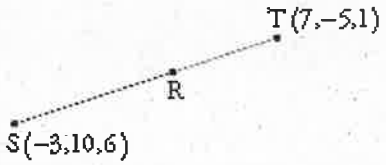
Expressions and Functions Assessment Standard 1.4

1. (a) Points A, B and C have coordinates (-4, -3, 1), (0, -1, 0) and (4, 1, -1) respectively.

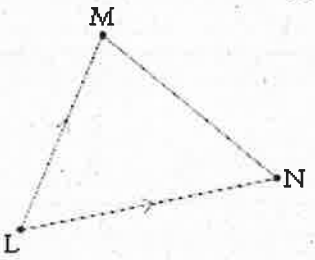
(i) Write down the components of $\overrightarrow{AC}$.

(ii) Hence show that the points A, B and C are collinear.

(b) The point R divides $\overrightarrow{ST}$ in the ratio 3:2, as shown below. Find the coordinates of R.



2. The diagram shows triangle LMN where $\overrightarrow{LM} = \begin{pmatrix} 3 \\ 4 \\ 2 \end{pmatrix}$ and $\overrightarrow{LN} = \begin{pmatrix} -2 \\ 4 \\ 5 \end{pmatrix}$.



(a) Find the value of $\overrightarrow{LM}.\overrightarrow{LN}$

(b) Use your answer from part (a) to find the size of the angle MLN.

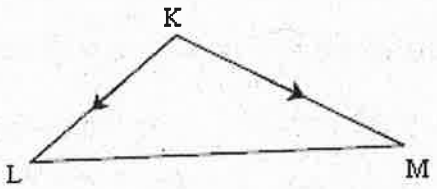
3. (a) Points S, T and U have components (1, 2, -5), (-3, 4, 1) and (-5, 5, 4) respectively.

(i) Write down the coordinates of $\overrightarrow{ST}$.

(ii) Hence show that the points S, T and U are collinear.

(b) The point P divides $\overrightarrow{AB}$ in the ratio 1:4, as shown below. Find the coordinates of P.

4. The diagram shows triangle KLM where $\overrightarrow{KL} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$ and $\overrightarrow{KM} = \begin{pmatrix} -4 \\ -3 \\ 2 \end{pmatrix}$.



(a) Find the value of $\overrightarrow{KL}.\overrightarrow{KM}$

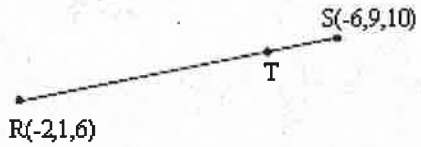
(b) Use your answer from part (a) to find the size of the angle LKM.

5.(a) Points P, Q and R have coordinates (2, -3, 3), (6, -2, 0) and (14, 0, -6) respectively.

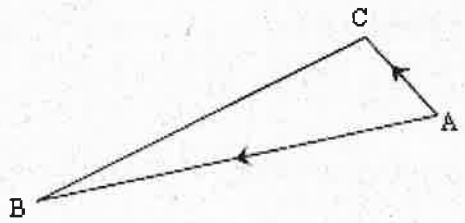
(i) Write down the components of $\overrightarrow{PQ}$.

(ii) Hence show that the points P, Q and R are collinear.

(b) The point T divides $\overrightarrow{RS}$ in the ratio 3:1, as shown below. Find the coordinates of T.



6. The diagram shows triangle ABC where $\overrightarrow{AB} = \begin{pmatrix} 5 \\ 0 \\ 12 \end{pmatrix}$ and $\overrightarrow{AC} = \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}$.



(a) Find the value of $\overrightarrow{AB}.\overrightarrow{AC}$

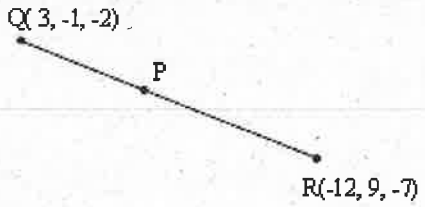
(b) Use your answer from part (a) to find the size of the angle BAC.

7.(a) Points E, F and G have coordinates (1, 4, -2), (-1, 8, -1) and (-5, 16, 1) respectively.

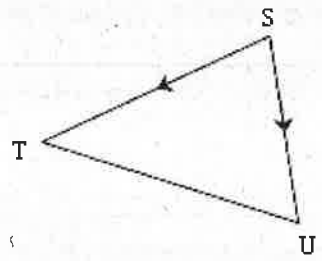
(i) Write down the components of $\overrightarrow{EF}$.

