Higher Mathematics

Supplementary Material

Applications

Straight Line

Higher Mathematics Supplementary Resources

Section A

This section is designed to provide examples which develop routine skills necessary for completion of this section.

R1 I have revised National 5 straight line.

1. Find the gradient of the line joining each pair of points

(a) T$(3, 2)$ and R$(4, 4)$

(b) A$(-1, 3)$ and Q$(4, 8)$

(c) C$(-3, -2)$ and S$(7, 3)$

(d) V$(0, 3)$ and L$(-3, 9)$

(e) B$(1, 4)$ and H$(-1, -2)$

(f) G$(-3, 4)$ and W$(-1, 8)$

(g) K$(9, -2)$ and N$(5, -12)$

(h) X$(-7, -4)$ and E$(-3, -2)$

2. Write down the gradient and y-intercept of each the line.

(a) $y = 3x + 2$

(b) $y = \frac{5}{8}x - 7$

(c) $y = 2 - 3x$

(d) $y = 4 - \frac{3}{4}x$

(e) $y = x - 3$

(f) $y = \frac{1}{2}x + 9$

(g) $y = 7 - 2x$

(h) $y = 11 - \frac{1}{3}x$

(i) $y = 5x + 1$

(j) $y = \frac{7}{9}x - 4$

(k) $y = 1 - x$

(l) $y = 43 - \frac{5}{7}x$

3. Rearrange the equation of each line so that it is in the form $y = mx + c$ and write down its gradient and y-intercept.

(a) $3y - 5x = 3$

(b) $4x + 3y = 9$

(c) $2x - y = -12$

(d) $5y + 2x = 0$

(e) $2y - 6x + 15 = 0$ **(f)** $4x - 3y - 7 = 0$

(g) $5x + 2y + 6 = 0$ **(h)** $8y + 4x - 11 = 0$

4. Write down the equation, in the form $y = mx + c$ where possible, of each straight line described.

(a) The straight line with gradient of -2 and passing through the point $(3, -2)$.

(b) A straight line passes through the point $(0, 7)$, with a gradient of 6.

(c) A straight line parallel to the x-axis and passes through $(-2, 4)$.

(d) A straight line passes through the point $(0, 11)$, with a gradient of -2.

(e) A straight line parallel to the y-axis and passes through $(5, 1)$.

(f) A straight line has a gradient of $\frac{1}{2}$ and passes through the point $(-1, 4)$.

(g) The straight line with gradient of 4 and passing through the point $(-1, 9)$.

(h) A straight line passes through the point $(0, -3)$, with a gradient of 2.

(i) A straight line parallel to the x-axis and passes through $(4, -3)$.

(j) A straight line passes through the point $(0, -4)$, with a gradient of $\frac{2}{3}$.

(k) A straight line parallel to the y-axis and passes through $(-3, -1)$.

(l) A straight line has a gradient of $-\frac{1}{2}$ and passes through the point $(-1, 4)$.

R2 I can find the Distance between 2 points- using the Distance Formula.

1. Use the distance formula to calculate the length of the straight line joining each pair of points. Leave your answer as a surd.

(a) A$(1, 5)$ and B$(3, 3)$

(b) P$(-7, 1)$ and Q$(3, 8)$

(c) C$(-3, -5)$ and D$(7, 1)$

(d) V$(0, 3)$ and W$(-7, 9)$

(e) G$(7, 3)$ and H$(-1, -2)$

(f) R$(-2, 3)$ and S$(-1, 8)$

(g) K$(9, -5)$ and L$(2, -12)$

(h) X$(-7, -3)$ and Y$(-1, -2)$

R3 I can use the Midpoint Formula.

1. Find the midpoint of each pair of points

(a) A$(-3, 1)$ and B$(1, 3)$

(b) P$(1, 4)$ and Q$(9, 8)$

(c) C$(3, -3)$ and D$(-6, 1)$

(d) V$(-7, 1)$ and W$(3, 9)$

(e) G$(2, 4)$ and H$(-2, -2)$

(f) R$(-6, 2)$ and S$(-2, 8)$

(g) K$(-3, -3)$ and L$(3, -11)$

(h) X$(0, -4)$ and Y$(-4, -2)$

2. The Line CD has the midpoint $(5, 3)$ and the point C has coordinates $(-3, 2)$.

Find the coordinates of D.

3. The Line AB has the midpoint $(-2, 7)$ and the point A has coordinates $(3, -7)$.

Find the coordinates of B.

4. The Line EF has the midpoint $(-5, 3)$ and the point F has coordinates $(3, 11)$.

Find the coordinates of E.

R4 I can calculate the gradient of perpendicular lines.

1. Write down the gradient of the line perpendicular to the gradient given

(a) $m = 3$ **(b)** $m = -2$ **(c)** $m = 6$

(d) $m = \frac{1}{3}$ **(e)** $m = -\frac{1}{4}$ **(f)** $m = \frac{1}{5}$

(g) $m = -\frac{2}{3}$ **(h)** $m = \frac{5}{4}$ **(i)** $m = -\frac{3}{5}$

2. Write down the gradient of the line perpendicular to the given line

(a) $y = 5x + 2$ **(b)** $y = \frac{2}{3}x - 7$

(c) $y = 2 - 3x$ **(d)** $y = 4 - \frac{1}{2}x$

(e) $y = 3x - 3$ **(f)** $y = x + 9$

(g) $y - 4x + 12 = 0$ **(h)** $3x - y - 8 = 0$

(i) $3x - 2y + 7 = 0$ **(j)** $8y + 4x - 2 = 0$

R5 I can find the point of intersection of straight lines.

1. Find the point of intersection between each pair of lines

(a) $3x + 4y = -7;$ and $2x + y = -3$

(b) $y = -x + 12;$ and $y = x - 4$

(c) $y = -x;$ and $4x + 3y = 3$

(d) $2x - 5y = 1;$ and $4x - 3y = 9$

(e) $y = -x + 2;$ and $y = 2x - 2$

(f) $x + y = 5;$ and $x - y = 2$

(g) $2x + 4y = 7;$ and $4x - 3y = 3$

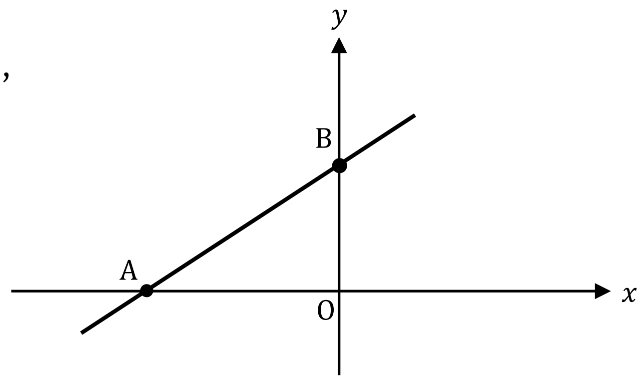
(h) $2x + 5y = 16;$ and $x - y = 1$

Section B

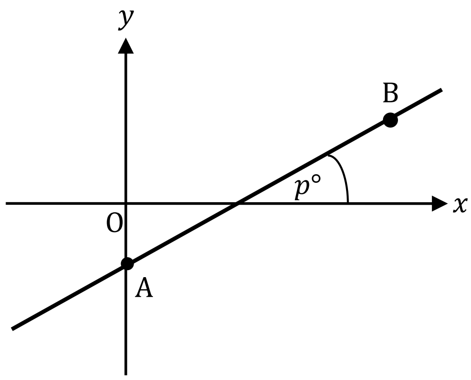
This section is designed to provide examples which develop Course Assessment level skills

NR1 I can apply $m = tan\theta$ in the context of a problem.

1. Find the equation of the line AB, where A is the point $(-3, 0)$ and the angle BAO is $30°$.



2. Find the size of the angle $p°$ that the line joining the points $A(0, -2)$ and $B(4\sqrt{3},\ 2)$ makes with the positive direction of the x-axis.



3. A straight line has equation $3x + 2y - 1 = 0$.

This line is inclined to the x-axis by an angle of $a°$.

Find the size of angle $a°$.

NR2 I can solve straight line problems involving parallel and perpendicular lines.

1. Find the equation of the straight line through the point $(-1, 5)$ which is parallel to the line with equation $3x - y + 1 = 0$.

2. Find the equation of the straight line which passes through the point $(-1, 4)$ and is perpendicular to the line with equation $4x + y - 3 = 0$.

3. Find the equation of the straight line which is parallel to the line with equation $2x + 3y = 6$ and which passes through the point $(2, -1)$.

4. The point P has coordinates $(1, 12)$. The straight lines with equations $x + 3y - 7 = 0$ and $2x + 5y = 11$ intersect at Q.

(a) Find the gradient of PQ.

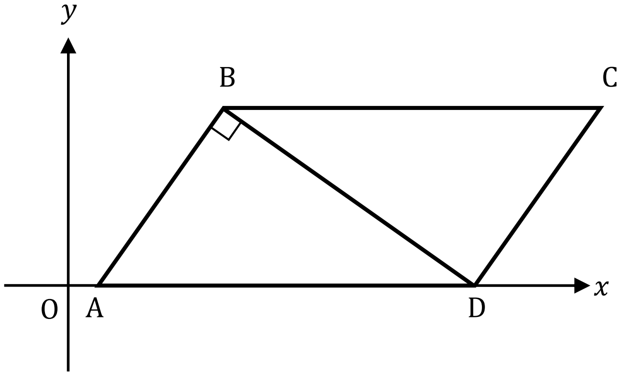
(b) Hence show that PQ is perpendicular to only one of the lines.

5. ABCD is a parallelogram.

A is the point $(3, 0)$, B is the point $(5, 6)$ and D lies on the x-axis. The diagonal BD is perpendicular to side AB.

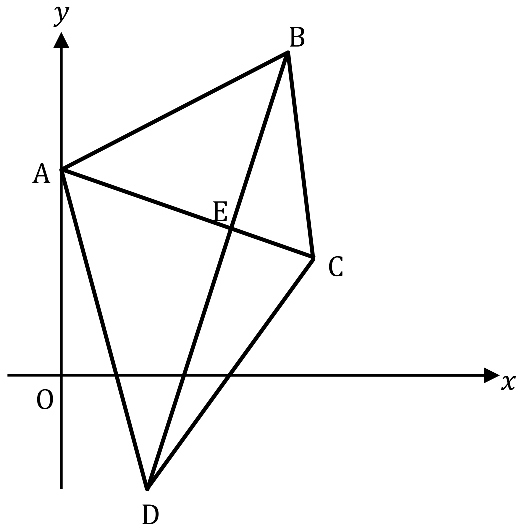
(a) Show that the equation of BD is $x + 3y - 23 = 0$.

(b) Hence find the coordinates of C and D.



NR3 I know the properties of: midpoints; altitudes; medians; perpendicular bisectors and can apply these in problems (including points of intersection).

1. A quadrilateral has vertices $A(-2, 8)$, $B(6, 12)$, $C(7, 5)$ and $D(1, -3)$ as shown in the diagram.



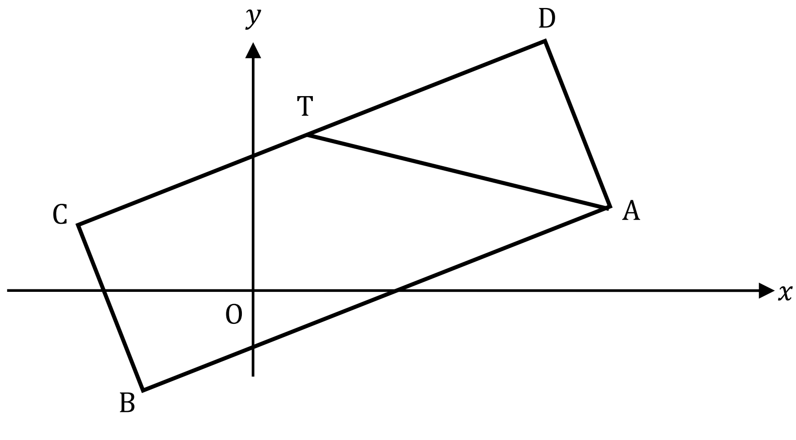
(a) Find the equation of diagonal BD.

(b) The equation of diagonal AC is $x + 3y = 22$. Find the coordinates of E, the point of intersection of the diagonals.

(c) **(i)** Find the equation of the perpendicular bisector of AB.

(ii) Show that this line passes through E.

