Homework 7 Solutions

1) $\underline{a} = a\underline{i}$

$\underline{v} = at\underline{i} + C$

at $t = 0$ $\underline{v} = U\underline{i} \Rightarrow C = U\underline{i}$

$v = at + U$

$s = \frac{1}{2}at^2 + Ut + C$

at $t = 0$ $s = 0 \Rightarrow C = 0$

$s = \frac{1}{2}at^2 + Ut$

b) H

$s = H$
$t = 6$
$U = 0$
$a = 9.8$

$H = \frac{1}{2} \times 9.8 \times 6^2$

$H = 176.4\text{m}$

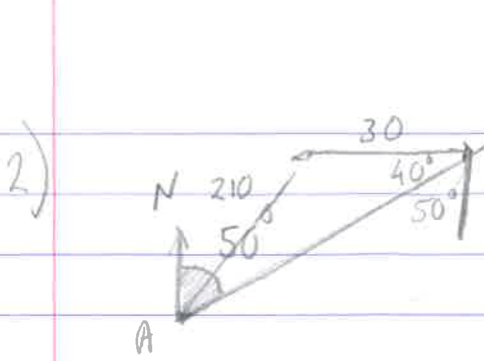
$\frac{1}{2}H = 88.2\text{m}$

$88.2 = \frac{1}{2} \times 9.8t^2$

$t = \sqrt{18}$

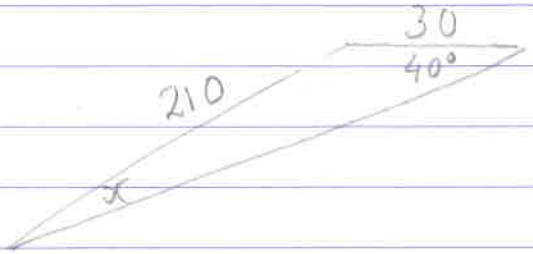
$t = 4.2\text{secs}$

2)



wind blowing from the west.

explanation of values used for second mark.

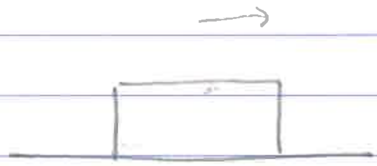


$$\frac{\sin x}{30} = \frac{\sin 40}{210}$$

$$x = 5.3°$$

so bearing $= 50° - 5.3° = 044.7°$

3)



$$ma = -2m\left(1 + \tfrac{t}{4}\right)$$

$$a = -2\left(1 + \tfrac{t}{4}\right)$$

$$a = -2 - \tfrac{t}{2}$$

$$v = -2t - \tfrac{t^2}{4} + c$$

at $t = 0$ $v = 12 \Rightarrow c = 12$

$$v = -2t - \tfrac{t^2}{4} + 12$$

stops when $v = 0$ $\quad \tfrac{t^2}{4} + 2t - 12 = 0$

$$t^2 + 8t - 48 = 0$$

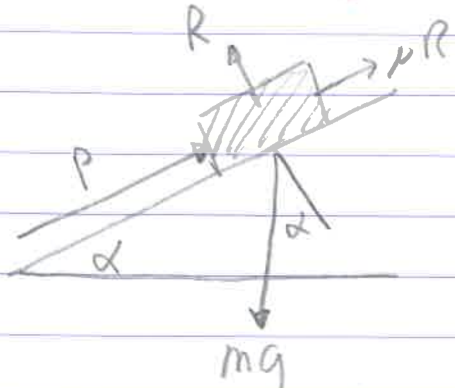
$$(t+12)(t-4) = 0$$

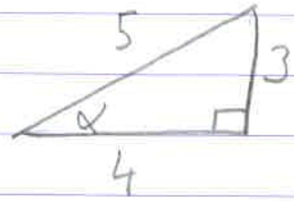
$$t = 4$$

b) $s = -t^2 - \frac{t^3}{12} + 12t$

$t = 4 \Rightarrow s = 26.7\text{m}$

4)