(ii) Hence show that the points E, F and G are collinear.

(b) The point P divides $\overrightarrow{QR}$ in the ratio 2:3, as shown below. Find the coordinates of P.



8. The diagram shows triangle STU where $\overrightarrow{ST} = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}$ and $\overrightarrow{SU} = \begin{pmatrix} -2 \\ 2 \\ 0 \end{pmatrix}$.



(a) Find the value of $\overrightarrow{ST}.\overrightarrow{SU}$

(b) Use your answer from part (a) to find the size of the angle TSU.

9.(a) Given that A(3, *t*, 5), B(5, 3, 2) and C(9, 13, -4) are collinear, find the value of *t*.

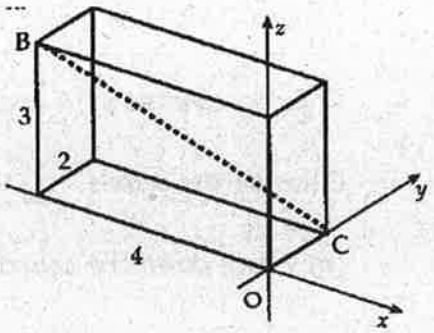
(b) Find the ratio in which B divides AC.

10.(a) Given that P(2, 5, 3), Q(3, 1, p) and R(6, -11, 23) are collinear, find the value of *p*.

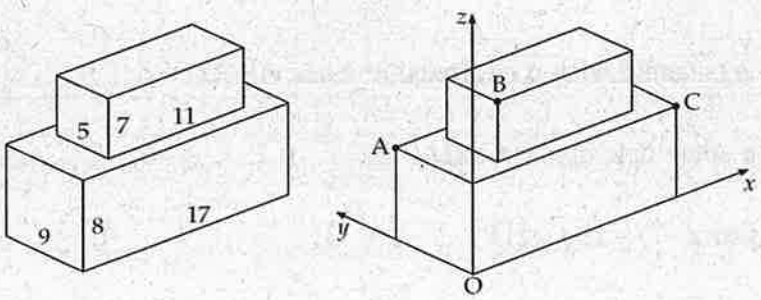
(b) Find the ratio in which Q divides PR.

11. A cuboid crystal is placed relative to the coordinate axes as shown.

Write down the components of $\overrightarrow{BC}$.



12. A cuboid measuring 11 cm by 5 cm by 7 cm is placed centrally on top of another cuboid measuring 17 cm by 9 cm by 8 cm.
Coordinates axes are taken as shown.



The point A has coordinates (0, 9, 8) and C has coordinates (17, 0, 8).

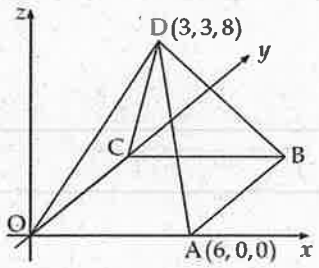
Write down the coordinates of B.

Write down the components of $\overrightarrow{BA}$ and $\overrightarrow{BC}$.

13. The diagram shows a square-based pyramid of height 8 units.

Square OABC has a side length of 6 units.

The coordinates of A and D are (6, 0, 0) and (3, 3, 8).



C lies on the *y*-axis.

(*a*) Write down the coordinates of B.

(*b*) Determine the components of $\overrightarrow{DA}$ and $\overrightarrow{DB}$.

14. VABCD is a pyramid with a rectangular base ABCD.

Relative to some appropriate axes,

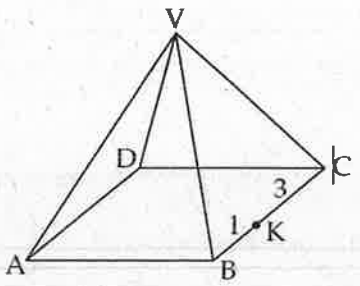
$\overrightarrow{VA}$ represents $-7\boldsymbol{i} - 13\boldsymbol{j} - 11\boldsymbol{k}$

$\overrightarrow{AB}$ represents $6\boldsymbol{i} + 6\boldsymbol{j} - 6\boldsymbol{k}$

$\overrightarrow{AD}$ represents $8\boldsymbol{i} - 4\boldsymbol{j} + 4\boldsymbol{k}$.

K divides BC in the ratio 1 : 3.

Find $\overrightarrow{VK}$ in component form.



