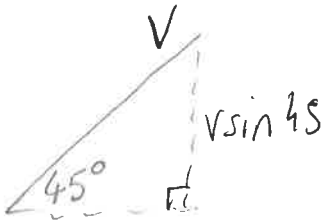


# Homework 8 solutions

(1)

1) a)



$$u = v \sin 45$$

$$v = 0$$

$$t = 5$$

$$v = u + at$$

$$0 = v \sin 45 - 5g \quad \checkmark$$

$$v = \frac{5g}{\sin 45}$$

$$\underline{v = 69.3 \text{ ms}^{-1}} \quad \checkmark$$

b) Total time of flight = 10 sec  $\checkmark$

$$R = (v \sin 45) \times 10 \quad [\text{distance} = \text{speed} \times \text{time}]$$

$$\underline{R = 490 \text{ m}} \quad \checkmark$$

2 a)

$$a_c = a$$

$$v_c = at + c \quad \text{at } t=0 \quad v=0 \Rightarrow c=0$$

$$v_c = at$$

$$s_c = \frac{1}{2} at^2 + c \quad \text{at } t=0 \quad s=0 \Rightarrow c=0$$

$$\underline{s_c = \frac{1}{2} at^2} \quad \checkmark$$

$$a_L = \frac{1}{2} a$$

$$v_L = \frac{1}{2} at + c \quad \text{at } t=0 \quad v=U \Rightarrow c=U$$

$$v_L = \frac{1}{2} at + U$$

$$\underline{s_L = \frac{1}{4} at^2 + Ut} \quad \checkmark$$

draws level when  $s_c = s_k$

$$\frac{1}{2}at^2 = \frac{1}{4}at^2 + Ut \quad \checkmark$$

$$\frac{1}{4}at^2 = Ut$$

$$\frac{1}{4}at^2 - Ut = 0$$

$$t\left(\frac{1}{4}at - U\right) = 0$$

$$t = 0 \quad \text{and}$$

$$\frac{1}{4}at = U \quad \checkmark$$

$$t = \frac{4U}{a} \text{ seconds}$$

b)  $s_c = \frac{1}{2}at^2$

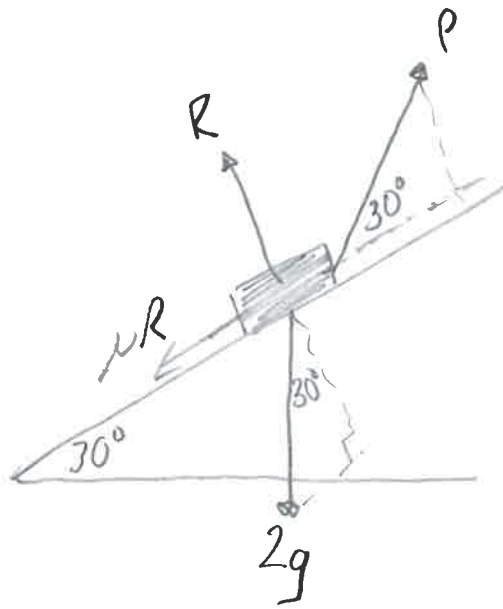
$$s_c = \frac{1}{2}a\left(\frac{4U}{a}\right)^2$$

$$s_c = \frac{16U^2a}{2a^2}$$

$$s_c = \frac{8U^2}{a} \quad \checkmark$$

(3)

3c)



$$R + P \sin 30^\circ = 2g \cos 30^\circ$$

$$R = 2g \cos 30^\circ - P \sin 30^\circ \quad \checkmark$$

$$\text{Frictional force} = \mu R \quad \mu = \frac{1}{2}$$

$$= \frac{1}{2} (2g \cos 30^\circ - P \sin 30^\circ) \quad \checkmark$$

$$= \frac{1}{2} \left( \sqrt{3}g - \frac{1}{2}P \right)$$

$$\mu R = \frac{1}{4} (2\sqrt{3}g - P) \quad \checkmark$$

$$b) \quad \mu R + 2g \sin 30^\circ = P \cos 30^\circ \quad \checkmark \quad [\text{resolving in direction of slope}]$$