2. The diagram shows rectangle ABCD with $A(7, 1)$ and $D(5, 5)$.



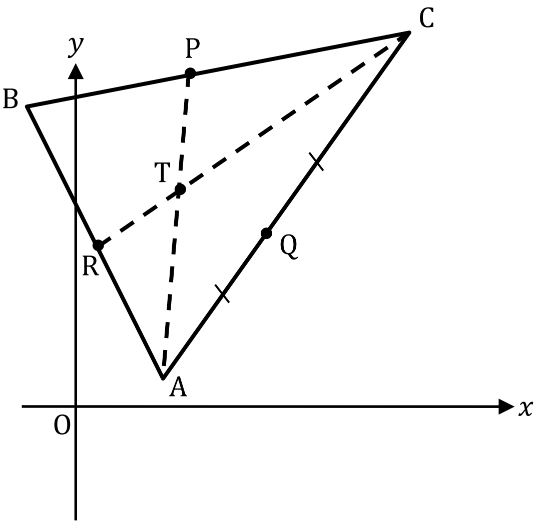
(a) Find the equation of AD.

(b) The line from A with equation $x + 3y = 10$ intersects with CD at T.

Find the coordinates of T.

(c) Given that T is the midpoint of CD, find the coordinates of C and B.

3. Triangle ABC has vertices A$(4,1)$, B$(-4,17)$ and C$(18,21)$.

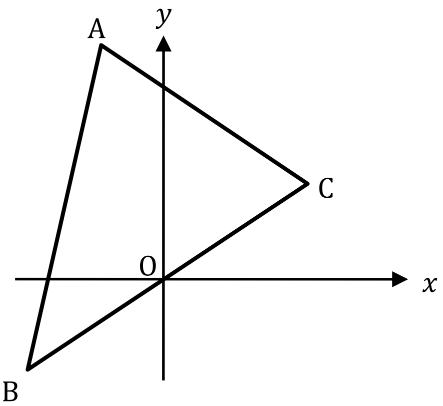


Medians AP and CR intersect at the point T$(6,13)$.

(a) Find the equation of median BQ.

(b) Verify that T lies on BQ.

4. Triangle ABC has vertices A$(-2,6)$, B$(-4,-2)$ and C$(4,2)$ as shown. Find

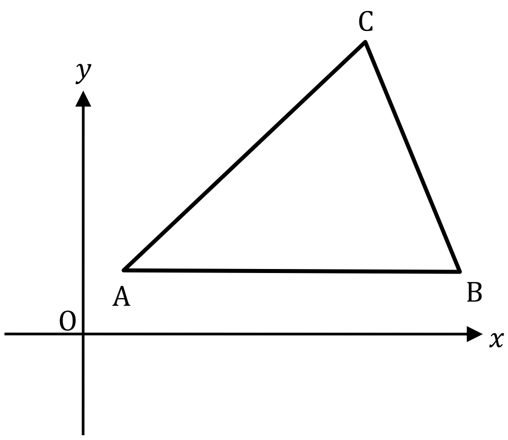


(a) the equation of the line p, the median from C of triangle ABC.

(b) the equation of the line q, the perpendicular bisector of BC.

(c) the coordinates of the point of intersection of the lines p and q.

5. Triangle ABC has vertices A$(1,2)$, B$(11,2)$ and C$(7,6)$ as shown.

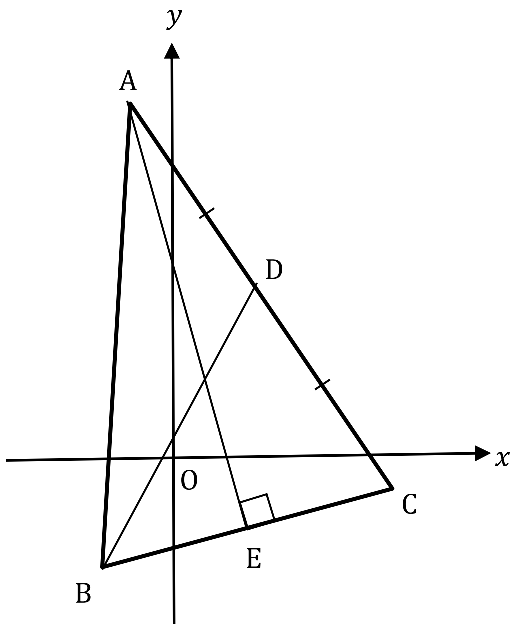


(a) Write down the equation of l_1, the perpendicular bisector of AB.

(b) Find the equation of l_2, the perpendicular bisector of AC.

(c) Find the point of intersection of the lines l_1 and l_2.

6. Triangle ABC has vertices $A(-2, 12)$, $B(-3, -5)$ and $C(6, -2)$ as shown.

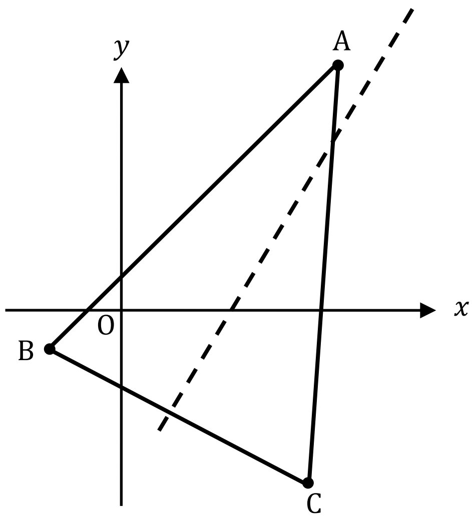


(a) Find the equation of the median BD.

(b) Find the equation of the altitude AE.

(c) Find the coordinates of the point of intersection of BD and AE.

7. The vertices of triangle ABC are $A(7, 8)$, $B(-3, -2)$ and $C(5, -6)$ as shown.



The broken line represents the perpendicular bisector of BC.

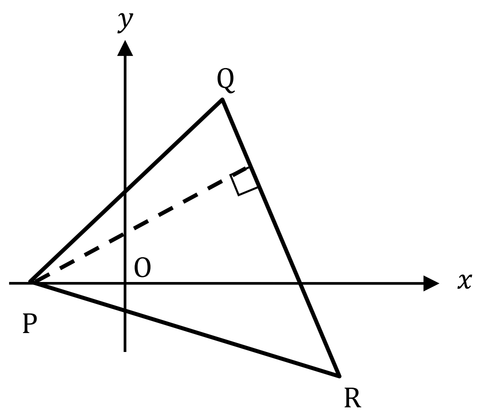
(a) Show that the perpendicular bisector of BC is $y = 2x - 6$.

(b) Find the equation of the median from C.

(c) Find the coordinates of the point of intersection of the perpendicular bisector of BC and the median from C.

8. Triangle PQR has vertex P on the x-axis as shown.

Q and R are the points $(3, 6)$ and $(7, -2)$ respectively. The equation of PQ is $3x - 7y + 12 = 0$



(a) State the coordinates of P.

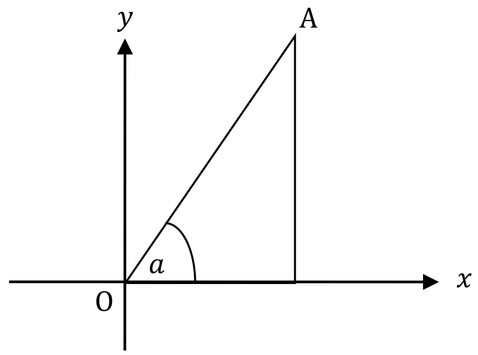
(b) Find the equation of the altitude from P.

(c) The altitude from P meets the line QR at T. Find the coordinates of T.

NR4 I have experience of cross topic exam standard questions.

Straight line and trigonometry

1. **(a)** The diagram below show a right angled triangle, where the line OA has equation $5x - 3y = 0$.

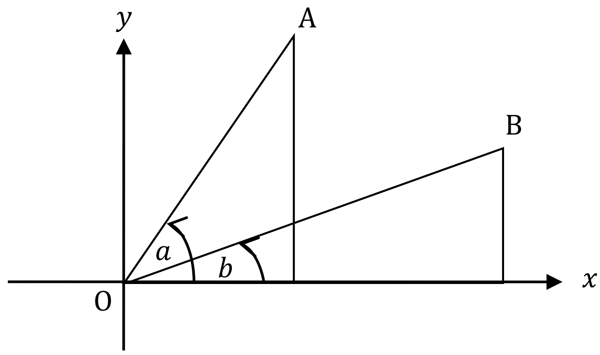


(i) Show that $\tan a = \frac{5}{3}$.

(ii) Find the value of $\sin a$ and $\cos a$.

(b) A second right angled triangle is added as shown.

The line OB has equation $x - 2y = 0$.



Find values of $\sin b$ and $\cos b$.

(c) **(i)** Find the value of $\sin(a - b)$.

(ii) Find the value of $\cos(a + b)$.

Section A

This section is designed to provide examples which develop routine skills necessary for completion of this section.

R1 I can use the distance formula to find the distance between two points.

1. Find the distance between each of the pairs of points below. Leave your answer as a surd where appropriate.

(a) $A(3,4)\ \ B(6,8)$ **(b)** $C(-2,0)\ \ D(3,12)$ **(c)** $E(3,-1)\ \ F(0,-5)$

(d) $G(0,4)\ \ H(-3,-7)$ **(e)** $J(-3,9)\ \ K(3,-1)$ **(f)** $P(2,-5)\ \ Q(-1,7)$

2. A circle has diameter AB where $A(6,-2)\ \ B(-3,5)$. Find the size of the radius of this circle.

3. The centre of three concentric circles is $(-5,3)$. Find the radius of each circle if:

(a) The smallest circle goes through the point $(-3,2)$.

(b) The middle circle goes through the point $(-7,5)$.

(c) The largest circle goes through the point $(8,7)$.

4. Two circles of the same size have centres $(-3,4)$ and $(2,-7)$ and touch at a single point.

Find the size of the radii of the circles.

R2 I can determine the equation of a circle given its centre and radius using $(x-a)^2+(y-b)^2=r^2$

1. Find the equation of the circles with:

(a) Centre $(3, 4)$ and radius 5

(b) Centre $(0, 0)$ and radius 8

(c) Centre $(0, 2)$ and radius 7

(d) Centre $(3, -8)$ and radius 6

(e) Centre $(-1, -5)$ and radius 10

(f) Centre $(-2, 0)$ and radius 3

2. Find the equation of the circles with:

(a) Centre $(0, 0)$ and diameter 3

(b) Centre $(-1, 4)$ and diameter 6

(c) Centre $(0, -5)$ and diameter 12

(d) Centre $(9, -7)$ and diameter 2

(e) Centre $(-3, -7)$ and diameter 5

(f) Centre $(4, 0)$ and diameter 7

R3 **I can determine the centre and radius of a circle given its equation using $\boldsymbol{x^2 + y^2 + 2gx + 2fy + c = 0}$.**

State the centre and radius of each of these circles.

1. $x^2 + y^2 + 4x + 8y - 5 = 0$

2. $x^2 + y^2 - 2x + 10y + 3 = 0$

3. $x^2 + y^2 - 10x - 4y - 9 = 0$

4. $x^2 + y^2 - 25 = 0$

5. $x^2 + y^2 + x - 6y + 8 = 0$

6. $x^2 + y^2 + 3x - y - 6 = 0$

R4 I can determine if a point lies inside, outside or on the circle.

In each example below, the equation of a circle and a point are given. In each case, state whether the point does or does not lie on the circumference of the given circle.

1. $x^2 + y^2 + 4x - 6y - 16 = 0$ and $(0, -2)$.

2. $x^2 + y^2 - 2x + 10y + 3 = 0$ and $(3, 4)$.

3. $x^2 + y^2 - 10x - 4y - 9 = 0$ and $(-2, -5)$.

4. $x^2 + y^2 - 2x + 4y - 15 = 0$ and $(-1, 2)$.

5. $x^2 + y^2 - 10x - 12y + 61 = 0$ and $(-5, -6)$.

6. $x^2 + y^2 + 10x - 9y - 18 = 0$ and $(2, 5)$.

R5 I can use $g^2 + f^2 - c$ to determine whether an equation in the form $x^2 + y^2 + 2gx + 2fy + c = 0$ is an equation of a circle or not.

Which of the equation below represent the equation of a circle?

