Relationships and Calculus Assessment Standard 1.1

1. Show that $(x + 2)$ is a factor of $f(x) = x^3 - 2x^2 - 4x + 8$ and hence factorise $f(x)$ fully. Hence solve the equation $x^3 - 2x^2 - x - 3 = 3x - 11$.

2. Show that $(x + 2)$ is a factor of $g(x) = x^3 + 4x^2 + x - 6$ and hence factorise $g(x)$ fully. Hence solve the equation $x^3 + 4x^2 + 2x + 1 = x + 7$.

3. Show that $(x - 1)$ is a factor of $f(x) = x^3 + 3x^2 - 4$ and hence factorise $f(x)$ fully. Hence solve the equation $x^3 + 3x^2 + x - 3 = x + 1$.

4. Show that $(x + 3)$ is a factor of $f(x) = x^3 + x^2 - 5x + 3$ and hence factorise $f(x)$ fully. Hence solve the equation $x^3 + x^2 + x - 3 = 6x - 6$.

5. For what values of k does the equation $x^2 - 2x + k = 0$ have

   (a) 2 real, distinct roots (b) equal roots (c) no real roots?

6. Show that the line $y = x + k$ meets the parabola $y = x^2 - 3x$ where $x^2 - 4x - k = 0$. Use the discriminant to find the value of k for which the line is a tangent to the parabola.

7. Show that the line $y = 3x + k$ meets the parabola $y = x^2 + 4$ where $x^2 - 3x + (4 - k) = 0$. Use the discriminant to find the value of k for which the line is a tangent to the parabola.

8. Calculate the range of values of k so that the graph of $y = 4x^2 - kx + 25$ does not cut or touch the x-axis.

9. The tangent line $y = 5x - 3$ meets the curve $y = x^3 + x^2$ at A(1, 2) and at another point B. Show that the tangent line and curve meet where $x^3 + x^2 - 5x + 3 = 0$ and hence find the coordinates of the point B.

10. The tangent line $y = 3x - 2$ meets the curve $y = x^3$ at A(1, 1) and at another point B. Show that the tangent line and curve meet where $x^3 - 3x + 2 = 0$ and hence find the coordinates of the point B.

Relationships and Calculus Assessment Standard 1.1 Answers

1. $f(x) = (x + 2)(x - 2)(x - 2)$ Solution : $x = -2, 2, 2$
2. $g(x) = (x + 2)(x + 3)(x - 1)$ Solution : $x = -2, -3, 1$
3. $f(x) = (x + 2)(x + 2)(x - 1)$ Solution : $x = -2, -2, 1$
4. $f(x) = (x + 3)(x - 1)(x - 1)$ Solution : $x = -3, 1, 1$
5. (a) $k < 1$ (b) $k = 1$ (c) $k > 1$
6. $k = -4$
7. $k = \frac{7}{4}$
8. $-20 < k < 20$
9. B(-3, -18)
10. B(-2, -8)

Relationships and Calculus Assessment Standard 1.4

1. Find $\int 2 + \frac{6}{x^3}\,dx$, where $x \neq 0$.

2. Find $\int \frac{1}{x^3}\,dx$, where $x \neq 0$.

3. Find $\int \frac{3}{x^4} + 1\,dx$, where $x \neq 0$.

4. Find $\int \frac{12}{x^5}\,dx$, where $x \neq 0$.

5. (a) Find $\int \frac{\sqrt{3}}{2} \cos x\,dx$.

   (b) Integrate $3\sin x$ with respect to x.

   (c) Evaluate $\int_4^6 (x-3)^3\,dx$

6. (a) Find $\int \frac{1}{2} \cos x\,dx$.

   (b) Integrate $\sin 4x$ with respect to x.

   (c) Evaluate $\int_2^4 (x-2)^3\,dx$

7. (a) Find $\int 2\,sinx$ dx.

(b) Integrate $\frac{1}{2}$ cosx with respect to x.

