

Chapter 7

Exercise 7A

- 1 a yes
 b yes
 c no
 d yes
 e no
 f yes
- 2 a 3
 b 1
 c 7
 d 5
 e 2

- 3 yes
 4 no
 5 no
 6 yes
 7 no
 8 no
 9 yes

Exercise 7B

- 1 $(x - 2)(x - 1)(x - 4)$
 2 $(x + 1)(x - 1)(x + 4)$
 3 $(x - 2)(x - 3)(x - 1)$
 4 $(x + 1)(x + 8)(x - 1)$
 5 $(x + 3)(x + 1)(x - 4)$
 6 $(x + 2)(x - 1)(x + 4)x$
 7 $(x - 3)(x + 3)(x - 2)$
 8 $(x + 5)(x - 2)(x - 2)x$

Exercise 7C

- 1 a $(x + 1)(x - 1)(x - 1)$
 b $(x + 1)(x + 1)(x - 1)$
 c $(x + 3)(x + 1)(x - 1)$
 d $(x + 5)(x - 1)(x - 1)$
 e $(x + 2)(x - 2)(x - 2)$
 f $(x - 3)(x - 2)(x - 1)$

- 2 a $(x - 3)(x + 2)(x + 3)$
 b $(x - 1)(x + 2)(x + 4)$
 c $(x - 3)(x - 2)(x + 1)$
 d $(x - 3)(x - 1)(x + 4)$
 e $(x - 4)(x + 1)(x + 5)$
 f $(x - 3)(x - 2)(x + 5)$
- 3 a $(x - 1)(x - 3)x$
 b $(x - 1)x(x + 1)$
 c $(x - 4)(x - 1)(x + 2)x$
 d $(x + 1)(x + 1)(x - 1)(x - 1)$

Exercise 7D

- 1 a $2\left(\frac{1}{2}\right)^3 + 11\left(\frac{1}{2}\right)^2 + 4\left(\frac{1}{2}\right) - 5 = 0$
 b $(x + 1)(x + 5)(2x - 1)$
- 2 a $(x - 1)(x + 1)(2x + 1)$
 b $(x - 2)(x - 1)(3x + 1)$
 c $(x - 3)(x - 2)(2x - 1)$
 d $(x + 1)(x - 1)(4x + 3)$
 e $(5x + 2)(x + 2)(x - 3)$
 f $(4x + 1)(x - 4)(x + 3)$
 g $(x - 1)(2x + 1)(3x + 2)$
 h $(2x - 1)(x + 2)(3x + 1)$
 i $(x + 2)(x^2 - x + 3)$
 j $(x - 1)(2x^2 - 3x + 3)$
- 3 a $(x - 2)(x - 2)(x + 2)(x + 2)$
 b $(x - 1)(x - 1)(x - 1)(x + 1)$
 c $(x - 2)(x - 1)(x + 1)(x + 1)$
 d $(x - 3)(x - 3)(x - 1)(x + 2)$
- 4 a $3x^2(x - 1)(x + 1)$
 b $(x - 3)(x - 2)(x - 1)x$
 c $(x - 2)(x - 1)(x + 1)(x + 2)$
 d $(x - 3)(x - 2)(x - 1)(x + 3)$
 e $(x + 1)(x - 1)(x + 1)(x + 5)$
 f $(2x + 1)(x - 4)(x^2 - x + 1)$
 g $(x - 2)(x + 2)(x^2 + 3)$
 h $(2x - 1)(x - 1)(2x + 1)(x + 2)$

- i** $2(x-2)(x-4)(x+1)(x+4)$
j $(x-3)(x+1)(x^2-3)$
5 $(x-8)(x+1)(x-3)$
6 5
7 a $x-3$
b $x \text{ min} = 4.$
 Area = $\frac{13}{3} \text{ cm}^3$

Exercise 7E

- 1** $p = 5$
2 $q = 20$
3 $k = 1$
 $(x-1)(x+1)(2x+1)$
4 $a = 7$
 $(x-2)(x-1)(x+3)(x+7)$
5 $k = 3$
 $(x+1)(x-1)(2x+3)$
6 $a = 2$
 $b = 2$
7 $p = -8$
 $q = 12$
 $(x-2)(x-1)(x+2)(x+3)$

Exercise 7F

- 1 a** 2
b -10
c 85
d $\frac{-3}{4}$
e $\frac{8}{3}$
2 a 2
b -7
c 8
3 a 3
 $b = -1$
4 a -1
 $b = 1$
5 a $x^2 + 7x + 11$
 23

- b** $x^2 - x$
 4
c $2x^2 - 3x + 6$
 0
d $4x^2 + 4x + 2$
 3

Exercise 7G

- 1 a** (1, 0)
 (2, 0)
b (-5, 0)
 (-1, 0)
 (3, 0)
c (-5, 0)
 (-1, 0)
 (5, 0)
d (-1, 0)
 (-3, 0)
 (-8, 0)
e (6, 0)
 (-3, 0)
 (2, 0)
f (2, 0)
 (-0.5, 0)
 (1, 0)
g (-2, 0)
 (-1, 0)
 (1, 0)
 (2, 0)
2 Months 0, 3 and 5
3 a A(2, 0)
 B(3, 0)
b 72 m
- Exercise 7H**
- 1 a** $f(x) = x^3 - 6x^2 + 11x - 6$
b $f(x) = 2x^3 + 4x^2 - 10x - 12$