1. $x^2 + y^2 + 4x - 6y - 3 = 0$

2. $x^2 + y^2 - 8x + 6y = 0$

3. $x^2 + y^2 - 4x - 6y + 16 = 0$

4. $x^2 + y^2 + 2x - 8y + 24 = 0$

5. $x^2 + y^2 + 14x + 6y + 54 = 0$

6. $x^2 + y^2 - 6x + 2y + 15 = 0$

7. $x^2 + y^2 - 2x - 14y + 14 = 0$

8. $x^2 + y^2 - 4x + 10y + 37 = 0$

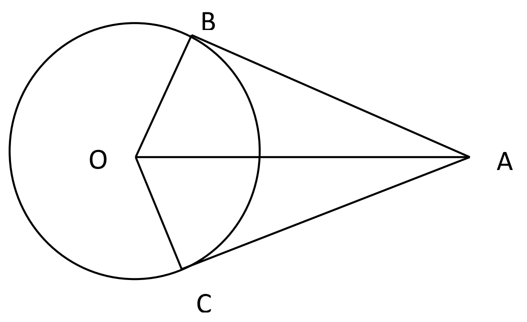
Section B

This section is designed to provide examples which develop Course Assessment level skills

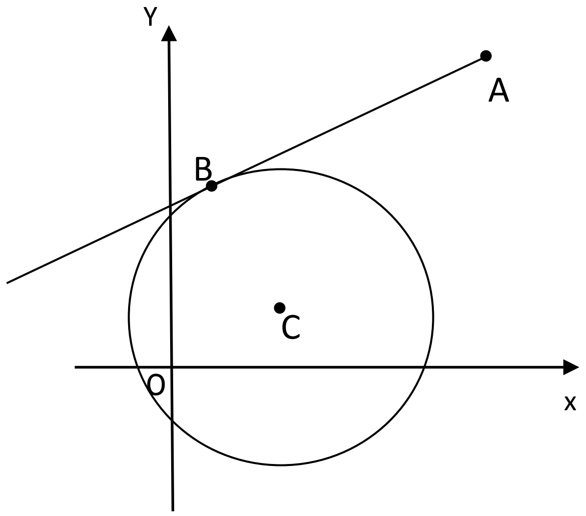
NR1	I can use circle equations in mixed problems.

1. The points A(6,3,1), B(8,4,-9) and C(3,1,k) lie on the circumference of a semicircle with AB as diameter. Find all the possible values of k.

2. In the diagram AB and AC are tangents from the point A(9,0) to the circle $x^2 + y^2 = 16$, with centre O. Find the area of the kite ABOC.



3. AB is a tangent at B to the circle with centre C and equation $(x-2)^2 + (y-2)^2 = 25$. The point A has coordinates (10,8). Find the area of triangle ABC.



4. A sports club awards trophies in the form of paperweights bearing the club crest. Diagram 1 shows the front view of one of these paperweights. Each is made from two different types of glass. The two circles are concentric and the base line is a tangent to the inner circle.

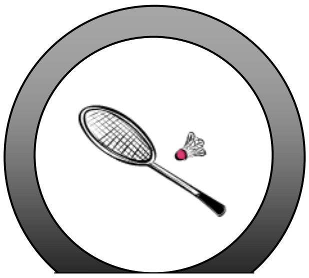


Diagram 1

(a) Relative to x, y coordinate axes, the equation of the outer circle is $x^2 + y^2 - 8x + 2y - 19 = 0$ and the equation of the base line is $y = -6$.
Show that the equation of the inner circle is
$x^2 + y^2 - 8x + 2y - 8 = 0$.

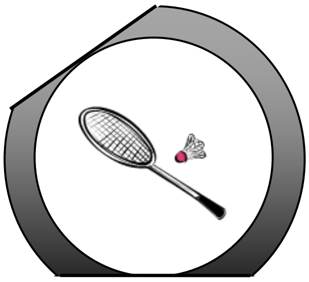


Diagram 2

(b) An alternative form of the paperweight is made by cutting off a piece of glass from the original design along a second line with equation $3x - 4y + 9 = 0$ as shown above in diagram 2.
Show that this line is a tangent to the inner circle and state the coordinates of the point of contact.

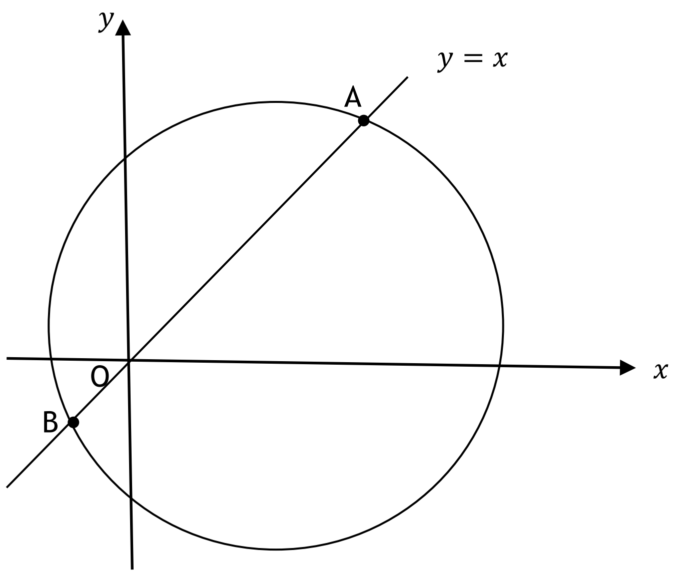
NR2 I can determine the point of intersection of a line and a circle or two circles.

1. A circle C_1 has centre A(1, 3) and radius $\sqrt{5}$. A circle C_2 has centre B(9, 7) and radius $3\sqrt{5}$.

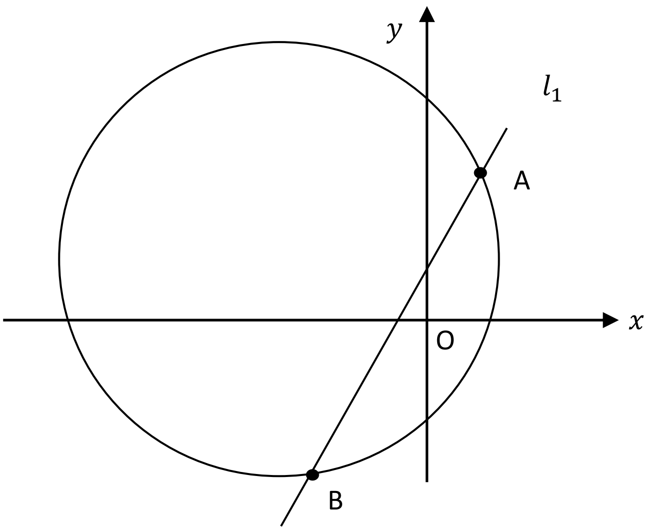
(a) Verify that C_1 touches C_2.
(b) Find the coordinates of X, the point of contact of C_1 and C_2.
(c) Find the equation of the common tangent to C_1 and C_2 drawn through X.

2. The straight line $y = x$ cuts the circle $x^2 + y^2 - 6x - 2y - 24 = 0$ at A and B.

(a) Find the coordinates of A and B.
(b) Find the equation of the circle which has AB as diameter.

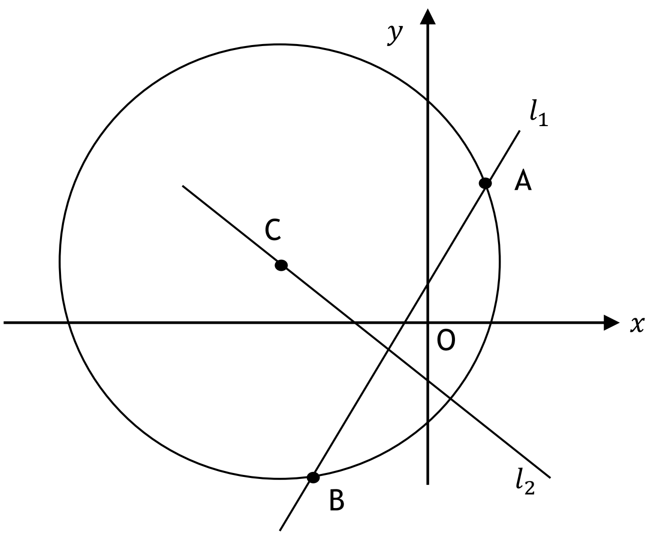


3. Diagram 1 shows a circle with equation $x^2 + y^2 + 10x - 2y - 14 = 0$ and a straight line, l_1, with equation $y = 2x + 1$.
The line intersects the circle at A and B.



(a) Find the coordinates of the points A and B.

(b) Diagram 2 shows a second line, l_2, which passes through the centre of the circle, C, and is at right angles to the line l_1.

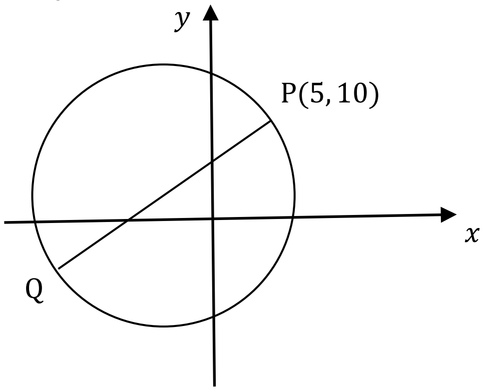


(i) Write down the coordinates of C.

(ii) Find the equation of the line l_2.

4. **(a)** Show that the point P(5,10) lies on circle C_1 with equation $(x+1)^2 + (y-2)^2 = 100$.

(b) PQ is a diameter of this circle as shown in the diagram. Find the equation of the tangent at Q.



(c) Two circles, C_2 and C_3 touch C_1 at Q.

The radius of each of these circles is twice the radius of C_1.
Find the equations of circles C_2 and C_3.

5. Circle C_1 has equation $(x+1)^2 + (y-1)^2 = 121$.
A circle C_2 with equation $x^2 + y^2 - 4x + 6y + p = 0$ is drawn inside C_1.
The circles have no points of contact.
What is the range of values of p?

6. **(a)** Relative to a suitable set of coordinate axes, Diagram 1 shows the line $2x - y + 5 = 0$ intersecting the circle $x^2 + y^2 - 6x - 2y - 30 = 0$ at the points P and Q.

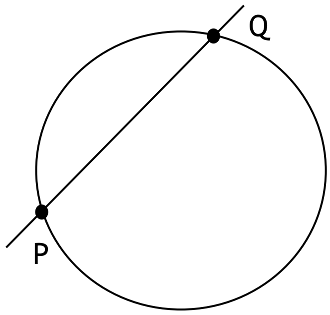


Diagram 1

Find the coordinates of P and Q.

(b) Diagram 2 shows the circle from **(a)** and a second congruent circle, which also passes through P and Q.

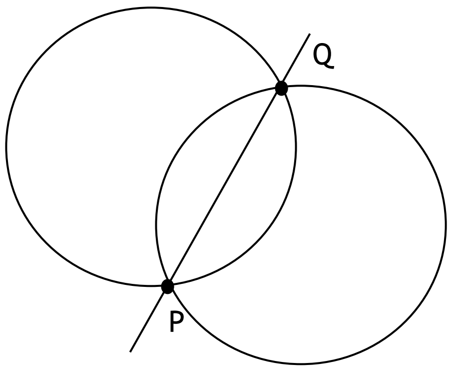


Diagram 2

Determine the equation of this second circle.

NR3 I can determine the point of intersection of a tangent to a circle using the criteria for tangency.

1. Find the possible values of k for which the line $x - y = k$ is a tangent to the circle $x^2 + y^2 = 18$.

2. Show that the line $x + y = 10$ is a tangent to the circle $x^2 + y^2 - 2x - 10y + 18 = 0$ and find the coordinates of the point of contact.

3. **(a)** Show that the equation of the circle which passes through (0, 0), (4, 0) and (0, -2) is $x^2 + y^2 - 4x + 2y = 0$.

   (b) Show that the line with equation $y = 2x - 10$ is a tangent to this circle and state the coordinates of the point of contact.

4. A circle, centre C, has equation $x^2 + y^2 - 4x + 6y - 12 = 0$.

   (a) Find the equation of the tangent at the point A(5, 1) on this circle.

   (b) Show that the line through the point P(1, 4) at right angles to the tangent has equation $3y - 4x = 8$ and show that this line is also a tangent to the circle.

5. A, B and C are the points (-1, 1), (1, 2) and (4, 1) respectively. AP is a diameter of a circle, centre B.

   (a) State the equation of the circle.

   (b) Prove that CP is a tangent to the circle.

   (c) D is the point (0, -1). Prove that CD is the other tangent to the circle from C.

NR4 I can determine the equation of a tangent to a circle.

1. Find the equation of the tangent to the circle $x^2 + y^2 - 3x + y - 16 = 0$ at the point (4,3).

2. **(a)** Find the equation of the circle, centre (9,-1), which passes through the point A (3,8).

(b) Obtain the equation of the tangent to this circle at A.

(c) Prove that this tangent passes through the centre of the circle with equation $x^2 + y^2 + 6x - 8y + 12 = 0$.

3. Prove that the line with equation $5x + y - 10 = 0$ is a tangent to the circle with equation $x^2 + y^2 - 16x + 8y + 54 = 0$ and find the coordinates of the point of contact.

