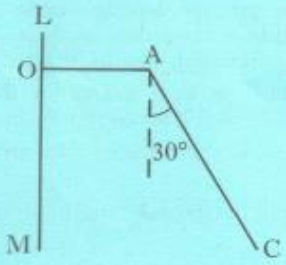
Homework 9

1)

A planet, radius 25 000 kilometres, rotates about its axis once every 14 hours. The magnitude of the gravitational acceleration on the surface of the planet is $10{\cdot}8\,\mathrm{m\,s^{-2}}$. A satellite moves in such a way that it remains at the same height above a fixed point on the equator of the planet. Calculate this height, assuming that Newton's inverse square law of gravitation applies. **6**

2)

At a fairground the "chair o' planes" are small chairs attached by chains 2·5 metres long to the ends of horizontal arms 1·5 metres long, radiating from a central pillar about which the arms rotate. To simplify the model, ignore the weight of the chains and consider a stage in the motion when the horizontal arms are rotating uniformly with the chains making an angle of 30° to the vertical, in the vertical plane containing the horizontal arm, as shown in the diagram.



LM represents the central pillar, OA a horizontal arm and AC the chain attaching a chair to this arm. Assume that the chain AC is in the vertical plane containing the arm OA.

For the particular stage of the motion described above in this simplified model, calculate the speed of the chairs and the time for one complete revolution. **6**

3)

A bend on a racetrack is circular and is banked at an angle α to the horizontal such that a car can take the bend at speed V with no sideways friction between the wheels and the track. Express V in terms of α, r and g, where r is the radius of the bend and g is the magnitude of the acceleration due to gravity. **3**

A car taking the bend at speed v is on the point of skidding outwards. Show that

$$v^2 = \frac{gr(\tan\alpha + \mu)}{1 - \mu\tan\alpha},$$

where μ is the coefficient of friction between the car tyres and the surface of the track. **4**

In dry conditions, $v = 2V$ and $\mu = \frac{3}{4}$. Use the previous results to find the value of $\tan\alpha$. **3**

In wet conditions, the coefficient of friction is reduced to a new value μ', such that a car at rest on the wet track is on the point of slipping **down** the banking. Find the value of μ'. **2**

Show that the maximum speed for taking the bend without skidding in wet conditions is approximately 82% of the maximum speed in dry conditions. **3**