(c) Evaluate $\int_1^2 (x+3)^4$ dx

8. (a) Find $\int -3sinx$ dx.

(b) Integrate cos 4x with respect to x.

(c) Evaluate $\int_1^3 (2x+1)^3$ dx

Relationships and Calculus Assessment Standard 1.4 Answers

1.	$2x - 3x^{-2} + c$			
2.	$-\frac{1}{2}x^{-2} + c$			
3.	$-x^{-3} + x + c$			
4.	$-3x^{-4} + c$			
5.	(a)	$\frac{\sqrt{3}}{2}$ sinx + c	(b) -3 cos x + c	(c) 20
6.	(a)	$\frac{1}{2}$ sinx + c	(b) $-\frac{1}{4}$ cos 4x + c	(c) 4
7.	(a)	-2cosx + c	(b) $\frac{1}{2}$ sin x + c	(c) 420.2
8.	(a)	3 cosx + c	(b) $\frac{1}{4}$ sin 4x + c	(c) 290

Relationships and Calculus Assessment Standard 1.2

1. Solve algebraically the equation $\sqrt{2}\sin 2x = 1$ for $0 \leq x < \pi$.

2. Solve algebraically the equation $2\sin 2x = \sqrt{3}$ for $0 \leq x < \pi$.

3. Solve algebraically the equation $\sqrt{2}\cos 2x = 1$ for $0 \leq x < \pi$.

4. Solve algebraically the equation $\sqrt{3}\tan 2x = 1$ for $0 \leq x < \pi$.

5.(a) Express $\sin 15^\circ \cos x^\circ + \cos 15^\circ \sin x^\circ$ in the form $\sin(A + B)^\circ$.

(b) Use your answer from part (a) to solve the equation

$$\sin 15^\circ \cos x^\circ + \cos 15^\circ \sin x^\circ = \frac{\sqrt{3}}{2} \text{ for } 0 < x < 360.$$

6.(a) Express $\cos x^\circ \cos 30^\circ - \sin x^\circ \sin 30^\circ$ in the form $\cos(A + B)^\circ$.

(b) Use your answer from part (a) to solve the equation

$$\cos x^\circ \cos 30^\circ - \sin x^\circ \sin 30^\circ = \frac{1}{4} \text{ for } 0 < x < 360.$$

7.(a) Express $\sin x^\circ \cos 20^\circ - \cos x^\circ \sin 20^\circ$ in the form $\sin(A - B)^\circ$.

(b) Hence solve the equation $\sin x^\circ \cos 20^\circ - \cos x^\circ \sin 20^\circ = \frac{4}{9}$ for $0 < x < 180$.

8. Solve the equation $\sin x^\circ \cos 35^\circ + \cos x^\circ \sin 35^\circ = \frac{7}{11}$ for $0 < x < 180$.

9. Solve the following equations for $0 \le x \le 360$:

(a) $\sin 2x^{o} - \cos x^{o} = 0$

(b) $\sin 2x^{o} - 3\sin x^{o} = 0$

(c) $\cos 2x^{o} + \sin x^{o} = 0$

(d) $\cos 2x^{o} + \cos x^{o} + 1 = 0$

(e) $\cos 2x^{o} + 3\cos x^{o} + 2 = 0$

(f) $\sin x^{o} - 2\cos 2x^{o} = 1$

10. $\sin x + \sqrt{3}\cos x$ can be written as $2\cos(x - \frac{\pi}{6})$.

Solve $5\sin 2x + 5\sqrt{3}\cos 2x = 5$, where $0 < x < \pi$.

11. $\sqrt{3}\sin x^{o} - \cos x^{o}$ can be written as $2\sin(x - 30)^{o}$.

Solve $4 + 5\cos 2x^{o} - 5\sqrt{3}\sin 2x^{o} = -1$, where $0 \le x^{o} \le 90$.

12. $\cos x - \sqrt{3}\sin x$ can be written in the form $2\cos(x + \frac{\pi}{3})$.