4. **(a)** Find the coordinates of the centre, C, and the radius of the circle whose equation is $x^2 + y^2 - 2x - 4y - 3 = 0$.

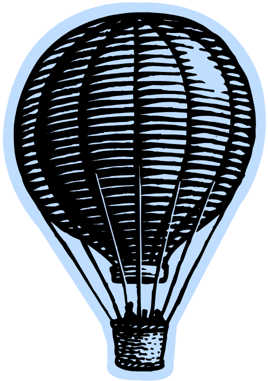
(b) If a tangent to the circle at the point A(3,4) is drawn:

(i) Find the equation of the tangent at A.

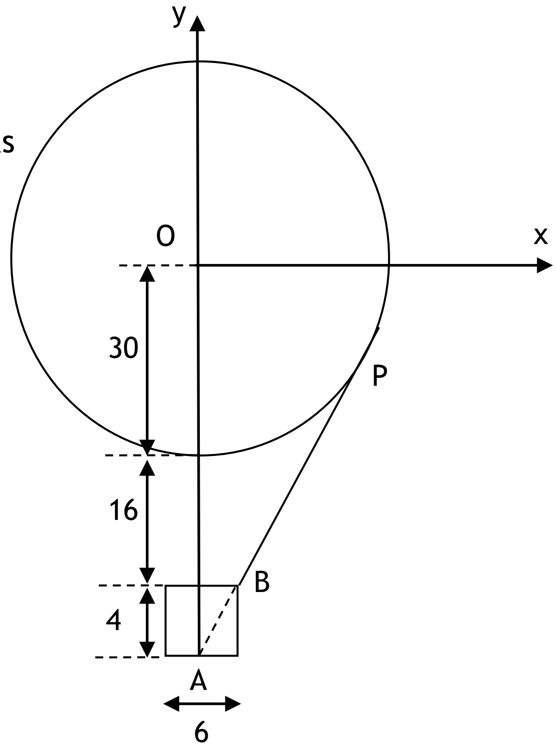
(ii) Prove that the point P(7,0) lies on the tangent.

(iii) Find the equation of the circle which passes through points C, A and P.

5.



A spherical hot-air balloon has a radius 30 feet. Cables join the balloon to the gondola which is cylindrical with diameter 6 feet and height 4 feet. The top of the gondola is 16 feet below the bottom of the balloon.



Coordinate axes are chosen as shown in the diagram.

One of the cables is represented by PB and PBA is a straight line.

(a) Find the equation of the cable PB.

(b) State the equation of the circle representing the balloon.

(c) Prove that this cable is a tangent to the balloon and find coordinates of P

NR5 I have experience of cross topic exam standard questions.

1. A and B are the points $(2, 2)$ and $(4, 8)$ respectively.

(a) Find the equation of the perpendicular bisector of AB.

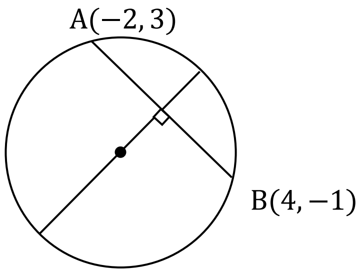
(b) Given that C, a point in the first quadrant equidistant from both axes, is the centre of the circle passing through A and B, find

(i) the coordinates of C;

(ii) the equation of the circle.

(c) Prove that the line $7x - y - 2 = 0$ is a tangent to this circle and state the coordinates of the point of contact.

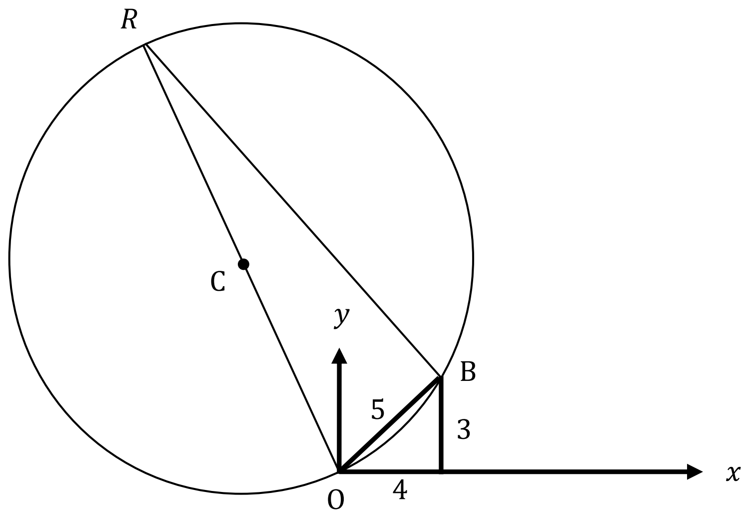
2. A circle passes through A$(-2, 3)$ and B$(4, -1)$.



Find the equation of the diameter which is perpendicular to the chord AB.

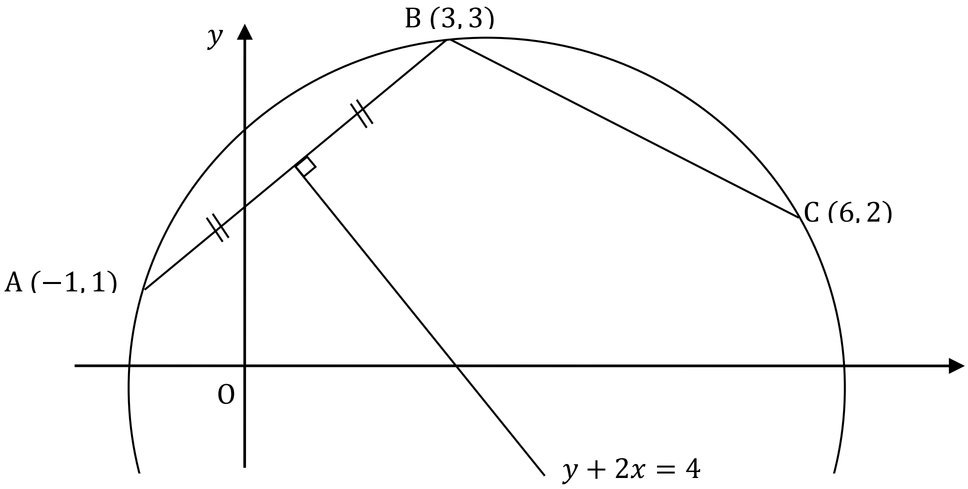
3. The right-angled triangle OAB with sides of length 3 cm, 4 cm and 5 cm is placed with one vertex at the origin O as shown in the diagram.

A circle of centre C with diameter RO of length 13 cm is drawn and passes through O and B.



What is the gradient of the line RO?

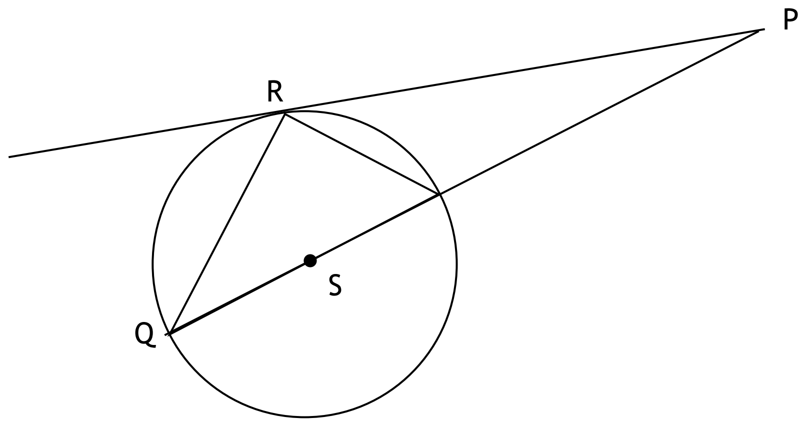
4. A is the point $(-1, 1)$, B is $(3, 3)$ and C is $(6, 2)$. The perpendicular bisector of AB has equation $y + 2x = 4$.



(a) Find the equation of the perpendicular bisector of BC.

(b) Find the centre and equation of the circle which passes through A, B and C.

5. P is a point on the tangent at R to a circle centre S. PS extends to cut the circle at Q.



If angle $\mathrm{RPQ} = 2\theta°$ and $\mathrm{PQ} = d$ units, prove that:

(a) angle $\mathrm{SQR} = (45 - \theta)°$

(b) by applying the Sine Rule to triangle PRQ,

$$\mathrm{RP} = \frac{d(\cos\theta° - \sin\theta°)}{\cos\theta° + \sin\theta°}$$

Section A

This section is designed to provide examples which develop routine skills necessary for completion of this section.

R1 I have revised percentage appreciation and depreciation questions using real life examples.

1. £3000 is invested at an interest rate of 4% per annum.

(a) What is the value of the investment after 5 years?

(b) After how many years will the investment be over £4000?

2. The population of Mexico in 2000 was 104 million. At the time, the rate of population was set to increase by 1.2% per annum.

If this rate remained at 1.2%, what was the predicted population in 2010. Round your answer to 3 significant figures.

3. A patient was injected with 120ml of a drug to reduce inflammation. Every hour 8% of the drug passes out of her bloodstream. Calculate the amount of the drug in the bloodstream after 6 hours.

Round your answer to 2 significant figures.

4. An investigation was carried out into the effect of temperature on fermentation in yeast.

The volume of gas produced (cm^3) by fermenting yeast increases by 23% as the temperature increases by 1°C.

When the temperature was 15°C the volume of gas was $5cm^3$. Calculate the volume when the temperature was 20°C.

Round your answer to 1 decimal place.

5. A student carried out an investigation into the effect of temperature on the growth of her Asiatic Lilly Plant. When the temperature was 10°C the height of the Lilly plant was 20cm.

It is expected to grow at a rate of 1% each week for the next four weeks. Calculate the height of the Lilly plant after four weeks.

6. A record of a human male's height from the age of 1 to age 15 was recorded. Each year his height increased by 6%. If his starting height was 80cm, what was his height aged 15?

Round your answer to the nearest cm

7. The pH level in milk changes as it sours. At the start of an experiment the pH level was 6.7. It is expected to decrease by 0.5% each hour for the next 50 hours.

Calculate the pH level after 50 hours to 2 significant figures.

8. The number of bacteria grown in a fermenter over a 24 hour period were recorded. The results calculated are 15% growth in the first 4 hour cycle, then 55% in the second four hour cycle then 80% in the third four hour cycle. The number of bacteria at the start of the experiment was 20 billion/mm^3. Calculate the bacteria present at the end of the third cycle.

Round your answer to 2 significant figures.

R2 I can set up a recurrence relation from given information.

1. A family take out a loan of £3000. The interest charged on this works out as 1.2% per calendar month. They set up a payment plan of £500 per month.

(a) Write down a recurrence relation for the amount they owe.

(b) How much will the family owe after 3 months?

(c) How many payments will it take for the loan to be repaid?

2. An investor saves £50000 in an account, gaining 4.5% interest per year. They withdraw £1800 every year.

(a) Write down a recurrence relation for the amount of money in the account.

(b) Find how much they would have in this savings account after 5 years.

3. The air pressure in a used car tyre was 35 p.s.i. This is above its recommended minimum pressure of 30 p.s.i. The tyre loses 12% of its air pressure every month. The owner has been refilling the tyre with air at a rate of 3 p.s.i. every month.

(a) Find a recurrence relation showing the air pressure of the tyre.

(b) What is the air pressure of the tyre after 2 months?

(c) After how many months would the tyre end up below the recommended minimum pressure?

R3 I can calculate a required term in a recurrence relation.

1. For each recurrence relation find, rounding you answer to 2 decimal places where applicable;

(a) $u_2 : \; u_{n+1} = 0.2u_n + 4, \;\; u_0 = 3$

(b) $u_3 : \; u_{n+1} = 0.1u_n + 5, \;\; u_0 = 7$

(c) $u_4 : \; u_{n+1} = -0.5u_n + 20, \;\; u_0 = 16$

(d) $u_3 : \; u_{n+1} = -u_n - 7, \;\; u_0 = 1$

(e) $u_2 : \; u_n = 0.9u_{n-1} + 450, \;\; u_0 = 2$

2. A sequence in defined by the recurrence relation $u_{n+1} = 0.3u_n + 6, \;\; u_0 = 100$

(a) Calculate the value of u_4

(b) Calculate the value of u_{10} , round your answer to 2 decimal places.

3. A sequence in defined by the recurrence relation $v_{n+1} = 1.2v_n - 8, \;\; v_0 = 150$

(a) Calculate the value of v_3

(b) Calculate the value of v_{11} , round your answer to 2 decimal places.