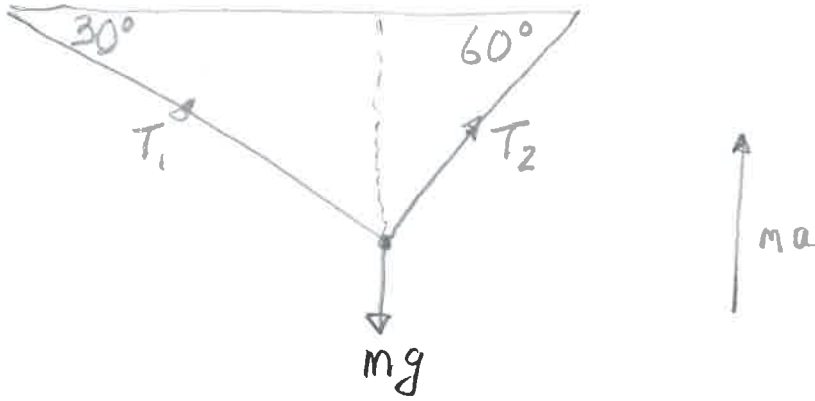
$$\frac{1}{4} (2\sqrt{3}g - P) + 2g \sin 30^\circ = P \cos 30^\circ$$

$$\frac{\sqrt{3}}{2}g - \frac{1}{4}P + g = \frac{\sqrt{3}}{2}P$$

$$P \left( \frac{\sqrt{3}}{2} + \frac{1}{4} \right) = \left( \frac{\sqrt{3}}{2} + 1 \right) g$$

$$P = \frac{\left( \frac{\sqrt{3}}{2} + 1 \right) g}{\left( \frac{\sqrt{3}}{2} + \frac{1}{4} \right)} \quad \checkmark = \underline{\underline{16.4N}} \quad \checkmark$$

4a)



$$T_1 \cos 30^\circ = T_2 \cos 60^\circ \checkmark$$

$$T_2 = \frac{\cos 30^\circ}{\cos 60^\circ} T_1$$

$$\underline{T_2 = \sqrt{3} T_1} \checkmark \text{ so } T_2 > T_1$$

$$b) \quad ma = T_1 \sin 30^\circ + T_2 \sin 60^\circ - mg \checkmark$$

$$ma = \frac{1}{\sqrt{3}} T_2 \sin 30^\circ + T_2 \sin 60^\circ - mg \checkmark$$

$$ma + mg = \frac{2\sqrt{3}}{3} T_2 \checkmark$$

$$\underline{T_2 = \frac{\sqrt{3}m(a+g)}{2}} \checkmark$$

5

5a)

$$a_A = -\frac{2}{5}t$$

$$v_A = -\frac{1}{5}t^2 + C \quad \text{at } t=0 \quad v=10 \Rightarrow C=10$$

$$v_A = -\frac{1}{5}t^2 + 10$$

$$s_A = -\frac{1}{15}t^3 + 10t + C \quad \text{at } t=0 \quad s=0 \Rightarrow C=0$$

$$s_A = -\frac{1}{15}t^3 + 10t$$

$$s_A = \frac{1}{15}t(150 - t^2) \underline{i}$$

$$b) \quad s_B = \frac{1}{15}(45 + 75t - t^3) \underline{i} + 4 \underline{j}$$

$$v_B = \frac{1}{15}(75 - 3t^2) \underline{i}$$

comes to rest  $v_B = 0 \Rightarrow 3t^2 = 75$   
 $t^2 = 25$   
 $t = 5 \text{ seconds}$

$$\text{at } t=5 \quad s_B = \frac{59}{3} \underline{i} + 4 \underline{j}$$

$$|s_B| = \sqrt{\left(\frac{59}{3}\right)^2 + 4^2}$$
$$= 20.1 \text{ m}$$

$$\begin{aligned}
 \text{ii) } AS_0 &= S_A - S_B \\
 &= \frac{1}{15}t(150 - t^2)\underline{i} - \left[ \frac{1}{15}(45 + 15t - t^3)\underline{i} + 4\underline{j} \right] \\
 &= \left[ \left( 10t - \frac{1}{15}t^3 \right) - \left( 3 + 5t - \frac{1}{15}t^3 \right) \right] \underline{i} - 4\underline{j} \\
 &= \underline{\underline{[5t - 3] \underline{i} - 4\underline{j}}} \quad \checkmark
 \end{aligned}$$

$$\begin{aligned}
 |AS_0|^2 &= (5t - 3)^2 + 4^2 \\
 &= 25t^2 - 30t + 9 + 16 \\
 &= \underline{\underline{25t^2 - 30t + 25}} \quad \checkmark
 \end{aligned}$$

$$\frac{d|AS_0|^2}{dt} = 50t - 30$$

$$\text{at min } \frac{d|AS_0|^2}{dt} = 0 \Rightarrow 50t - 30 = 0$$

$$\underline{\underline{t = \frac{3}{5}}} \quad \checkmark$$

$$\begin{aligned}
 \text{at } t = \frac{3}{5} \quad |AS_0| &= \sqrt{25t^2 - 30t + 25} \\
 &= \underline{\underline{4m}} \quad \checkmark
 \end{aligned}$$