- c $f(x) = 3x^3 - 12x^2 - 12x + 48$
 d $f(x) = x^3 + x^2 - 8x - 12$
 e $f(x) = 2x^3 - 2x^2 - 16x + 24$
 f $f(x) = -x^3 - 4x^2 + x + 4$
 g $f(x) = -2x^3 + 4x^2 + 10x - 12$
 h $f(x) = x^4 - 5x^2 + 4$
 i $f(x) = x^4 - 2x^2 + 1$
 2 $f(x) = \frac{1}{2}x^3 - \frac{3}{4}x^2 - \frac{11}{4}x + \frac{3}{2}$
 3 $\frac{1}{32500} = (-x^4 + 150x^3 - 6875x^2 + 93750x)$
 4 $f(x) = 2x^3 - 8x^2 + 2x + 12$
 $a = -1$

Exercise 7I

- 1 a $x = -4, -1, 2$
 b $x = -4, -2, -1$
 c $x = -1, 3, 5$
 d $x = -4, 2$
 2 a $x = -3, -1, 1$
 b $x = -3, -2, 0, 5$
 c $x = -\frac{5}{2}, -1, 2$
 d $x = -2, 1, 2, 3$
 e $-2, -1, \frac{1}{2}, 3$
 f $-2, -\frac{1}{2}, \frac{1}{3}, 2$
 3 a $x = -4, -3, 1$
 b $x = -6, -2, 1$
 c $x = -2, 3, 7$
 d $x = -1, -\frac{2}{3}, 3$
 e $x = -5, -1, 1, 4$
 f $x = -\frac{1}{2}, \frac{1}{2}, 1$

4 $a = -3$
 $x = -2, -\frac{1}{2}, 1$

5 $k = 3$
 $x = -3$

Factorising gives $(x + 3)(x^2 + 3)$ and we can see second term cannot be zero.

- 6 a $h(1) = 0$
 b $-(t - 8)(t - 6)(t - 3)(t - 1)$
 c $t = 3$

- 7 a 4 months
 b Factors are $(t - 4)(2t^2 - 8t + 11)$ and second term has no roots.
 c £786
 8 a $v(x) = (x + 18)(x + 12)(x + 8)$
 b $+1728 + 456x + 38x^2 + x^3 = 3456$
 $x^3 + 38x^2 + 456x - 1737 = 0$
 c Factor above expression to get
 $(x - 3)(x^2 + 41x + 579)$
 Second term has no roots so dimensions are (with $x = 3$) 11, 15, 21
 9 $4 \times 4 \times 8$
 10 $x < -1 \parallel -1 < x < 1 \parallel x > 3$
 where \parallel means OR

Exercise 7J

- 1 $(-2, -8), (3, 7), (4, 10)$
 2 a $(-6, -241), (-3, -52), (2, -17)$
 b $(-7, 72), (-3, 40), (1, 8)$
 c $(-3, 19), (1, 3), (3, 55)$
 3 a $(-1, -20), (4, -10), (7, -4)$
 b $(-2, -19), (-1.5, -9.875), (1, 2)$
 c $(1, 0), (2, -10), (3, 30), (4, 180)$
 d $(-1, 3), (-0.5, 0.875), (0.5, 1.125), (1, 5)$
 e $(+\frac{1}{3}, \frac{284}{27}), (-\sqrt{5}, 15 - 11\sqrt{5}), (\sqrt{5}, 15 + 11\sqrt{5})$

Exercise 7K

- 1 a $k = \{-8, 8\}$
 b $k = 40$
 c $k = \{0, 5\}$
 d $k = \{-1, 4\}$
 e $k = \{-1, 3\}$
 2 a $k = 4$
 b $x = (-\frac{2}{3})$
 3 a $x < -1 \parallel x > 3$
 b $-3 \leq x \leq \frac{1}{2}$

- c** $-1 < x < 4$
d $x < -2 \parallel x > 8$
e $x \leq -\frac{1}{3} \parallel x \geq \frac{1}{3}$
f $x \leq \frac{1}{3} \parallel x \geq \frac{1}{2}$
- 4 a** $k \leq \frac{1}{3}$
b $k \leq 3$
c $k \leq 0 \parallel k \geq \frac{1}{3}$
- 5** $k > 0$
6 $-4 \leq k \leq 1$
7 Discriminant ≥ 0
 $(3k - 2)^2 - 4 \times 2k \times (k - 2) \geq 0$
 $k^2 + 4k + 4$
 $= (k + 2)^2 \geq 0$
 So roots are always real.

- 8** $k < 0 \parallel k > 4$
- 9 a** 2
b $(x - 1)(3x^2 + 4x + 10)$
 second term has no real roots
 $(1, -8)$
- 10 a** $a = -1$
 $b = -2$
b roots are
 $1, \frac{1+\sqrt{5}}{2}, \frac{1-\sqrt{5}}{2}$
 only rational root is 1.
 $(1, 0)$