Solve $\cos 2x - \sqrt{3}\sin 2x = 1$, $0 \le x \le \pi$

Relationships and Calculus Assessment Standard 1.2 Answers

1. $\frac{\pi}{8}, \frac{3\pi}{8}$

2. $\frac{\pi}{6}, \frac{\pi}{3}$

3. $\frac{\pi}{8}, \frac{7\pi}{8}$

4. $\frac{\pi}{12}, \frac{7\pi}{12}$

5. (a) $\sin(x + 15)^o$ (b) $x = 45^o$ or 105^o

6. (a) $\cos(x + 30)^o$ (b) $x = 45.5^o$ or 254.5^o

7. (a) $\sin(x - 20)^o$ (b) $x = 46.4^o$ or 173.6^o

8. 4.5^o or 105.5^o

9.(a) $\sin 2x - \cos x = 0$

$2 \sin x \cos x - \cos x = 0$

$\cos x\,(2 \sin x - 1) = 0$

$\cos x = 0$ or $2 \sin x - 1 = 0$

$x^o = 30^o, 90^o, 150^o, 270^o$

(b) $x^o = 0^o, 180^o, 360^o$

(c) $x^o = 90^o, 210^o, 330^o$

(d) $x^o = 90^o, 120^o, 240^o, 270^o$

(e) $x^o = 120^o, 180^o, 240^o$

(f) $x^o = 48.6^o, 131.4^o, 270^o$

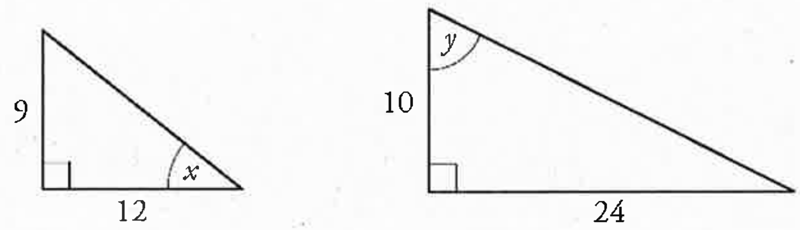
10. $x = \frac{\pi}{4}, \frac{11\pi}{12}$

11. $x^o = 30^o, 90^o$

12. $x = 0, \frac{2\pi}{3}$

Expressions and Functions Assessment Standard 1.2

1. The diagram below shows two right-angled triangles.

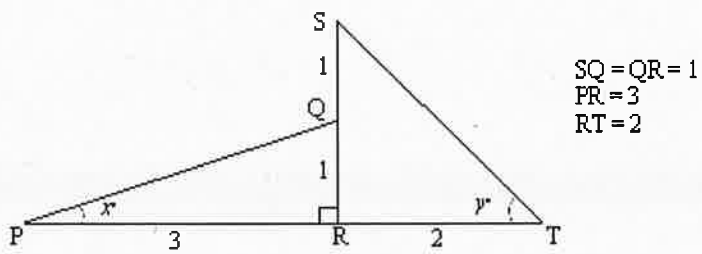


(a) Write down the values of $\sin x^\circ$ and $\cos y^\circ$.

(b) By expanding $\cos(x + y)^\circ$ show that the exact value of $\cos(x + y)^\circ$ is $\frac{-16}{65}$.

