{m\,s^{-1}}$, where $\mathbf{i}$ is the unit vector in the direction of motion. The driver of the car applies the brakes which produce a retarding force $-2m\left(1+\frac{t}{4}\right)\mathbf{i}$ newtons, where t is the time measured in seconds from the moment that the brakes are applied. The brakes are applied until the car is stationary.

Determine:

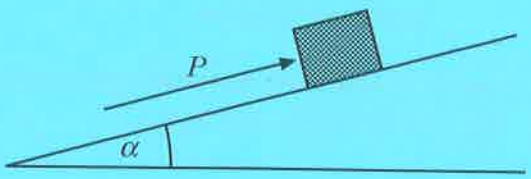
(*a*) the time taken for the car to stop; **5**

(*b*) the stopping distance. **2**

[Turn over

Marks

E4. A block of wood of mass m kg is at rest on a plane inclined at α to the horizontal as shown below, where $\tan\alpha = \frac{3}{4}$. A force of magnitude P newtons acting on the block parallel to the inclined plane, up the line of greatest slope, is just sufficient to prevent the block from sliding **down** the plane. The coefficient of friction between the block and the plane is μ.



(*a*) Show that

$$P = \frac{mg}{5}(3-4\mu),$$

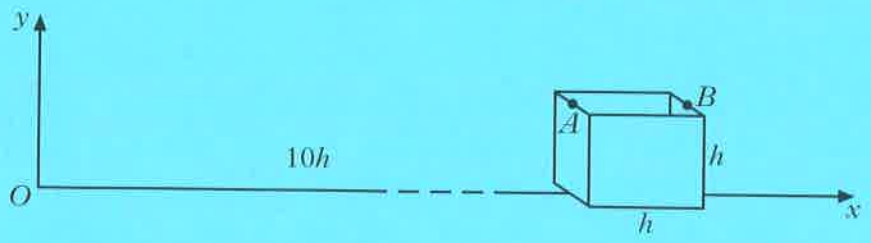
where g m s^{-2} is the magnitude of the acceleration due to gravity. **3**

(*b*) The force acting on the block parallel to the inclined plane is increased to $2P$ newtons and the block is now on the point of moving **up** the plane. Show that

$$P = \frac{mg}{10}(3+4\mu),$$

and hence find the value of μ. **4**

E5. A competition is held at a school gala. The object is to hit a golf ball from a point O on a horizontal playing field directly into an open box situated at a distance $10h$ metres away. The box is a cube with edges h metres long. A and B are the midpoints of the upper edges of the box as shown in the diagram in which AB is in the same plane as the x and y axes.



One of the pupils, Joanna, hits the ball from O, in the vertical plane OAB, imparting a speed of V m s^{-1} to the ball with angle of projection 45°.

(*a*) Using the coordinate system shown in the diagram, show that the equation of the trajectory of the ball is

$$y = x - \frac{gx^2}{V^2},$$

where g m s^{-2} is the magnitude of the acceleration due to gravity. **4**

E5. (continued)

(*b*) Obtain an expression for V, in terms of g and h, for the ball to hit A. **3**

(*c*) Suppose that Joanna succeeds in hitting the ball into the box. Show that the speed of projection satisfies

$$\frac{10}{3} < \frac{V}{\sqrt{gh}} < \frac{11}{\sqrt{10}}.$$ **3**

[END OF SECTION E]

[END OF QUESTION PAPER]