Expressions and Functions Assessment Standard 1.4 Answers

1. (a) (i) $\overrightarrow{AC} = \begin{pmatrix} 8 \\ 4 \\ -2 \end{pmatrix} = 2\begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix}$

(ii) $\overrightarrow{AB} = \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix}$. Since $2\overrightarrow{AB} = \overrightarrow{AC}$ and A is a common point, A, B and C are collinear.

(b) R(3, 1, 3)

2. (a) $\overrightarrow{LM}.\overrightarrow{LN} = 20$ (b) Angle MLN = 56.4°

3. (a) (i) $\overrightarrow{ST} = \begin{pmatrix} -4 \\ 2 \\ 6 \end{pmatrix}$ (ii) proof (b) P(4, -1, -2)

4. (a) $\overrightarrow{KL}.\overrightarrow{KM} = -5$ (b) 104.4°

5. (a) (i) $\overrightarrow{PQ} = \begin{pmatrix} 4 \\ 1 \\ -3 \end{pmatrix}$ (ii) proof (b) T(-5, 7, 9)

6. (a) $\overrightarrow{AB}.\overrightarrow{AC} = 22$ (b) 55.7°

7. (a) (i) $\overrightarrow{EF} = \begin{pmatrix} -2 \\ 4 \\ 1 \end{pmatrix}$ (ii) proof (b) P(-3, 3, -4)

8. (a) $\overrightarrow{ST}.\overrightarrow{SU} = 2$ (b) 79.1°

9. (a) $t = -2$ (b) AB : BC = 1 : 2

10. (a) $p = 8$ (b) PQ : QR = 1 : 3

11. B(-4, 0, 3), C(0, 2, 0) and $\overrightarrow{BC} = \begin{pmatrix} 4 \\ 2 \\ -3 \end{pmatrix}$

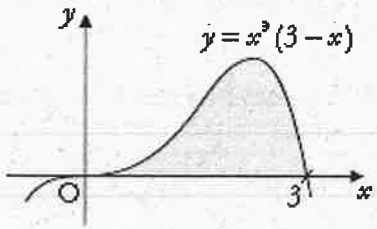
12. B(3, 2, 15), $\overrightarrow{BA} = \begin{pmatrix} -3 \\ 7 \\ -7 \end{pmatrix}$ and $\overrightarrow{BC} = \begin{pmatrix} 14 \\ -2 \\ -7 \end{pmatrix}$

13. B(6, 6, 0), $\overrightarrow{DA} = \begin{pmatrix} 3 \\ -3 \\ -8 \end{pmatrix}$ and $\overrightarrow{DB} = \begin{pmatrix} 3 \\ 3 \\ -8 \end{pmatrix}$

14. $\overrightarrow{VK} = \overrightarrow{VA} + \overrightarrow{AB} + \frac{1}{4}\overrightarrow{BC} = \begin{pmatrix} 1 \\ -8 \\ -16 \end{pmatrix}$

Applications Assessment Standard 1.4

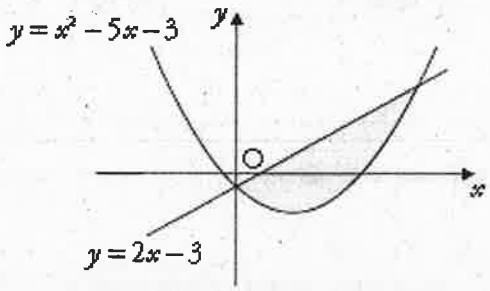
1. The curve $y = x^3(3 - x)$ is shown in the diagram below.



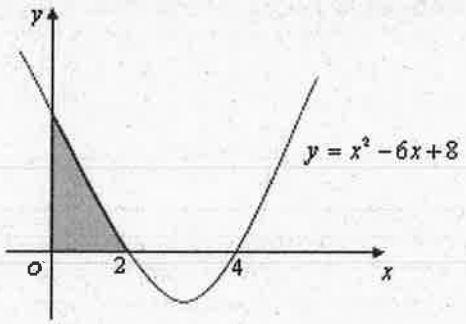
Calculate the shaded area enclosed between the curve and the x-axis between $x = 0$ and $x = 3$.

2. The diagram shows the line with equation $y = 2x - 3$ and the curve with equation $y = x^2 - 5x - 3$. The line and curve meet at the points where $x = 0$ and $x = 7$.

Calculate the shaded area shown in the diagram.