2. Express $12\cos x^0 + 5\sin x^\circ$ in the form $k\cos(x - a)^0$ where $k > 0$ and $0 \leq a \leq 360$.

3. The diagram below shows two right-angled triangles PQR and SRT.

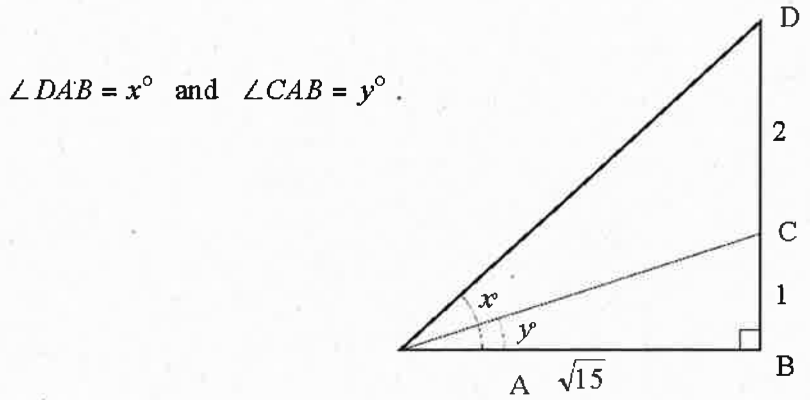


(a) Write down the values of $\cos x^\circ$ and $\sin y^\circ$.

(b) By expanding $\sin(x + y)^\circ$ show that the exact value of $\sin(x + y)^\circ$ is $\frac{8}{\sqrt{80}}$.

4. Express $2\cos x^0 + 5\sin x^\circ$ in the form $k\cos(x - a)^0$ where $k > 0$ and $0 \leq a \leq 360$.

5. The diagram below shows two right-angled triangles ABC and ABD.

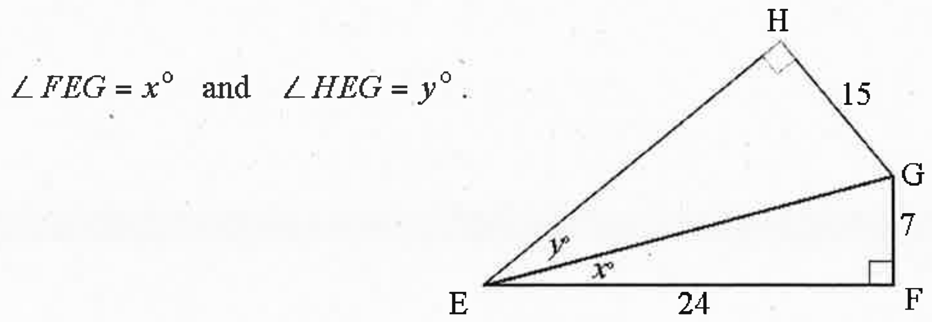


(a) Write down the values of $\cos x^\circ$ and $\sin y^\circ$.

(b) By expanding $\cos(x - y)^\circ$ show that the exact value of $\cos(x - y)^\circ$ is $\frac{18}{4\sqrt{24}}$.

6. Express $4\cos x^\circ + \sin x^\circ$ in the form $k\cos(x - a)^\circ$ where $k > 0$ and $0 \leq a < 360$.

7. The diagram below shows two right-angled triangles EFG and EHG.



(a) Write down the values of $\sin x^\circ$ and $\cos y^\circ$.

(b) By expanding $\cos(x + y)^\circ$ show that the exact value of $\cos(x + y)^\circ$ is $\frac{3}{5}$.

8. Express $7\sin x^\circ + 4\cos x^\circ$ in the form $k\cos(x - a)^\circ$ where $k > 0$ and $0 \leq a \leq 360$.

9. Show that $(\sin A + \cos A)^2 = 1 + \sin 2A$ and hence state the maximum value of $4(\sin A + \cos A)^2$.

10. Show that $\sin^3 x \cos x + \sin x \cos^3 x = \frac{1}{2}\sin 2x$ and hence state the minimum value of $8\sin^3 x \cos x + 8\sin x \cos^3 x$.

11. Show that $(\cos A + \sin A)(\cos A - \sin A) = \cos 2A$ and hence state the maximum value of $5(\cos A + \sin A)(\cos A - \sin A)$.