(c) Find the smallest value of n for which $v_n > 1500$

4. A sequence in defined by the recurrence relation $u_n = 1.05u_{n-1} - 20, \;\; u_1 = 200$

(a) Calculate the value of u_5

(b) Find the smallest value of n for which $u_n < 50$

Section B

This section is designed to provide examples which develop Course Assessment level skills

NR1	I can solve problems involving the limit of a recurrence relation.

1. A sequence is defined by the recurrence relation

$$u_{n+1} = ku_n - 5,\ u_0 = 0$$

(a) Given that $u_2 = -7$, find the value of k.

(b) (i) Why does this sequence tend to a limit as $n \to \infty$.

(ii) Find the value of this limit.

(Non-calculator)

2. A sequence is defined by the recurrence relation

$$u_{n+1} = \frac{1}{4}u_n + 20,\ u_0 = 0.$$

(a) Determine the value of u_1, u_2 and u_3.

(b) (i) Give a reason why this sequence has a limit.

(ii) Find the exact value of the limit.

(Non-calculator)

3. The terms of a sequence satisfy

$$u_{n+1} = ku_n + 4.$$

(a) Find the value of k which produces a sequence with a limit of 5.

(b) A sequence satisfies the recurrence relation

$$u_{n+1} = mu_n + 4,\ u_0 = 3.$$

(i) Express u_1 and u_2 in terms of m.

(ii) Given that $u_2 = 8$, find the value of m which produces a sequence with no limit.

(Non-calculator)

4. Two unique sequences are defined by the following recurrence relations

$$u_{n+1} = ku_n + 6 \quad \text{and} \quad u_{n+1} = k^2u_n + 9,$$

where k is a constant.

(a) If both sequences have the same limit, find the value of k.

(b) For both sequences $u_0 = 80$. Find the difference between their first terms.

(Non-calculator)

5. A recurrence relation is given as

$$u_{n+1} = 0 \cdot 3u_n + 21$$

(a) Given that $u_1 = 36$, find the initial value, u_0 of this sequence.

(b) Hence find the difference between u_0 and the limit of this sequence.

(Non-calculator)

6. Two sequences are generated by the recurrence relations

$$u_{n+1} = 0 \cdot 2u_n + 4 \cdot 8$$

$$v_{n+1} = kv_n + 4$$

The 2 sequences approach the same limit as $\rightarrow \infty$.

(a) Evaluate this limit.

(b) Hence determine the value of k.

(Non-calculator)

NR2	**Given $u_{n-1} = au_n + b$ (or equivalent), I can find the values of a and b from applying my knowledge of recurrence relations.**

1. A recurrence relation is defined by

$$u_{n+1} = au_n + b,$$

where $-1 < a < 1$ and $u_0 = 25$.

(a) If $u_1 = 30$ and $u_2 = 31$, find the values of a and b.

(b) Find the limit of this recurrence relation as $\rightarrow \infty$.

(Non-calculator)

2. A sequence is defined by

$$u_{n+1} = -\frac{1}{3}u_n$$

with $u_0 = -18$

(a) Write down the values of u_1 and u_2.

A second sequence is given by $3, 5, 11, 29, \ldots.$ It is generated by the recurrence relation

$$v_{n+1} = pv_n + q$$

with $v_0 = 3$.

(b) Find the values of p and q.

(c) Either the sequence in (a) or the sequence in (b) has a limit.

(i) Calculate the limit.

(ii) Why does the other sequence not have a limit?

(Non-calculator)

3. For the recurrence relation

$$u_{n+1} = mu_n + c,$$

it is known that $u_0 = 2$, $u_1 = 4$ and $u_2 = 7$

Find the values of m, c and u_3.

(Non-calculator)

4. The first three terms of a sequence are 5, 11 and 29.

The sequence is generated by the recurrence relation

$$u_{n+1} = au_n + b,$$

where $u_1 = 5$.

Find the values of a and b.

(Non-calculator)

5. A sequence of numbers is defined by the recurrence relation

$$u_{n+1} = ku_n + c,$$

where k and c are constants.

(a) Given that $u_1 = 100$, $u_2 = 90$ and $u_3 = 84$, find algebraically, the values of k and c.

(b) Hence find the limit of this sequence.

(Non-calculator)

6. For the recurrence relation

$$u_{n+1} = au_n + b,$$

it is known that u_0 = 6, u_1 = 12 and u_2 = 21.

(a) Find the values of a and b.

(b) Hence find the value of u_3.

(Non-calculator)

NR3 I can use recurrence relations to solve real life problems.

1. A scientist studying a large colony of bats in a cave has noticed that the fall in the population over a number of years has followed the recurrence relation

$$u_{n+1} = 0{\cdot}75u_n + 150,$$

where n is the time in years and 150 is the average number of bats born each year during a concentrated breeding week. He estimates the colony size at present to be 1800 bats with the breeding week just over.

(a) Calculate the estimated bat population in 4 years time immediately **after** that year's breeding.

(b) The scientist knows that if in the long term the colony drops, at any time, below 700 bats it is in serious trouble and will probably be unable to sustain itself.

Is this colony in danger of extinction?

Explain your answer with words and appropriate working.

(Calculator)

2. A new 24 volt lead acid battery is being tested as a possible power source for a battery-powered wheelchair.

The battery, which has an initial capacity of 22Ah (ampere hours), is being **artificially** drained over a 12 hour period to represent 1 month of use and an operating distance of 300 miles.

It has been found that by the end of each draining period the battery has lost 28% of its initial capacity at the start of that session.

After each 12 hour draining period the battery is hooked up to a super-charger for 12 hours which allows it to regain 4 Ah of capacity.

(a) What is the capacity of the battery immediately after its fifth re-charging period?

The battery is unusable if its capacity falls below 14.5 Ah.

(b) By considering the limit of a suitable sequence, make a comment on the durability and lifespan of the battery.

(Calculator)

3. A man decides to plant a number of fast-growing trees as a boundary between his property and the property of his next door neighbour. He has been warned, however, by the local garden centre that, during any year, the trees are expected to increase in height by 0.4 metres. In response to this warning he decides to trim 15% off the height of the trees at the start of any year.

(a) If he adopts the "15% pruning policy", to what height (to 2 decimal places) will he expect the trees to grow in the long run?

(b) His neighbour is concerned that the trees are growing at an alarming rate and wants assurances that the trees will grow no taller than 2 metres. What is the minimum percentage that the trees will need to be trimmed each year so as to meet this condition?

(Calculator)

4. A new '24 hour antibiotic' is being tested on a patient in hospital.

It is known that over a 24 hour period the amount of antibiotic remaining in the bloodstream is reduced by 70%. On the first day of the trial, an initial 220 mg dose is given to a patient at 7 a.m.

(a) After 24 hours and just prior to the second dose being given, how much antibiotic remains in the patient's bloodstream?

The patient is then given a further 220 mg dose at 7 a.m. and at this time each subsequent morning thereafter.

(b) A recurrence relation of the form un+1 = aun + b can be used to model this course of treatment.

Write down the values of a and b.

It is also known that more than 350 mg of the drug in the bloodstream results in unpleasant side effects.

(c) Is it safe to administer this antibiotic over an extended period of time?

(Calculator)

5. Biologists calculate that when the concentration of a particular chemical in a sea loch reaches 7 milligrams per litre (mg/l), the level of pollution endangers the life of the fish.

A factory wishes to release waste containing the chemical into the loch. It is claimed that the discharge will not endanger the fish.

The Local Authority is supplied with the following information:

- The loch contains none of this chemical at present.
- The factory manager has applied to discharge waste once per week which will result in an increase in concentration of 2.5mg/l of the chemical in the loch.
- The natural tidal action will remove 40% of the chemical from the loch every week.

(a) If the Local Authority allows the factory to go ahead with the discharge, what will the level of concentration of the chemical in the loch be in the long run?

(b) A local MSP is concerned that the level of discharge is too high and the factory is not granted permission. The factory manager is told that in the long run there has to be no more than 6mg/l of the chemical in the loch.

How much of the chemical can the factory now discharge every week?

(Calculator)

6. Certain radioisotopes are used as *tracers*, to track down diseased tissue within the body, and then be absorbed, to act as a long-term radiotherapy treatment. Their passage through the body and mass is ascertained by means of a Geiger-Muller counter.

During trials of a particular radioisotope the following information was obtained.

- *the isotope loses 55% of its mass every 12 hours*
- *the maximum recommended mass in the bloodstream is 120mgs*
- *100mgs is the smallest mass detectable by the Geiger-Muller counter*

An initial dose of 50mgs of the isotope is injected into a patient and top-up injections of 60mgs are given every 12 hours.

(a) After how many top-up injections will the Geiger-Muller counter be able to detect the isotope?

Your answer must be accompanied by appropriate working.

(b) **(i)** Set up a linear recurrence relation to model this situation.

(ii) Comment on the long-term suitability of this plan.

Your answer must be accompanied by appropriate working.

(Calculator)

NR4 I have experience of cross topic exam standard questions.

1. For $0 < x < \frac{\pi}{2}$, sequences can be generated using the recurrence relations

$$u_{n+1} = (\sin x)u_n + \cos 2x, \text{ with } u_0 = 1.$$

(a) Why do these sequences have a limit?

(b) The limit of one sequence generated by this recurrence relation is $\frac{1}{2}\sin x$. Find the value(s) of x.

Applications of Calculus

Higher Mathematics Supplementary Resources

Section B

This section is designed to provide examples which develop Course Assessment level skills

NR1 I can determine where the maximum/minimum values lie in a closed interval.

1. For each function determine the maximum AND minimum values within the given limits

(a) $f(x) = x^3 - 3x^2 - 9x + 27$ for $-4 \leq x \leq 3$

(b) $f(x) = x^3 - 3x$ for $-2 \leq x \leq 3$

(c) $f(x) = 3x^4 - 5x^3$ for $-2 \leq x \leq 2$

(d) $f(x) = 5x^3 - 3x^5 + 3$ for $0 \leq x \leq 5$

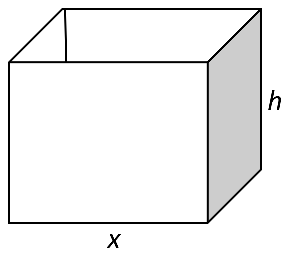
(e) $f(x) = x^3 - 3x^2$ for $-6 \leq x \leq 1$

(f) $f(x) = x^3 - 5x + 8$ for $-5 \leq x \leq 5$

NR2 I can solve optimization problems in context using differentiation.

1. A plastic box with a square base and an open top is being designed. It must have a volume of 108 cm^3 .

The length of the base is *x* cm and the height is *h* cm.



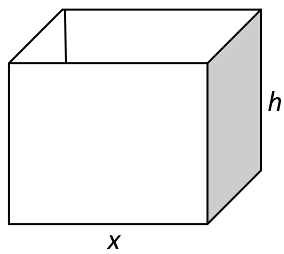
(a) Show that the total surface area *A* is given by

$$A(x) = x^2 + \frac{432}{x}$$

(b) Find the dimensions of the tray using the least amount of plastic

2. An open tank is to be designed in the shape of a cuboid with a square base. It must have a surface area of 100 cm^2 .

The length of the base is *x* cm.



(a) Show that the volume *V* is given by

$$A(x) = \frac{x}{4(100-x^2)}$$

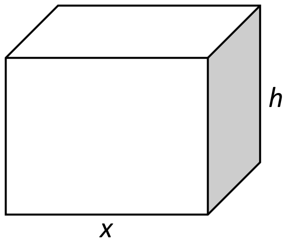
(b) Find the length of the base which gives the tank a maximum volume.

3. The height *h* m of a ball thrown upwards is given by the formula $h(x) = 20t - 5t^2$ where *t* is the time in seconds from when the ball is thrown.

(a) When does the ball reach its maximum height?

(b) Calculate the maximum height of the ball.

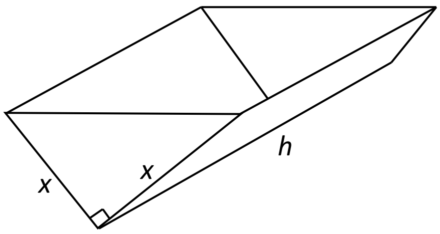
4. A Cuboid measures *x* by *x* by *h* units. The volume is 125 units[2].



(a) Show that the surface area of this Cuboid is given by $A(x) = 2x^2 + \frac{500}{x}$

(b) Find the value of *x* such that the surface area is minimised.

