Homework 9 solutions

1)

$a = \frac{K}{r_p^2}$ at surface of planet $a = 10.8 ms^{-2}$, $r_p = 2.5 \times 10^7 m$

$K = a r_p^2$

$K = 10.8 (2.5 \times 10^7)^2$

$K = 6.75 \times 10^{15}$

for satellite $a_s = w^2 r_s$

$a_s = \frac{K}{r_s^2}$

$\Rightarrow \quad w^2 r_s = \frac{K}{r_s^2}$

$r_s^3 = \frac{K}{w^2}$

$r_s = \sqrt[3]{\frac{K}{w^2}}$

$r_s = \sqrt[3]{\frac{6.75 \times 10^{15}}{1.5 \times 10^{-8}}}$

$r_s = 75730071.99 m$

$r_s = 75730 km$

$w = 2\pi / T$

$T = 14$ hours

$= 50400$ secs

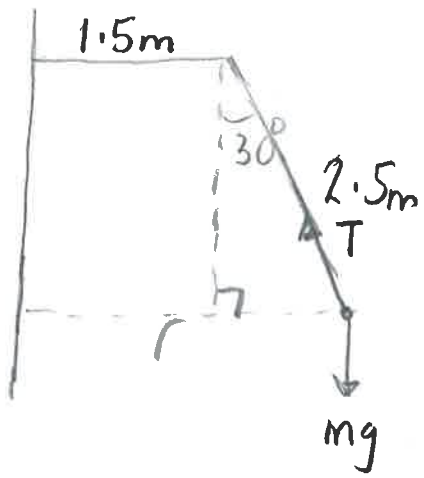
$w = 2\pi / 50400$

$w = 1.25 \times 10^{-4}$

$w^2 = 1.5 \times 10^{-8}$

$\Rightarrow$ distance above surface $= 75730 - 25000 = 50730 km$

2)



$r = 1.5 + 2.5\sin 30°$

$r = 2.75m$

resolve vertically

① $T\cos 30° = mg$

resolve horizontally

② $T\sin 30° = \frac{mv^2}{r}$

② ÷ ① ⇒ $\frac{T\sin 30°}{T\cos 30°} = \frac{mv^2/r}{mg}$

$\tan 30° = \frac{v^2}{gr}$

$v^2 = gr\tan 30°$

$v = \sqrt{9.8 \times 2.75 \tan 30°}$

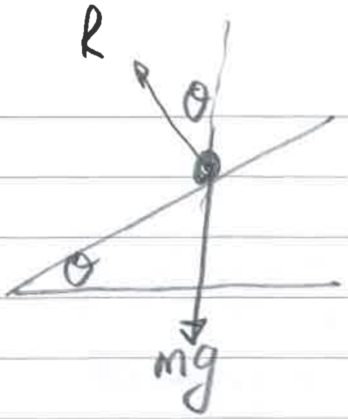
$v = 3.94 ms^{-1}$

$T = \frac{2\pi r}{v}$

$T = \frac{2\pi \times 2.75}{3.94}$

$T = 4.38$ seconds

3)

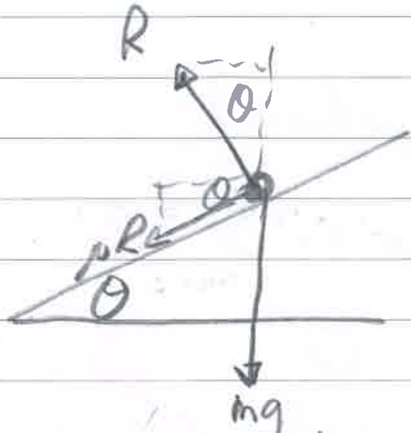


$$R\sin\theta = \frac{mV^2}{r}$$

$$R\cos\theta = mg$$

$$\tan\theta = \frac{v^2}{gr}$$

$$V = \sqrt{gr\tan\theta}$$



resolve horizontally

$$R\sin\theta + \mu R\cos\theta = \frac{mv^2}{r}$$

resolve vertically

$$R\cos\theta = mg + \mu R\sin\theta$$

$$R\sin\theta + \mu R\cos\theta = \frac{mv^2}{r}$$

$$R\cos\theta - \mu R\sin\theta = mg$$

$$\frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta} = \frac{v^2}{gr}$$ ÷ through by $\cos\theta$

$$\frac{\tan\theta + \mu}{1 - \mu\tan\theta} = \frac{v^2}{gr}$$

$$v^2 = \frac{gr(\tan\theta + \mu)}{1 - \mu\tan\theta}$$

$v = 2V$

$V = \sqrt{gr\tan\theta}$

or $v^2 = 4V^2$

$V^2 = gr\tan\theta$

$$\frac{gr(\tan\theta + \mu)}{1 - \mu\tan\theta} = 4gr\tan\theta$$

$\tan\theta + \mu = 4\tan\theta(1 - \mu\tan\theta)$

$\tan\theta + \mu = 4\tan\theta - 4\mu\tan^2\theta$

$4\mu\tan^2\theta - 3\tan\theta + \mu = 0$ $\qquad \mu = 3/4$

$\Rightarrow 3\tan^2\theta - 3\tan\theta + 3/4 = 0$

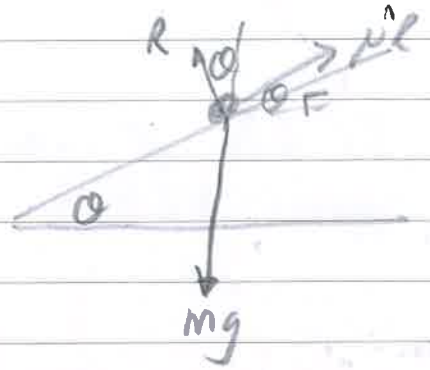
$12\tan^2\theta - 12\tan\theta + 3 = 0$

$4\tan^2\theta - 4\tan\theta + 1 = 0$

$(2\tan\theta - 1)(2\tan\theta - 1) = 0$

$\Rightarrow \underline{\underline{\tan\theta = 1/2}}$

For wet conditions



$R\cos\theta + \mu' R\sin\theta = mg$ $\qquad$ $R\sin\theta - \mu' R\cos\theta = \frac{mv^2}{r}$

$$\frac{R\sin\theta - \mu' R\cos\theta}{R\cos\theta + \mu' R\sin\theta} = \frac{mv^2/r}{mg}$$

$$\frac{\tan\theta - \mu'}{1+\mu'\tan\theta} = \frac{v^2}{gr} \qquad v = 0$$

$$\frac{\tan\theta - \mu'}{1+\mu'\tan\theta} = 0 \checkmark$$

$$\tan\theta - \mu' = 0$$

$$\mu' = \tan\theta$$

$$\mu' = 1/2 \checkmark$$

in dry conditions max speed (call it v_1)

$$v_1 = 2V = 2\sqrt{gr\tan\theta} \checkmark \qquad \tan\theta = 1/2$$

$$= \sqrt{2}\times\sqrt{gr}$$

in wet conditions max speed (call it v_2)

$$V_2 = \sqrt{\frac{gr(\tan\theta + \mu')}{1 - \mu'\tan\theta}} \Rightarrow V_2 = \sqrt{\frac{gr(0.5+0.5)}{1-0.5\times0.5}}$$

$$V_2 = 1.15\sqrt{gr} \checkmark$$

$V_2 = 1.15\sqrt{g}$ ✓

$V_1 = \sqrt{2}\sqrt{g}$ ✓

$V_2/V_1 = 0.816 \Rightarrow$ 81.6% of speed ✓
(approx 82%).