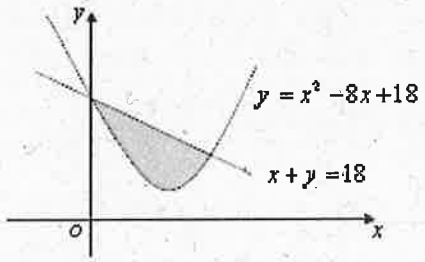


3. Calculate the shaded area shown in the diagram.

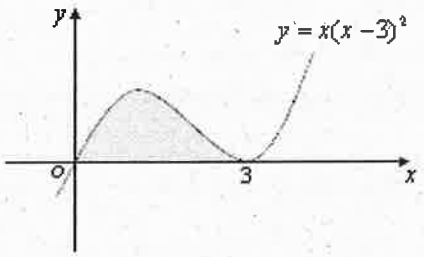


4. The diagram shows the line with equation $x + y = 18$ and the curve with equation $y = x^2 - 8x + 18$. The line and curve meet at the points where $x = 0$ and $x = 3$.

Calculate the shaded area shown in the diagram.

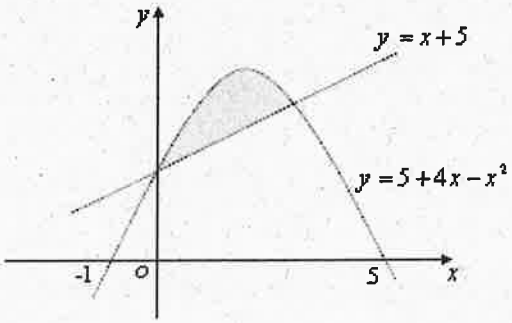


5. Calculate the shaded area shown in the diagram.

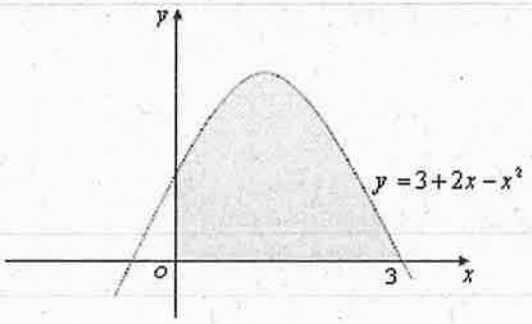


6. The diagram shows the line with equation $y = x + 5$ and the curve with equation $y = 5 + 4x - x^2$.

Calculate the shaded area shown in the diagram.

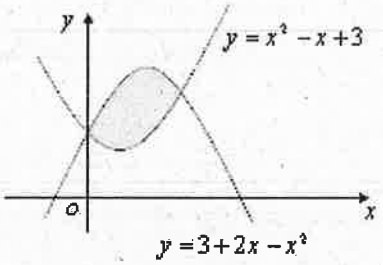


7. Calculate the shaded area shown in the diagram.

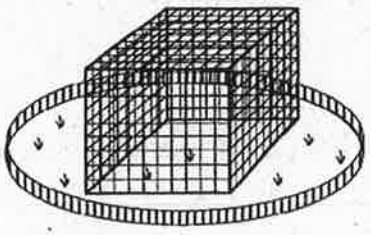


8. The diagram shows the curves with equations $y = x^2 - x + 3$ and $y = 3 + 2x - x^2$. The curves meet at the points where $x = 0$ and $x = 1.5$.

Calculate the shaded area shown in the diagram.

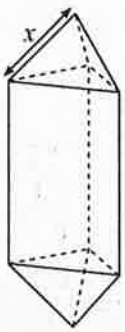


9. The owners of a zoo intend to build a new aviary in the shape of a cuboid with a square floor. The volume of the aviary will be 500 m^3. The length of one edge of the floor is x metres and the area A square metres of netting required is given by $A = x^2 + \frac{2000}{x}$.



Find the dimensions of the aviary to ensure that the cost of netting is minimised.

10. A goldsmith has built up a solid which consists of a triangular prism of fixed volume with a regular tetrahedron at each end. The surface area, A, of the solid is given by



$$A(x) = \frac{3\sqrt{3}}{2}\left(x^2 + \frac{16}{x}\right)$$

where x is the length of each edge of the tetrahedron.
Find the value of x which the goldsmith should use to minimise the amount of gold plating required to cover the solid.

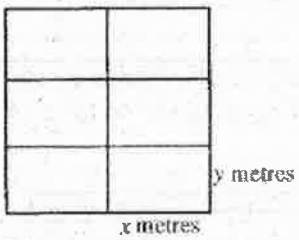
11. A child's beaker is in the shape of a cylinder with a hemispherical lid and a circular flat base. The radius of the cylinder is r cm and the height is h cm.