5. An open trough is in the shape of a triangular prism, the trough has a capacity of 256 000 cm^3.

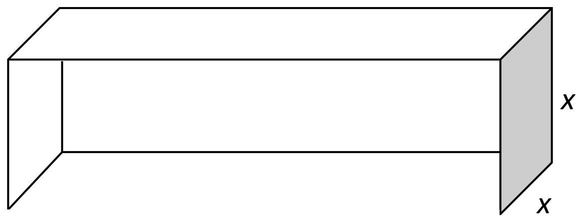


(a) Show that the surface area of this trough is given by

$$A(x) = x^2 + \frac{1024\,000}{x}$$

(b) Find the value of *x* such that the surface area is minimised.

6. A shelter consists of two square sides (x meters), a rectangular top and back. The total amount of material used to make the shelter is 96 m^2

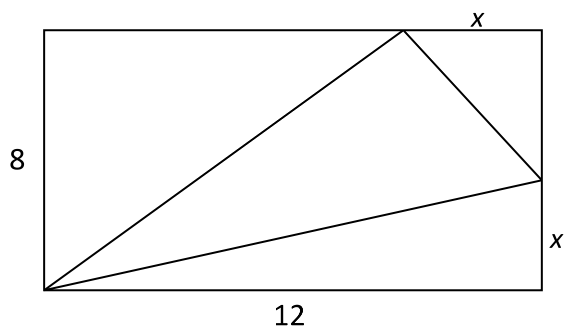


(a) Show that the volume of the shelter is given by

$$V(x) = x(48 - x^2)$$

(b) Find the dimensions of the shelter with the maximum volume.

7. A triangular piece of material is cut out of a rectangular sheet, the dimensions are shown on the diagram.



(a) Show that the area of the triangle is given by the formula

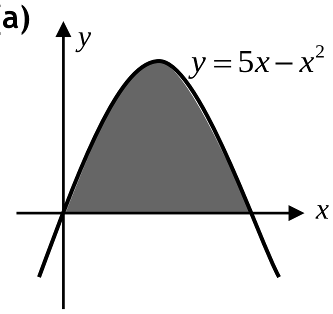
$$A(x) = 48 - 6x + \frac{1}{2}x^2$$

(b) Find the biggest area of triangle possible.

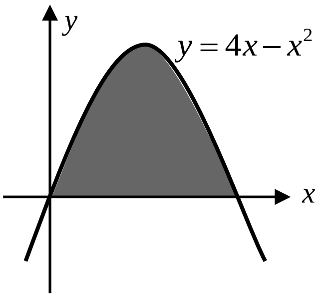
NR3 I can evaluate the area enclosed between a function and the x axis.

1. Find the shaded area in the following diagrams

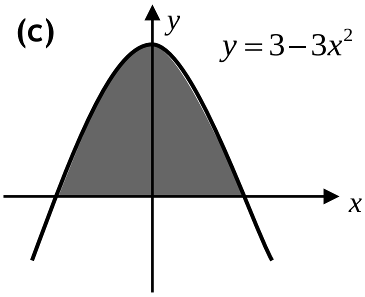
(a)



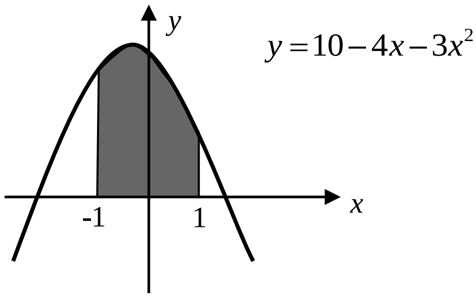
(b)



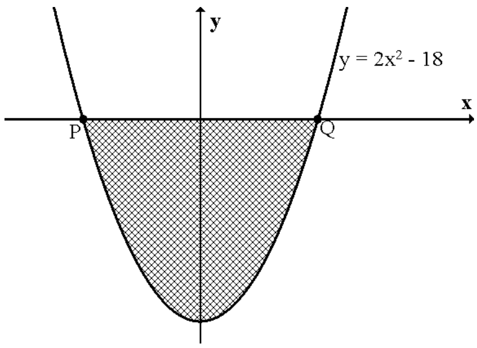
(c)



(d)



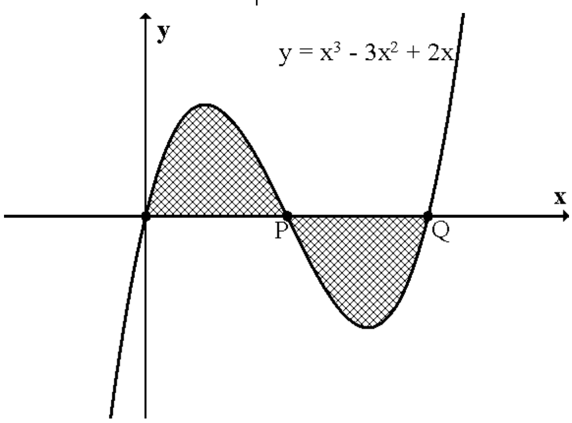
2. The diagram shows part of the graph of $y = 2x^2 - 18$.



(a) Find the coordinates of P and Q.

(b) Find the shaded area.

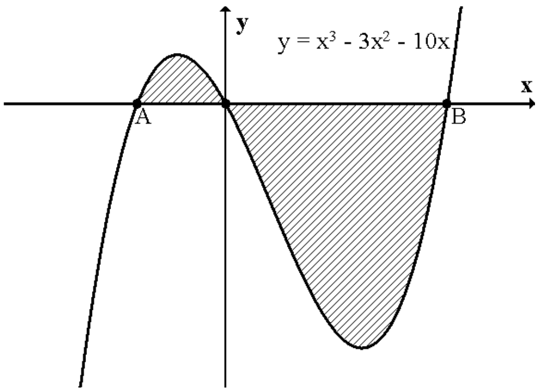
3. The diagram shows part of the graph of $y = x^3 - 3x^2 + 2x$.



(a) Find the coordinates of P and Q.

(b) Calculate the shaded area.

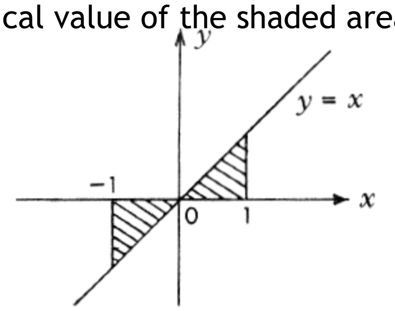
4. The diagram shows the graph of $y = x^3 - 3x^2 - 10x$.



(a) Find the coordinates of A and B.

(b) Calculate the shaded area.

5. Which of the following gives the numerical value of the shaded area?



(I) $\int_{-1}^{1} x\,dx$

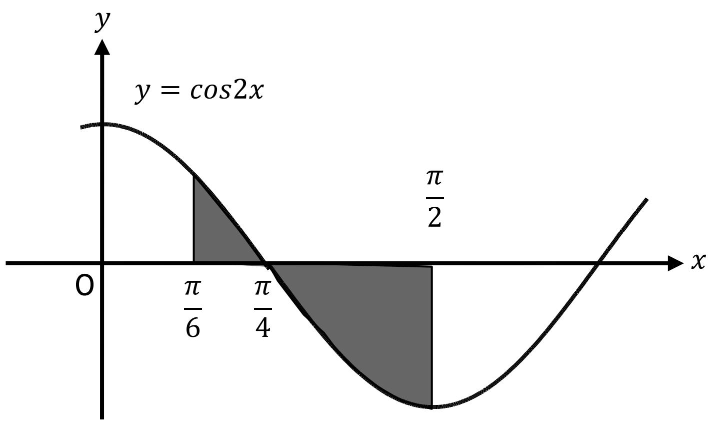
(II) $\int_{-1}^{0} x\,dx + \int_{0}^{1} x\,dx$

(III) $2\int_{0}^{1} x\,dx$

A (I) only B (II) only C (III) only D (I), (II) and (III)

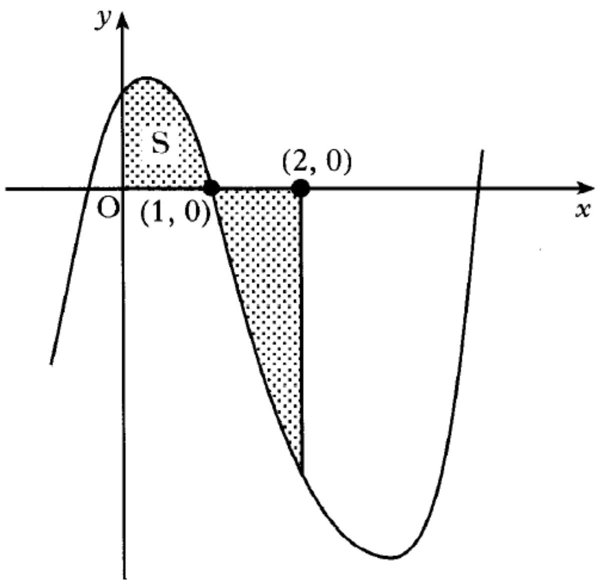
6. An artist has designed a "bow" shape which he finds can be modelled by the shaded area shown.

Calculate the area of this shape.



7. The graph below has equation $y = x^3 - 6x^2 + 4x + 1$.

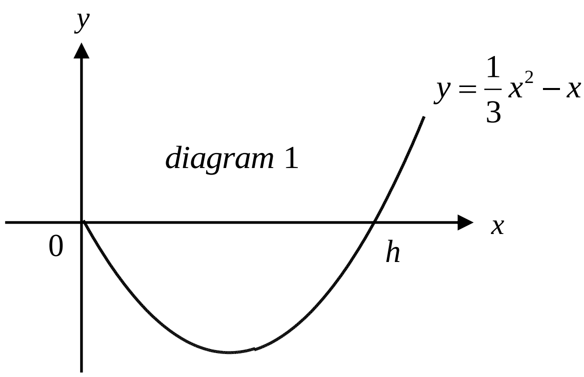
The total shaded aread is bounded by the curve, the x-axis, the y-axis and the line $x = 2$.



(a) Calculate the shaded area labelled S.

(b) Hence find the total shaded area.

8. Diagram 1 shows the profit/loss function for the manufacture of x thousand kitchen units.



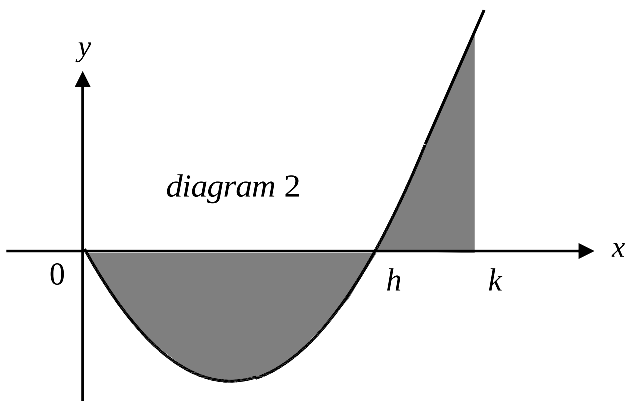
The profit/loss is measured in millions of £s and is represented by the area between the function and the x-axis .

Any area below the x-axis represents a loss; any area above the x-axis represents a profit.

The profit/loss function is given by $f(x) = \frac{1}{3}x^2 - x$ where $x \geq 0$.

(a) Find the value of $\boldsymbol{h}$.

(b) Diagram 2 (not drawn to scale) represents the breakeven situation where the initial loss made on selling the first ***h*** thousand units is exactly balanced by the later profit.

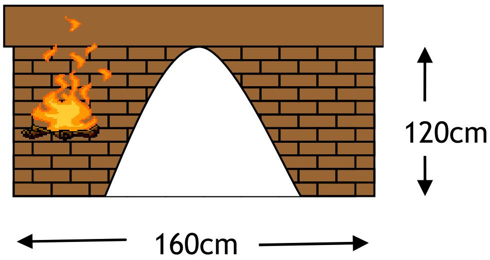


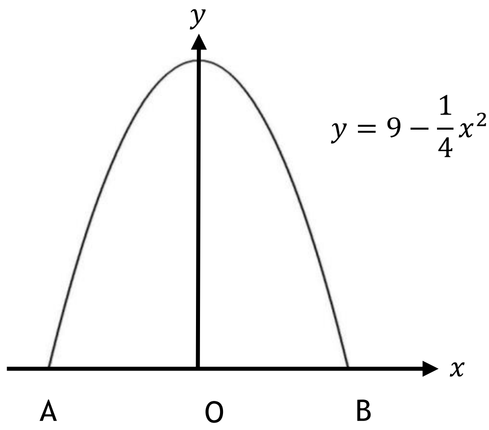
Calculate the value of $\boldsymbol{k}$.

9. The fire surround is rectangular, with a parabolic opening for the fire.

(a) Find the coordinates of A and B.

(b) Calculate the tiled area (1 unit = 10cm).