$\sin\alpha = 3/5$

$\cos\alpha = 4/5$

resolve parallel to slope

$P + \mu R = mg\sin\alpha$

$P + \mu mg \times 4/5 = mg \times 3/5$

$5P + 4\mu mg = 3mg$

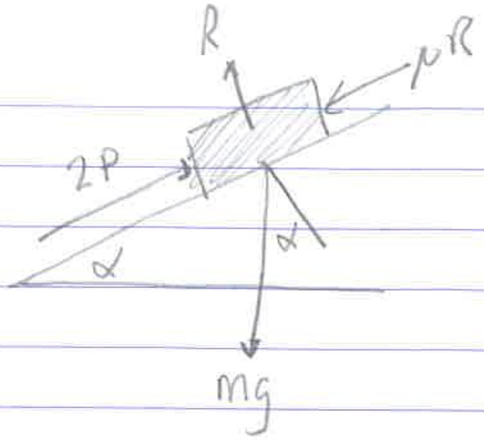
$5P = 3mg - 4\mu mg$

$P = \frac{mg}{5}(3 - 4\mu)$

resolve perpendicular to slope

$R = mg\cos\alpha$

b)



resolve parallel to slope

$2P = \mu R + mg\sin\alpha$ ✓

$2P = \mu mg\cos\alpha + mg\sin\alpha$

$2P = \frac{4}{5}\mu mg + \frac{3}{5}mg$ ✓

$P = \frac{mg}{10}(3+4\mu)$

resolve perpendicular to slope

$R = mg\cos\alpha$

$\frac{mg}{5}(3-4\mu) = \frac{mg}{10}(3+4\mu)$ ✓

$\frac{1}{5}(3-4\mu) = \frac{1}{10}(3+4\mu)$ ×10

$2(3-4\mu) = 3+4\mu$

$6-8\mu = 3+4\mu$

$12\mu = 3$

$\mu = 0.25$ ✓

5) $r = vt\cos\alpha\, \underline{i} + \left(vt\sin\alpha - \frac{1}{2}gt^2\right)\underline{j}$

$x = vt\cos\alpha$ $\qquad$ $y = vt\sin\alpha - \frac{1}{2}gt^2$

$t = \frac{x}{v\cos\alpha} \Rightarrow$ $\qquad$ $y = v\left(\frac{x}{v\cos\alpha}\right)\sin\alpha - \frac{1}{2}g\left(\frac{x}{v\cos\alpha}\right)^2$

$$y = x\tan\alpha - \frac{gx^2}{2v^2\cos^2\alpha}$$

$\alpha = 45°$ $\qquad$ $y = x\tan 45 - \frac{gx^2}{2v^2\cos^2 45}$

$$y = x - \frac{gx^2}{v^2}$$

b) at A $x = 10h$ $y = h$

$$y = x - \frac{gx^2}{v^2}$$

$$h = 10h - \frac{g \times (10h)^2}{v^2}$$

$$\frac{100gh^{\cancel{2}}}{v^2} = 9\cancel{h}$$

$$v^2 = \frac{100gh}{9}$$

$$v = \sqrt{\frac{100gh}{9}}$$

$$v = \frac{10\sqrt{gh}}{3}$$

c) at B $x = 11h$ $y = h$ $\Rightarrow$

$$h = 11h - \frac{g \times (11h)^2}{v^2}$$

$$\frac{121gh^{\cancel{2}}}{v^2} = 10\cancel{h}$$

$$v^2 = \frac{121gh}{10}$$

$$v = \frac{11\sqrt{gh}}{\sqrt{10}}$$

so $\frac{10\sqrt{gh}}{3} < v < \frac{11\sqrt{gh}}{\sqrt{10}}$ to land in the box

or $\frac{10}{3} < \frac{v}{\sqrt{gh}} < \frac{11}{\sqrt{10}}$