Expressions and Functions Assessment Standard 1.2 Answers

1.(a) $\sin x = \frac{9}{15} = \frac{3}{5}$, $\cos x = \frac{10}{26} = \frac{5}{13}$ (b) Proof

2. $k = 13$, $a^\circ = 22.6^\circ$

3.(a) $\sin y = \frac{2}{\sqrt{8}}$, $\cos x = \frac{3}{\sqrt{10}}$ (b) Proof

4. $k = \sqrt{29}$, $a^\circ = 68.2^\circ$

5.(a) $\cos x = \frac{\sqrt{15}}{\sqrt{24}}$, $\sin y = \frac{1}{4}$ (b) Proof

6. $k = \sqrt{17}$, $a^\circ = 14.0^\circ$

7.(a) $\sin x = \frac{7}{25}$, $\cos y = \frac{20}{25}$ (b) Proof

8. $k = \sqrt{65}$, $a^\circ = 60.3^\circ$

9. Max value of $4(\sin A + \cos A)^2$ = max value of $4(1 + \sin 2A) = 4(1 + 1) = 8$.

10. Min value of $8\sin^3 x \cos x + 8\sin x \cos^3 x$ = min value of $8(\frac{1}{2}\sin 2x) = 8 \times (-\frac{1}{2}) = -4$.

11. Max value of $5(\cos A + \sin A)(\cos A - \sin A)$ = max value of $5 \cos 2A = 5$.

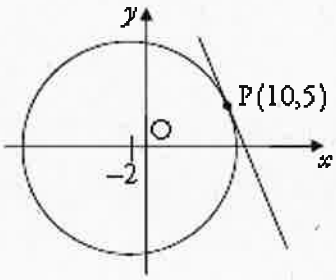
Applications Assessment Standard 1.2

1. (a) A circle has radius 7 units and centre (2, -3). Write down the equation of the circle.

   (b) A circle has equation $x^2 + y^2 - 10x + 6y - 3 = 0$. Write down its radius and the coordinates of its centre.

2. Show that the straight line $y = -2x - 3$ is a tangent to the circle with equation $x^2 + y^2 + 6x + 4y + 8 = 0$.

3. The point P(10, 5) lies on the circle with centre (-2, 0), as shown in the diagram below.



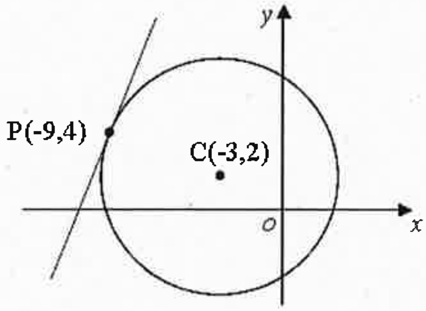
   Find the equation of the tangent to the circle at P.

4. (a) A circle has radius 6 units and centre C(4, -1). Write down the equation of the circle.

   (b) A circle has equation $x^2 + y^2 - 4x + 2y - 4 = 0$. Write down its radius and the coordinates of its centre.

5. Determine if the line $y = 5 - 2x$ is a tangent to the circle with equation $x^2 + y^2 + 6x - 2y - 10 = 0$.

6. The point P(-9, 4) lies on the circle with centre C(-3, 2), as shown in the diagram below.



Find the equation of the tangent to the circle at P.

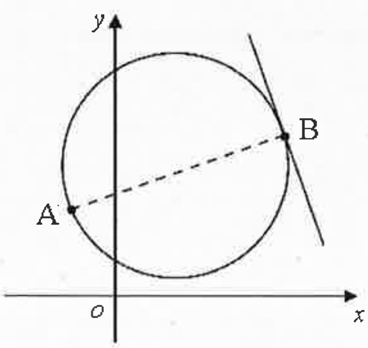
7. (a) A circle has radius 10 units and centre C(5, -2). Write down the equation of the circle.

(b) A circle has equation $x^2 + y^2 - 2x + 10y + 1 = 0$. Write down its radius and the coordinates of its centre.

8. Determine if the line $y = x - 10$ is a tangent to the circle with equation $x^2 + y^2 - 6x + 6y + 10 = 0$.

9. A circle has AB as a diameter, as shown in the diagram. A and B have coordinates (-2, 5) and (10, 8) respectively.

Find the equation of the tangent at B.