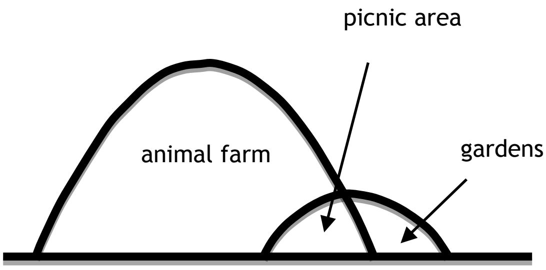


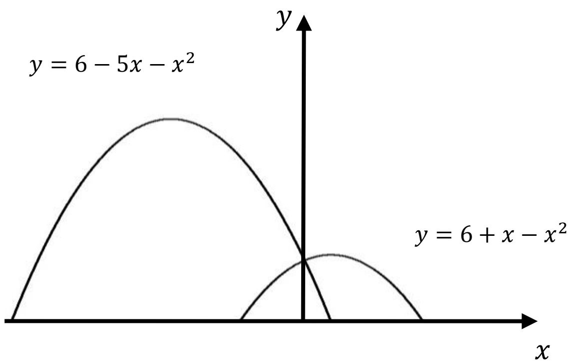


10. Millennium Park is based on two parabola shapes.

A plan is laid out in the coordinate diagram.

Calculate the areas of the three parts of the park on the diagram.

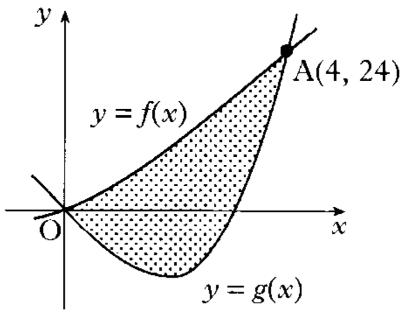




NR4 I can evaluate the area enclosed between two functions.

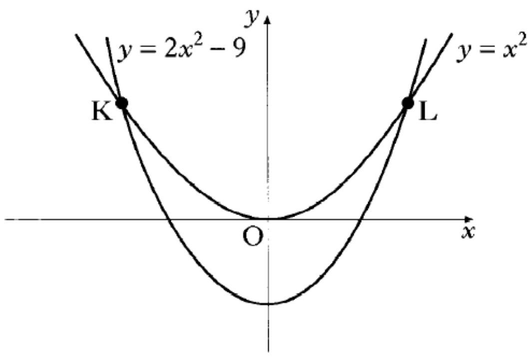
1. The incomplete graphs of $f(x) = x^2 + 2x$ and $g(x) = x^3 - x^2 - 6x$ are shown in the diagram. The graphs intersect at A(4, 24) and the origin.

Find the shaded area enclosed between the curves.

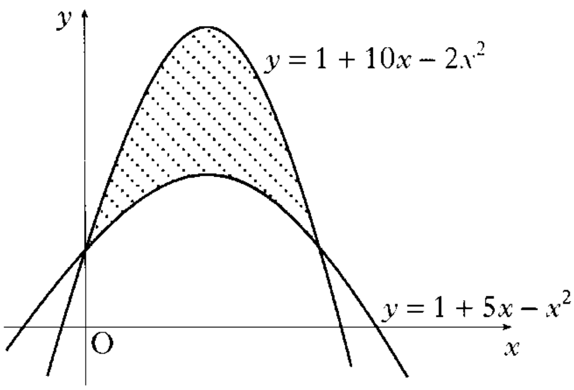


2. The curves with equations $y = x^2$ and $y = 2x^2 - 9$ intersect at K and L as shown.

Calculate the area enclosed between the curves.

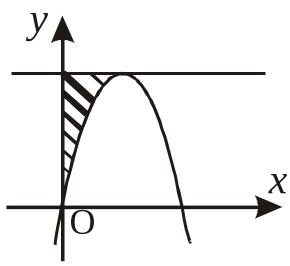


3. Calculate the shaded area enclosed between the parabolas with equations $y = 1 + 10x - 2x^2$ and $y = 1 + 5x - x^2$.



4. Calculate the area enclosed by the line $y = 3(x - 1)$ and the parabola $y = 3 + 2x - x^2$

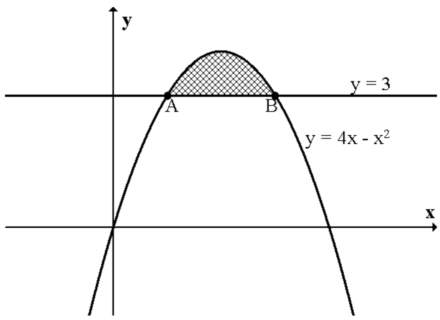
5. This diagram shows a rough sketch of the quadratic function $y = 6x - x^2$. The tangent at the maximum stationary point has been drawn.



(a) Explain clearly why the tangent has equation y = 9.

(b)Calculate the shaded area enclosed by the curve, the tangent and the y-axis.

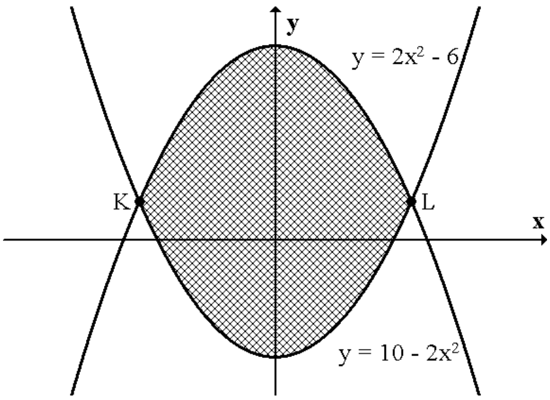
6. The diagram opposite shows the curve $y = 4x - x^2$ and the line $y = 3$.



(a) Find the coordinates of A and B.

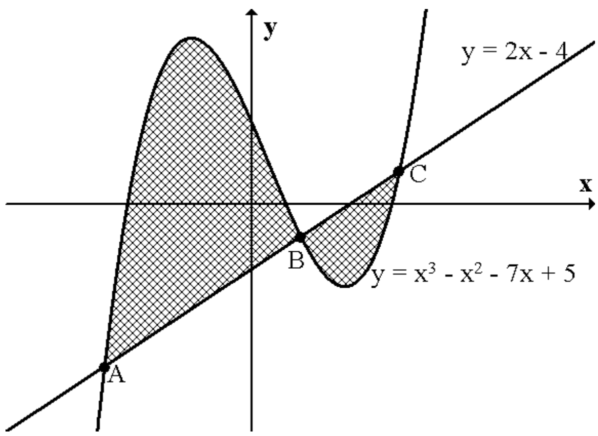
(b) Calculate the shaded area.

7. The curves with equations $y = 2x^2 - 6$ and $y = 10 - 2x^2$ intersect at K and L. Calculate the area enclosed by these two curves.



8. The curve $y = x^3 - x^2 - 7x + 5$ and the line $y = 2x - 4$ are shown opposite.

(a) B has coordinates (1,-2). Find the coordinates of A and C.
(b) Hence calculate the shaded area.

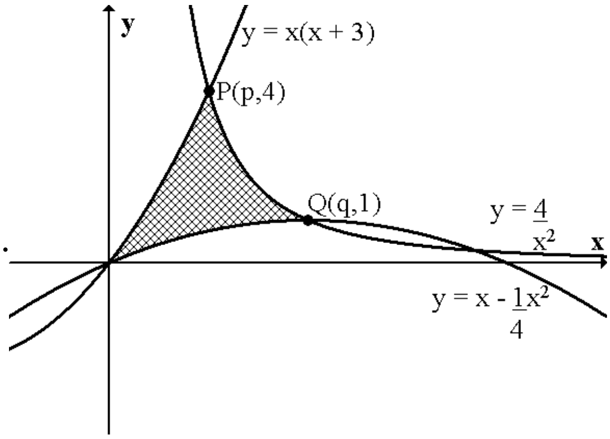


9. The diagram opposite shows an area enclosed by 3 curves:

$y = x(x+3)$, $y = \frac{4}{x^2}$ and $y = x - \frac{1}{4}x^2$

(a) P and Q have coordinates (p,4) and (q,1). Find the values of p and q.

(b) Calculate the shaded area.



10. This diagram shows 2 curves $y = f_1(x)$ and $y = f_2(x)$ which intersect at $x = a$.

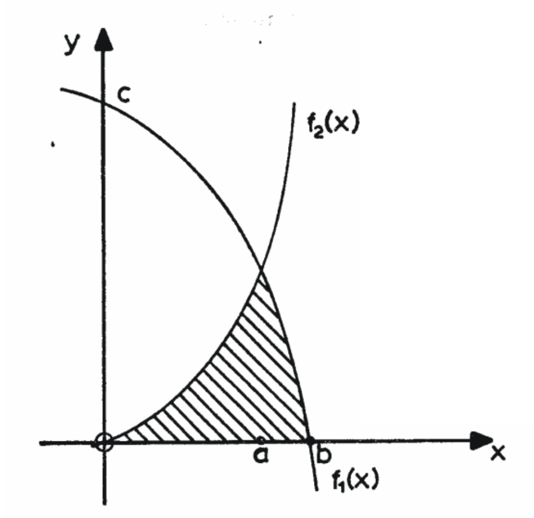
The area of the shaded region is

A $\int_0^b f_1(x)dx - \int_0^a f_2(x)dx$

B $\int_0^a f_2(x)dx + \int_a^b f_1(x)dx$

C $\int_0^a f_2(x)dx - \int_a^b f_1(x)dx$

D $\int_0^a f_1(x)dx + \int_a^b f_2(x)dx$

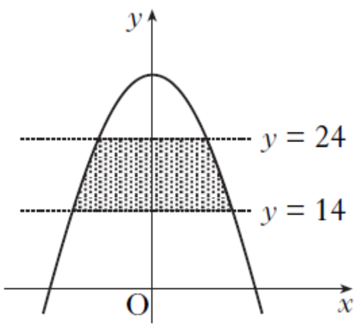


11. The parabola shown in the diagram has equation

$y = 32 - 2x^2$.

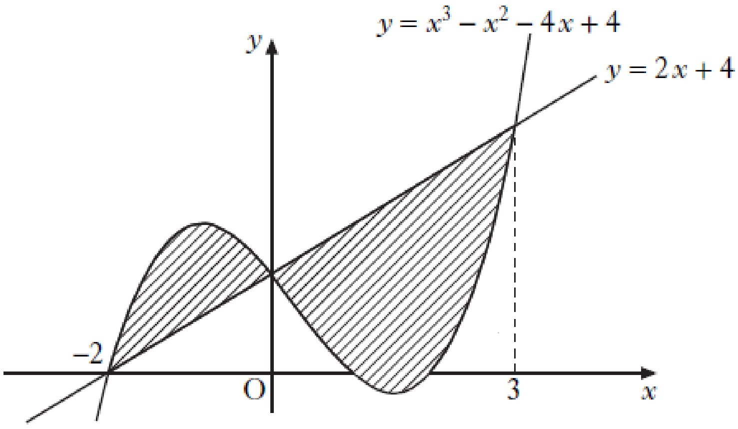
The shaded area lies between the lines $y = 14$ and $y = 24$.

Calculate the shaded area.



12. The diagram shows the curve with equation $y = x^3 - x^2 - 4x + 4$ and the line with equation $y = 2x + 4$.

The curve and the line intersect at the points (–2, 0), (0, 4) and (3, 10).



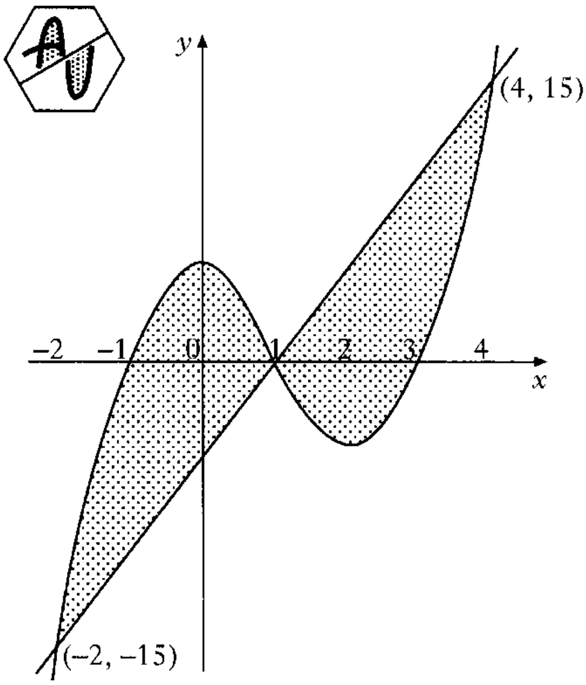
Calculate the total shaded area.