The surface area of plastic, A(r), needed to make the beaker is given by

$A(r) = 3\pi r^2 + \frac{800}{r}$.

Find the value of r which ensures that the surface area of plastic is minimised.

12. A zookeeper wants to fence off six individual animal pens. Each pen is a rectangle measuring x metres by y metres, as shown in the diagram.



(a) The total length of fencing is 360 m. Express the total length of fencing in terms of x and y.

(b) The total area, A m^2, of the six pens is given by $A(x) = 240x - \frac{16}{3}x^2$. Find the values of x and y which give the maximum area and write down this maximum area.

Applications Assessment Standard 1.4 Answers

1. $\int_0^3 x^3(3-x)dx = \int_0^3 (3x^3 - x^4)dx = \frac{243}{20} = 12\frac{3}{20}$

2. $\int_0^7 \big((2x-3) - (x^2 - 5x - 3)\big)dx$ Shaded area = $\frac{343}{6}$ units^2

3. Shaded area = $6\frac{2}{3}$ units^2

4. Shaded area = $\frac{343}{6}$ units^2

5. Shaded area = $6\frac{3}{4}$ units^2

6. Shaded area = $\frac{27}{6}$ units^2

7. Shaded area = 9 units^2

8. Shaded area = $\frac{9}{8}$ units^2

9. x = 10 so that l = 10m, b = 10m and h = 5m

10. x = 2

11. r = $\left(\frac{400}{3\pi}\right)^{1/3}$ = 3.5 cm

12. x = 22.5m, y = 20m and Area = (2x)(3y) = 2700m^2

Expressions and Functions Assessment Standard 1.1

1. (a) Simplify $\log_a 7 + \log_a 3$.

   (b) Simplify $\log_3 5 - 3\log_3 2$.

   (c) Evaluate $\log_2 2$.

2. (a) Given $x = \frac{\log_e 7}{\log_e 4}$, find an approximation for x.

   (b) Given $\log_{10} y = 3.1$, write an expression for the **exact** value of y.

   (c) Given $y = 10^{2.9}$, find an approximation for y.

3. (a) Simplify $\log_x 8 + \log_x 5$.

   (b) Simplify $5\log_9 3 - \log_9 27$.

4. (a) Given $x = \frac{\log_e 33}{\log_e 7}$, find an approximation for x.

   (b) Given $\log_{10} y = 2.5$, write an expression for the **exact** value of y.

   (c) Given $y = 10^{1.66}$, find an approximation for y.

5. (a) Simplify $\log_p 6 + \log_p 3$.

   (b) Simplify $2\log_2 6 - \log_2 9$.

6. (a) Given $x \log_e 9 = \log_e 11$, find an approximation for x.

   (b) Given $\log_3 y = 1.6$, write an expression for the **exact** value of y.

   (c) Given $y = 10^{0.8}$, find an approximation for y.

7. (a) Simplify $\log_y 16 - \log_y 8$.

   (b) Simplify $3\log_4 2 + \log_4 8$.

8. (a) Given $p \log_e 12 = 2\log_e 17$, find an approximation for p.

   (b) Given $\log_2 x = 3.4$, write an expression for the **exact** value of x.

   (c) Given $y = 4^{2.7}$, find an approximation for y.

9. Solve $e^x = 3{\cdot}5$.

10. Solve $e^x = 6{\cdot}1$.

11. Solve $e^x = 2{\cdot}8$.

12. Solve $e^x = 5{\cdot}7$.

Expressions and Functions Assessment Standard 1.1 Answers

1.	(a) $\log_a 21$	(b) $\log_3 \frac{5}{8}$	(c) $\log_2 2 = 1$
2.	(a) 1.404	(b) $y = 10^{3.1}$	(c) 794.3
3.	(a) $\log_x 40$	(b) 1	
4.	(a) 1.80	(b) $y = 10^{2.5}$	(c) 45.71
5.	(a) $\log_p 18$	(b) 2	
6.	(a) 1.09	(b) $3^{1.6}$	(c) 6.3
7.	(a) $\log_y 2$	(b) 3	
8.	(a) 2.28	(b) $x = 2^{3.4}$	(c) 42.2

9. 1.253 to 3 decimal places

10. 1.808 to 3 decimal places

11. 1.030 to 3 decimal places

12. 1.740 to 3 decimal places