10. (a) A circle has a radius of 1 unit and centre $C(-2, 6)$. Write down the equation of the circle.

(b) A circle has equation $x^2 + y^2 - 6x + 5 = 0$. Write down its radius and the coordinates of its centre.

11. Determine if the line $y = 17 - 4x$ is a tangent to the circle with equation $x^2 + y^2 + 8x + 2y - 51 = 0$.

12. A circle has as its centre the point $C(5, 1)$. The point $P(9, 3)$ lies on its circumference.

Find the equation of the tangent at P.

13. Determine whether circle A: $(x - 2)^2 + (y - 1)^2 = 15$ intersects with circle B: $(x + 4)^2 + (y - 3)^2 = 27$. Justify your answer.

14. Determine whether circle A: $(x - 2)^2 + (y - 3)^2 = 9$ intersects with circle B: $(x - 1)^2 + (y + 1)^2 = 16$. State whether they intersect at zero, one or two points and justify your answer.

15. Determine whether circle A: $(x - 3)^2 + (y - 4)^2 = 25$ intersects with circle B: $(x - 3)^2 + (y - 14)^2 = 25$. State whether they intersect at zero, one or two points and justify your answer. What does this mean geometrically?

16. Consider circles A: $(x - 18)^2 + (y - 20)^2 = 100$ and B: $(x - 15)^2 + (y - 16)^2 = 25$. Explain why these circles intersect at one common point.

Applications Assessment Standard 1.2 Answers

1. (a) $(x - 2)^2 + (y + 3)^2 = 49$ (b) Centre (5, -3). Radius = $\sqrt{37}$

2. Either discriminant = 0 or show that there is only one root, therefore line is a tangent.

3. $y - 5 = \frac{-12}{5}(x - 10)$

4. (a) $(x - 4)^2 + (y + 1)^2 = 36$ (b) Centre (2, -1). Radius = 3

5. Either discriminant = 0 or show that there is only one root, therefore line is a tangent.

6. $y - 4 = 3(x + 9)$

7. (a) $(x - 5)^2 + (y + 2)^2 = 100$ (b) Centre (1, -5). Radius = 5

8. Either discriminant = 0 or show that there is only one root, therefore line is a tangent.

9. $y - 8 = -4(x - 10)$

10. (a) $(x + 2)^2 + (y - 6)^2 = 1$ (b) Centre (3, 0). Radius = 2

11. Either discriminant = 0 or show that there is only one root, therefore line is a tangent.

12. $y - 3 = -2(x - 9)$

13. Circle A has centre (2, 1) and radius $\sqrt{15} = 3.9$

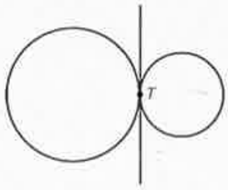
    Circle B has centre (-4, 3) and radius $\sqrt{27} = 5.2$

    The distance between the centres = $\sqrt{40} = 6.3 <$ sum of the radii, hence the circles intersect at two distinct points.

14. Circle A has centre (2, 3) and radius = 3

Circle B has centre (1, -1) and radius = 4

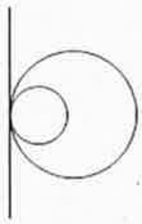
The distance between the centres = $\sqrt{17}$ = 4.1 < sum of the radii, hence the circles intersect at two distinct points.



15. Circle A has centre (3, 4) and radius = 5

Circle B has centre (3, 14) and radius = 5

The distance between the centres = 10 $\equiv$ sum of the radii, hence the circles intersect at one distinct point on a common tangent.



16. Circle A has centre (18, 20) and radius = 10

Circle B has centre (15, 16) and radius = 5

The distance between the centres = 5 < sum of the radii.

The distance between the centres = 5 $\leq$ each individual radii.

Hence the circles intersect at one distinct point on a common tangent.