13. A firm asked for a logo to be designed involving the letters A and U. Their initial sketch is shown in the hexagon.

A mathematical representation of the final logo is shown in the coordinate diagram.

The curve has equation $y = (x + 1)(x - 1)(x - 3)$ and the straight line has equation $y = 5x - 5$. The point (1, 0) is the centre of half-turn symmetry.

Calculate the total shaded area.



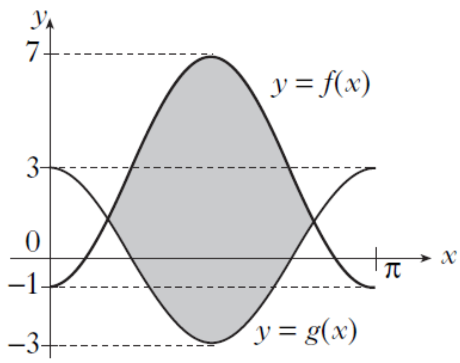
14. The graphs of $y = f(x)$ and $y = g(x)$ are shown in the diagram.

$f(x) = -4\cos(2x) + 3$ and $g(x)$ is of the form $g(x) = m\cos(nx)$.

(a) Write down the values of m and n.

(b) Find, correct to one decimal place, the coordinates of the points of intersection of the two graphs in the interval $0 \leq x \leq \pi$.

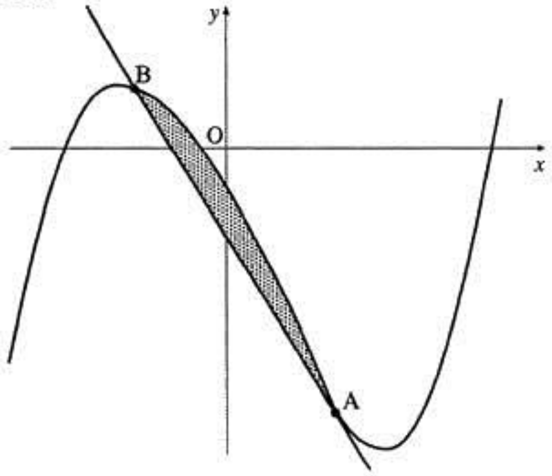
(c) Calculate the shaded area.



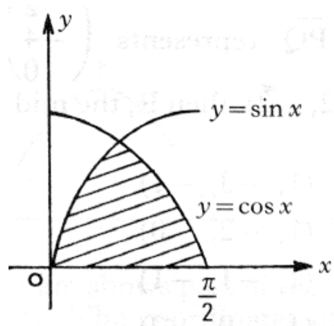
15. In the diagram below a winding river has been modelled by the curve $y = x^3 - x^2 - 6x - 2$ and a road has been modelled by the straight line AB. The road is a tangent to the river at the point A(1, – 8).

(a) Find the equation of the tangent at A and hence find the coordinates of B. **(8)**

(b) Find the area of the shaded part which represents the land bounded by the river and the road. **(3)**



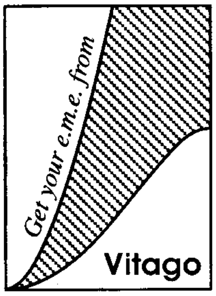
16. The diagram shows the graphs of the curve $y = 2\sqrt{x}$ and the straight line $y = x - 3$.



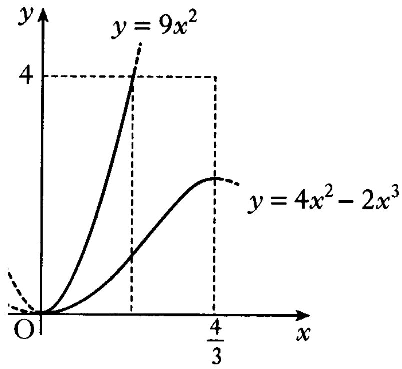
(a) Prove, by solving an equation, that the line and curve meet when x = 1 and x = 9.

(b) Hence find the shaded area between the curve and the line.

19. The diagram shows the front of a packet of Vitago, a new vitamin preparation to provide early morning energy. The shaded region is red and the rest yellow.



The design was created by drawing the curves $y = 9x^2$ and $y = 4x^2 - 2x^3$ as shown in the diagram below.

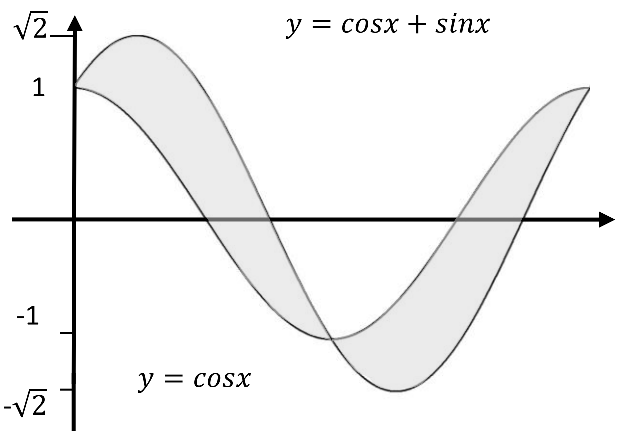


The edges of the packet are represented by the coordinate axes and the lines $x = \frac{4}{3}$ and $y = 4$.

Show that $\frac{160}{81}$ of the front of the packet is red.

20. a) Find the x-values of the three points of intersection for these curves, for $0 \leq x \leq 2\pi$.

b) Calculate the area enclosed by the curves.



NR5	I can solve differential equations of the form $\frac{dy}{dx} = f(x)$ and give a particular solution.

1. Given the gradient $\frac{dy}{dx}$ of the curve at the point (x, y) and a point on the curve, find the equation of each curve:

   a) $\frac{dy}{dx} = 3x^2 - 6x + 1$ (3,4)

   b) $\frac{dy}{dx} = 4x^3 - 6x^2$ (1,9)

2. Find the solution to the following differential equations:

   a) $\frac{dy}{dx} = 4x^3 + \frac{2}{x^3}$ and $y = 0 \; when \; x = 1$

   b) $\frac{dy}{du} = \frac{u^2+1}{u^2}$ and $y = 4 \; when \; u = 2$

3. A curve has gradient given by $\frac{dy}{dx} = \frac{1}{\sqrt{x}}$. The curve passes through the point (9,10). Find the equation of the curve.

4. The graph of $y = f(x)$ passes through the point $\left(\frac{\pi}{9}, 1\right)$.

   If $f'(x) = \sin(3x)$, express y in terms of x.

5. A curve for which $\frac{dy}{dx} = 3\sin(2x)$ passes through the point $\left(\frac{5}{12}\pi, \sqrt{3}\right)$.

   Find y in terms of x.

6. A point moves in a straight line such that its acceleration a is given by $a = 2(4 - t)^{\frac{1}{2}}$, $0 \le t \le 4$. If it starts at rest, find an expression for the velocity v where $a = \frac{dv}{dt}$.

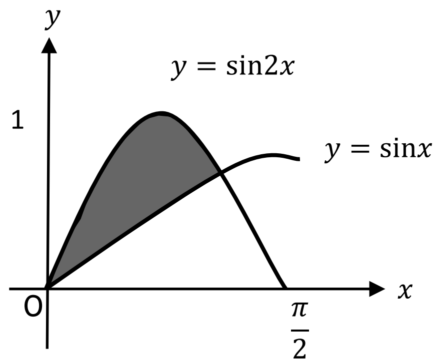
7. The curve $y = f(x)$ is such that $\frac{dy}{dx} = 4x - 6x^2$. The curve passes through the point $(-1, 9)$. Express y in terms of x.

NR6 I have experience of cross topic exam standard questions.

Integration and the addition formula

1. **a)** Write down a formula for $\sin 2x$, and use it to solve the equation $\sin 2x = \sin x$ for $0 \le x \le \frac{\pi}{2}$.

   b) Find the shaded area enclosed by the curves $y = \sin 2x$ and $y = \sin x$.



Integration and quadratic graphs

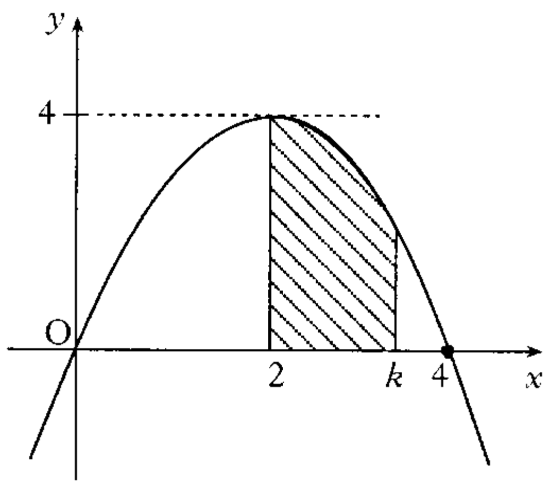
2. The parabola shown crosses the x-axis at $(0, 0)$ and $(4, 0)$, and has a maximum at $(2, 4)$.

The shaded area is bound by the parabola, the x-axis and the lines $x = 2$ and $x = k$.

(a) Find the equation of the parabola.

(b) Hence show that the shaded area, A, is given by

$$A = -\frac{1}{3}k^3 + 2k^2 - \frac{16}{3}.$$



Integration and the wave function

3. **(a)** The expression $3\sin x - 5\cos x$ can be written in the form $R\sin(x + a)$ where $R > 0$ and $0 \le a \le 2\pi$.

Calculate the values of R and a.

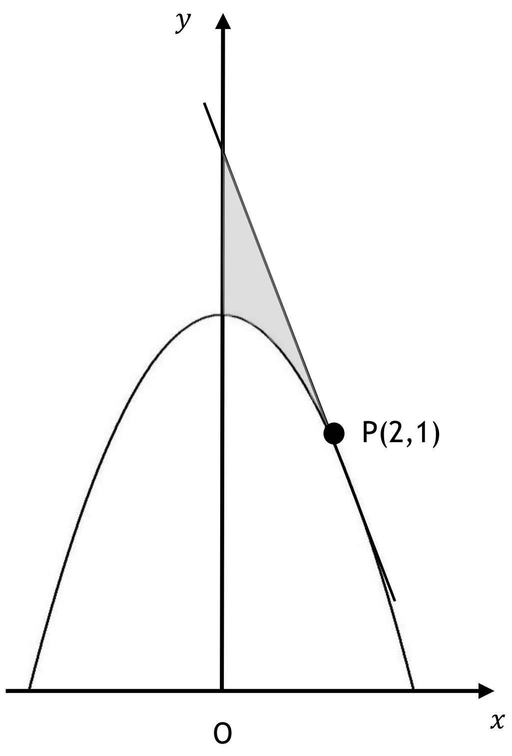
(b) Hence find the value of t, where $0 \le t \le 2$, for which

$$\int_0^t (3\sin x - 5\cos x)\,dx = 3$$

Integration and Differentiation

4. **(a)** Find the equation of the tangent to the parabola $y = 5 - x^2$ at P(2,1)

(b) Calculate the area of the shaded region bounded by the tangent, the parabola and the y axis.



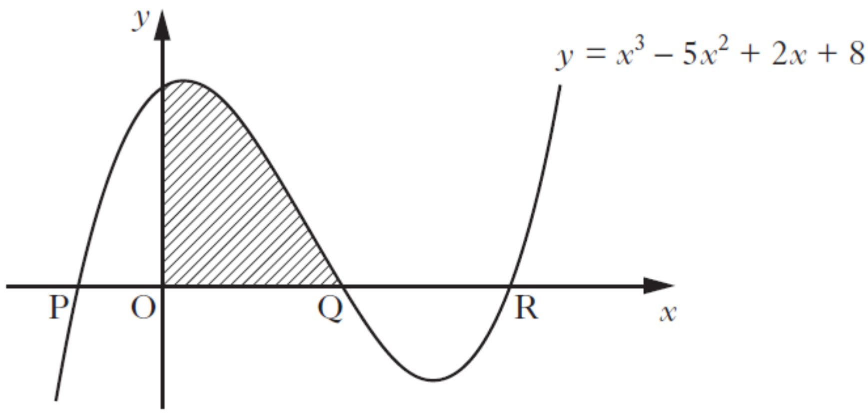
Integration and polynomials

5. **(a)** **(i)** Show that $(x - 4)$ is a factor of $x^3 - 5x^2 + 2x + 8$.

(ii) Factorise $x^3 - 5x^2 + 2x + 8$ fully.

(iii) Solve $x^3 - 5x^2 + 2x + 8 = 0$.

(b) The diagram shows the curve with equation $y = x^3 - 5x^2 + 2x + 8$



The curve crosses the x-axis at P, Q and R.

Determine the shade area.